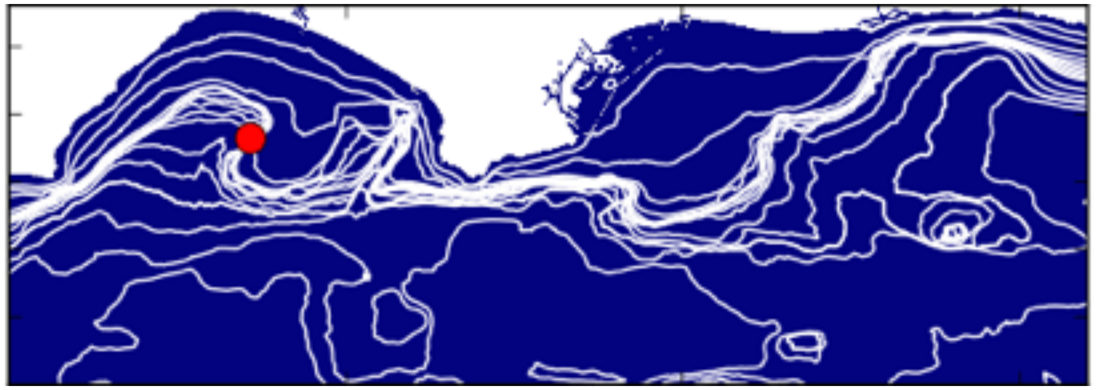
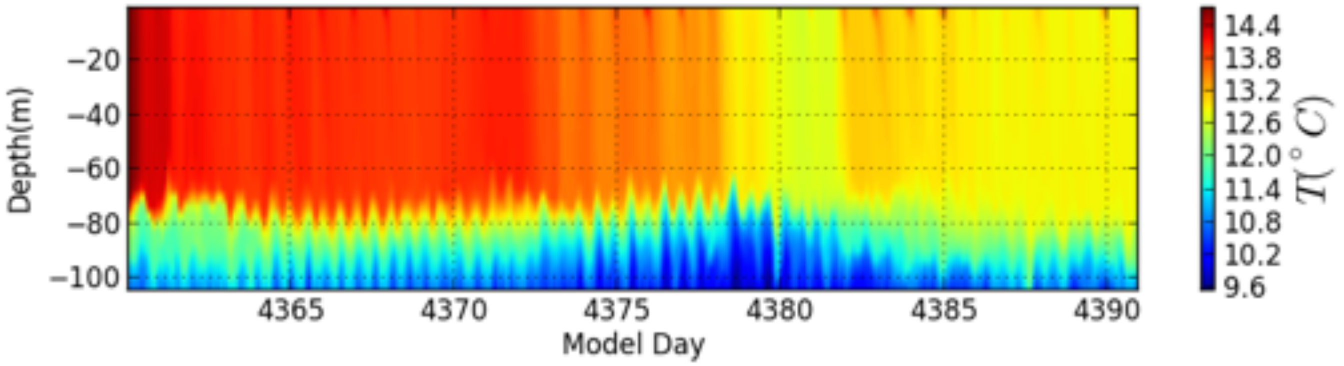
AOS 103

Week 8 Discussion

Data Interpretation Exercise

Data from a regional ocean model of Santa Monica Bay





What type of phenomenon do you think is apparent from the plot on the right (which is data taken from the location indicated by the red dot on the right)?

Waves

1) Characterize (i.e., fully describe) a generic wave with a sketch and variables/equations relating different aspects of the wave
2) What are the phase and group speeds for shallow water and deep water waves (both are surface waves)
3) Are shallow water or deep water waves dispersive (why or why not)?
4) How do long or short waves behave for shallow water and deep water waves
5) The following is the dispersion relation for a Rossby wave:

$$\omega = -\frac{\beta_0 k}{k^2 + \ell^2} .$$

Say something about Rossby waves based on this dispersion relation

1) Characterize (i.e., fully describe) a generic wave with a sketch and variables/equations relating different aspects of the wave

$$\eta = a\cos(kx - \omega t)$$

Linear wave equation

$$k = \frac{2\pi}{\lambda}$$

Wavenumber

*Note: wavenumber is inversely proportional to wavelength

$$\omega = c_p k = \frac{2\pi}{T}$$

Frequency

*Note: frequency is a function of wavenumber $\omega = \omega(k)$

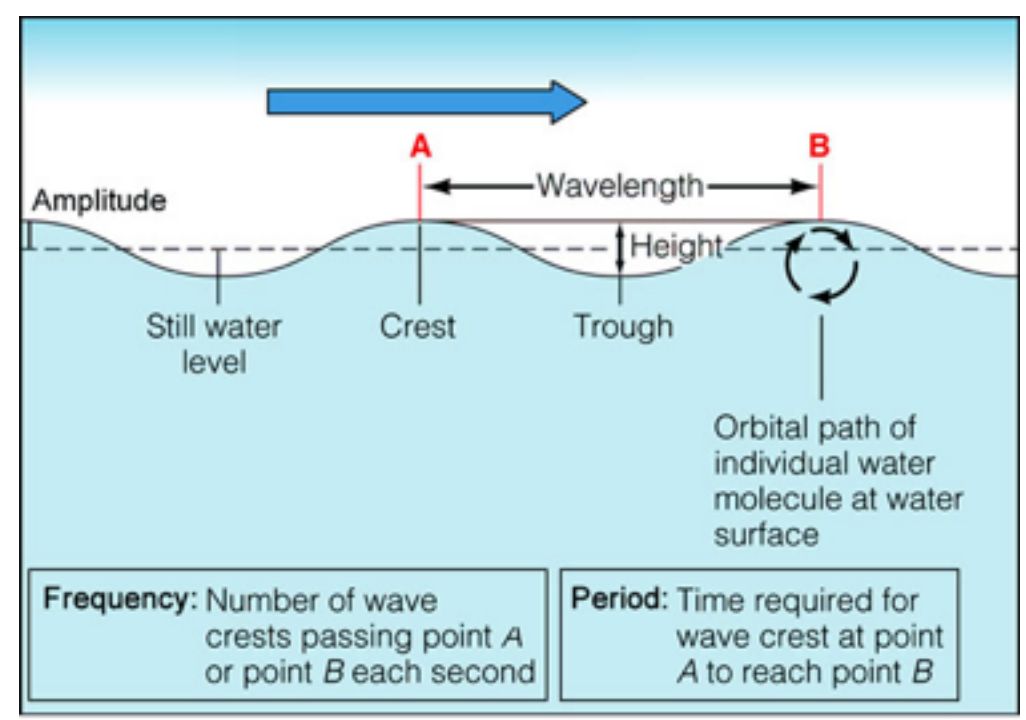
$$c_p = \frac{\omega}{k} = \frac{2\pi}{Tk}$$

Phase Speed

$$c_g = \frac{\partial \omega}{\partial k}$$

Group Speed

- t: Time [s]
- λ: Wave length [m]
- T: Wave period [s]
- ω: Frequency [1/s]
- C_p: Phase Speed of Wave [m/s]
- H: Water Depth [m]
- x, y, z: Space Coordinates [m]
- a: Amplitude [m]
- k: Wave number [1/m]
- η: Sea Surface Height [m]
- C_g: Group Speed of Wave [m/s]



2) What are the phase and group speeds for shallow water and deep water waves (both are surface waves)

Surface Wave: General Phase Speed

$$C_p = \frac{\omega}{k} = (g/k \tanh(kH))^{1/2}$$

***We can simplify (and categorize surface waves) via assumptions about the ratio of wavelength to depth**

Shallow Water Waves

$$\lambda > 20H$$

$$c_p = (gH)^{1/2}$$

$$\omega = C_p k = (gH)^{1/2}\, k$$

$$C_g = \frac{\partial \omega}{\partial k} = (gH)^{1/2}$$

Deep Water Waves

$$\lambda < 2H$$

$$c_p = \left(\frac{g}{k}\right)^{1/2}$$

$$\omega = C_p k = k\left(\frac{g}{k}\right)^{1/2} = (gk)^{1/2}$$

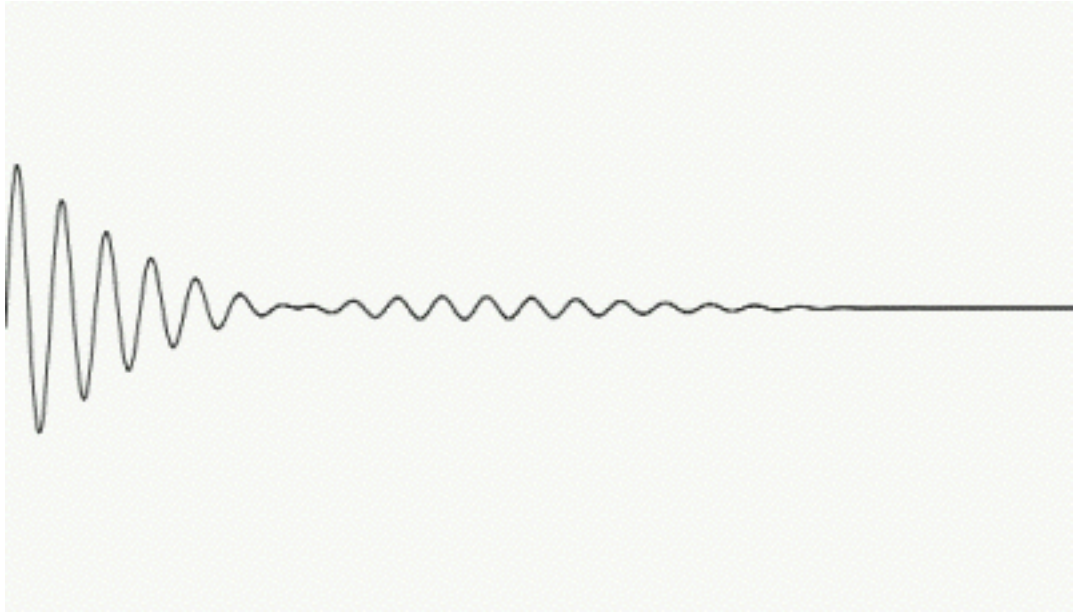
$$C_g = \frac{\partial \omega}{\partial k} = \left(\frac{g}{4k}\right)^{1/2}$$

3) Are shallow water or deep water waves dispersive (why or why not)?

A wave is dispersive if the phase velocity DOES NOT equal the group velocity

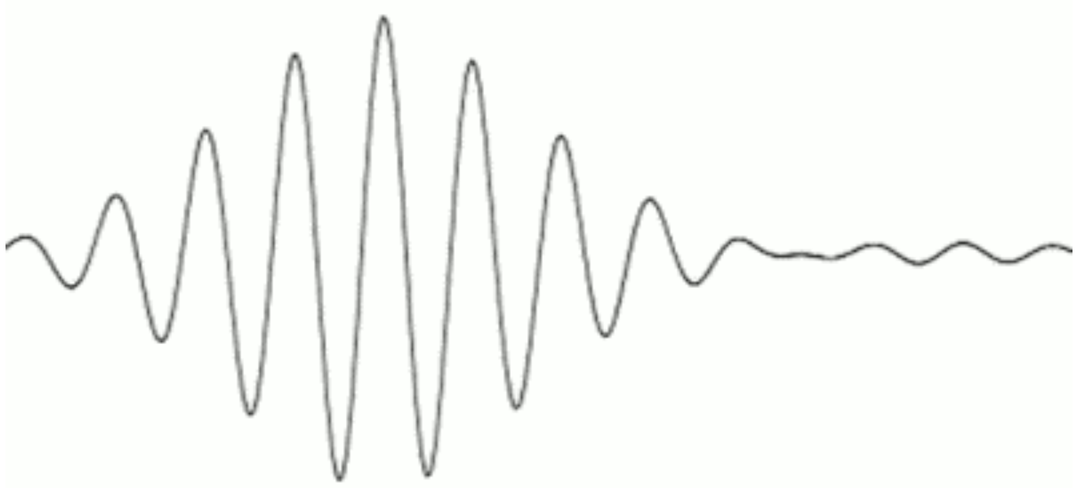
Non-dispersive wave packet (movie)

$$c_p = c_g$$



Dispersive wave packet (movie)

$$c_p \neq c_g$$



Google/wikipedia "wave dispersion" to see more movie examples

3) Are shallow water or deep water waves dispersive (why or why not)?

Shallow Water Waves

$$c_p = (gH)^{1/2}$$

$$C_g = \frac{\partial \omega}{\partial k} = (gH)^{1/2}$$

The phase and group speed are equal to each other so shallow water waves are non-dispersive

Deep Water Waves

$$c_p = \left(\frac{g}{k}\right)^{1/2}$$

$$C_g = \frac{\partial \omega}{\partial k} = \left(\frac{g}{4k}\right)^{1/2}$$

The phase and group speed are not equal to each other so deep water waves are dispersive

4) How do long or short waves behave for shallow water and deep water waves

Long waves: long wavelength, small wavenumber
Short waves: short wavelength, large wavenumber

$$k = \frac{2\pi}{\lambda}$$

Shallow Water Waves

$$c_p = (gH)^{1/2}$$

$$C_g = \frac{\partial \omega}{\partial k} = (gH)^{1/2}$$

Phase and group speed are independent of wavenumber(length) so long and short waves for shallow-water travel at the same speed at constant depth

Deep Water Waves

$$c_p = \left(\frac{g}{k}\right)^{1/2}$$

$$C_g = \frac{\partial \omega}{\partial k} = \left(\frac{g}{4k}\right)^{1/2}$$

Phase and group speeds are both inversely proportional to wavenumber.

Long waves (small k) travel faster than short waves (large k) in both phase and group speed (energy)

5) The following is the dispersion relation for a Rossby wave:

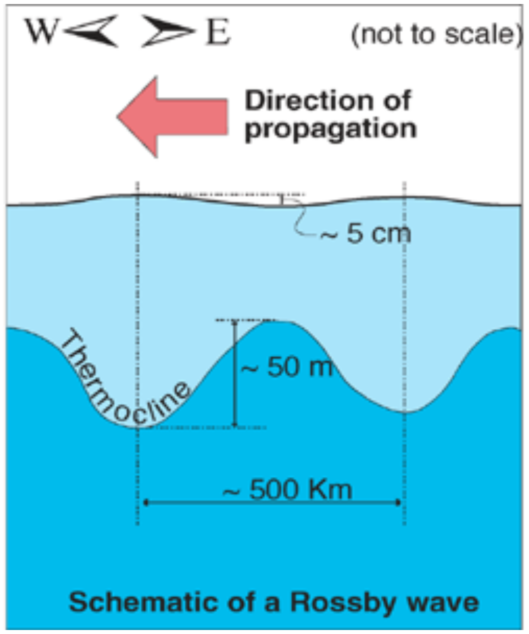
$$\omega = -\frac{\beta_0 k}{k^2 + \ell^2}.$$

Say something about Rossby waves based on this dispersion relation

We can determine the phase speed from the dispersion relation and see that long Rossby waves (small k,l) travel faster than short (large k,l) Rossby waves

$$c_p = \frac{\omega}{k} = \frac{-\beta_0}{k^2 + l^2}$$

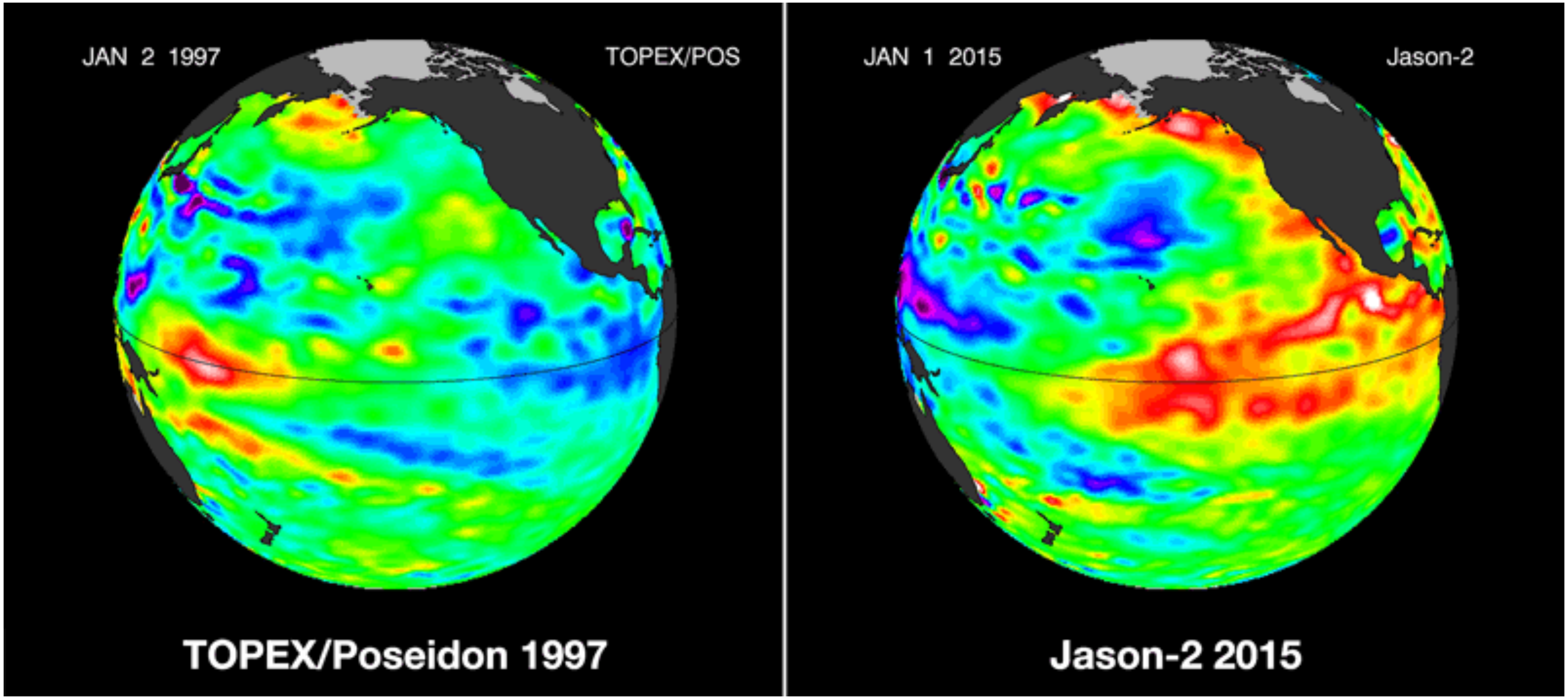
From this phase speed, we can determine that Rossby waves will always travel in the negative x-direction (i.e., to the west) unless they are doppler shifted by a strong enough opposing current



We can also determine the group speed, and see that Rossby waves are dispersive because the group speed and phase speed are not equal

$$C_g = \frac{\partial \omega}{\partial k} = \frac{\beta_0 \left(k^2 - l^2\right)}{\left(k^2 + l^2\right)^2}$$

Rossby and Kelvin Waves and El Nino

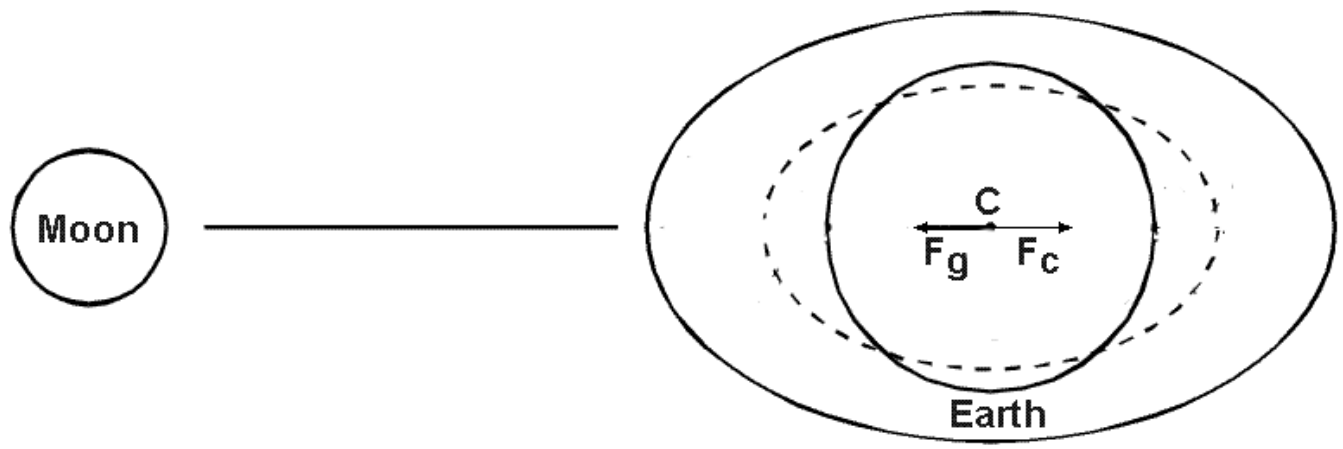


Tides

1) Explain why there is a net force on the earth from the moon (or sun) that causes the earth's "tidal bulge" and lead to oceanic tides (**equate the centrifugal and gravitational forces)

2) Explain the propagation of Kelvin waves via its interaction and manifestation in tides, sea surface, coastal boundary and geostrophic dynamics

Tides due to gravitational forces: moon and sun pulling on the Earth



$$F_g = \text{gravitational force}$$

$$F_c = \text{centrifugal force}$$

$$\text{Net Force} = \text{F}_\text{g} - \text{F}_\text{c}$$

$$F_g = \frac{G \times M_{moon} \times M_{earth}}{R^2}$$

How does the gravitational force behave with distance to the moon?

At center of mass of Earth:

$$F_g = F_c \longrightarrow \text{Net Force} = 0$$

At surface of Earth (closest to Moon):

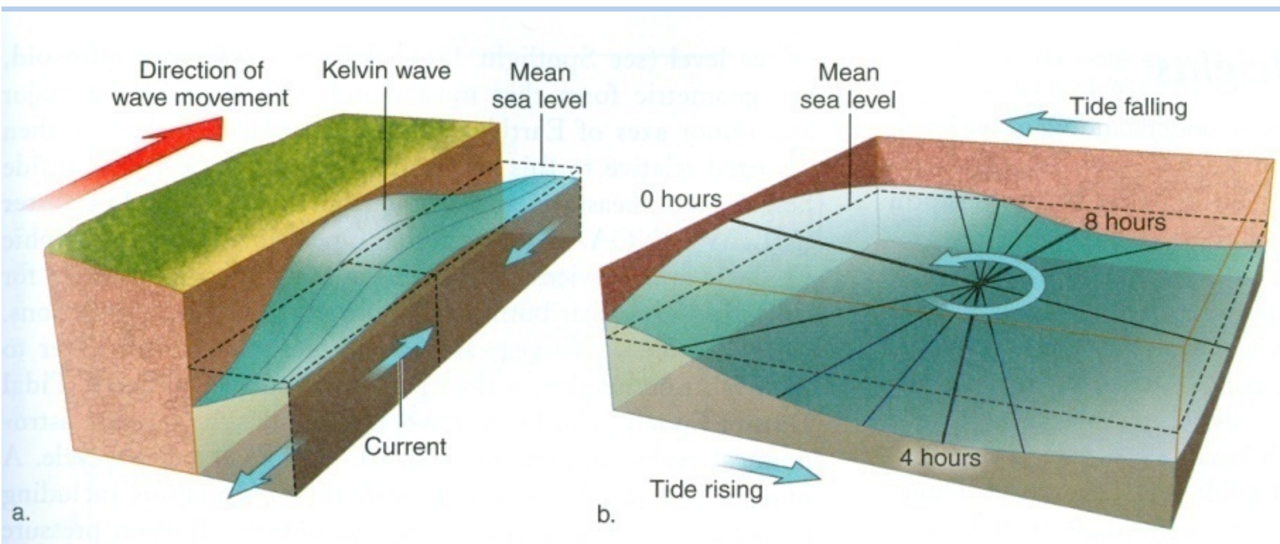
$$F_g > F_c \longrightarrow \text{Net Force} > 0$$

Net force that is non-zero causes the earth to bulge towards the moon (or sun)… this causes tides

Coastal Kelvin Wave: a tidal signature that propagates along the coast

Ingredients for a coastal Kelvin wave: Coriolis force, pressure gradient, topographic boundary (i.e. a coast, that we call a 'coastal waveguide')

**A 'wave' here can be considered a sea surface elevation that propagates (like waves you see at the beach, but here they are of much bigger scale and for our purposes, we will say that there is no wave breaking)



North Hemi example

-a Kelvin wave propagates northward along the coast,
-the Coriolis force wants to deflect this wave to the RIGHT
-BUT, there is a coastal boundary, so water piles up at the coast
-This pile of up of water is manifested in a sea surface elevation, which we know causes pressure gradients
-The pressure gradient force leads to a geostrophic current (PGF = CF)

$$c^2 = gH$$

$$c = \sqrt{gH} = \text{wave speed}$$

g = gravity
H = depth (water column thickness)

Rule of thumb: Kelvin waves propagate with the coast to the right in the NH (so to the left in the SH)