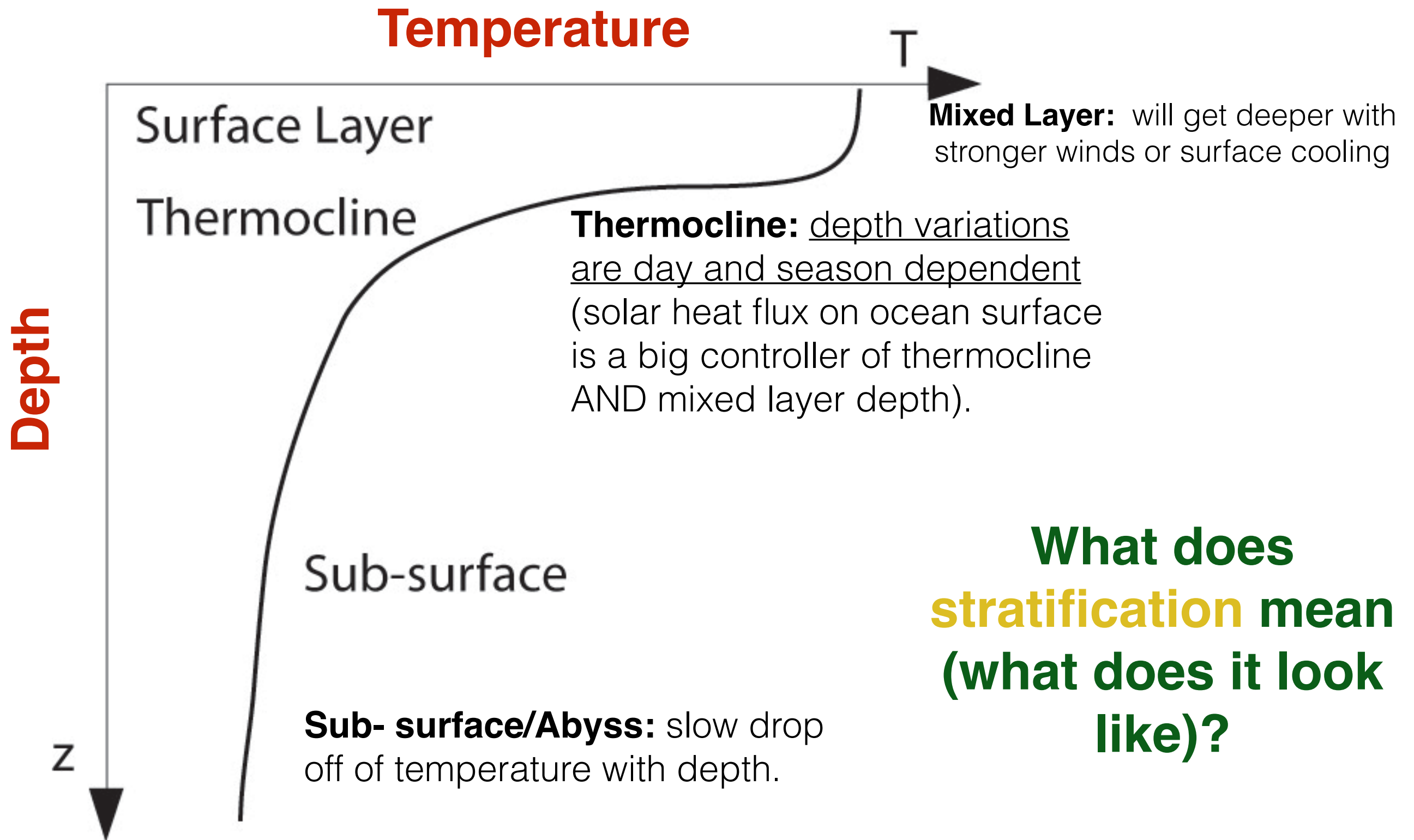


AOS 103

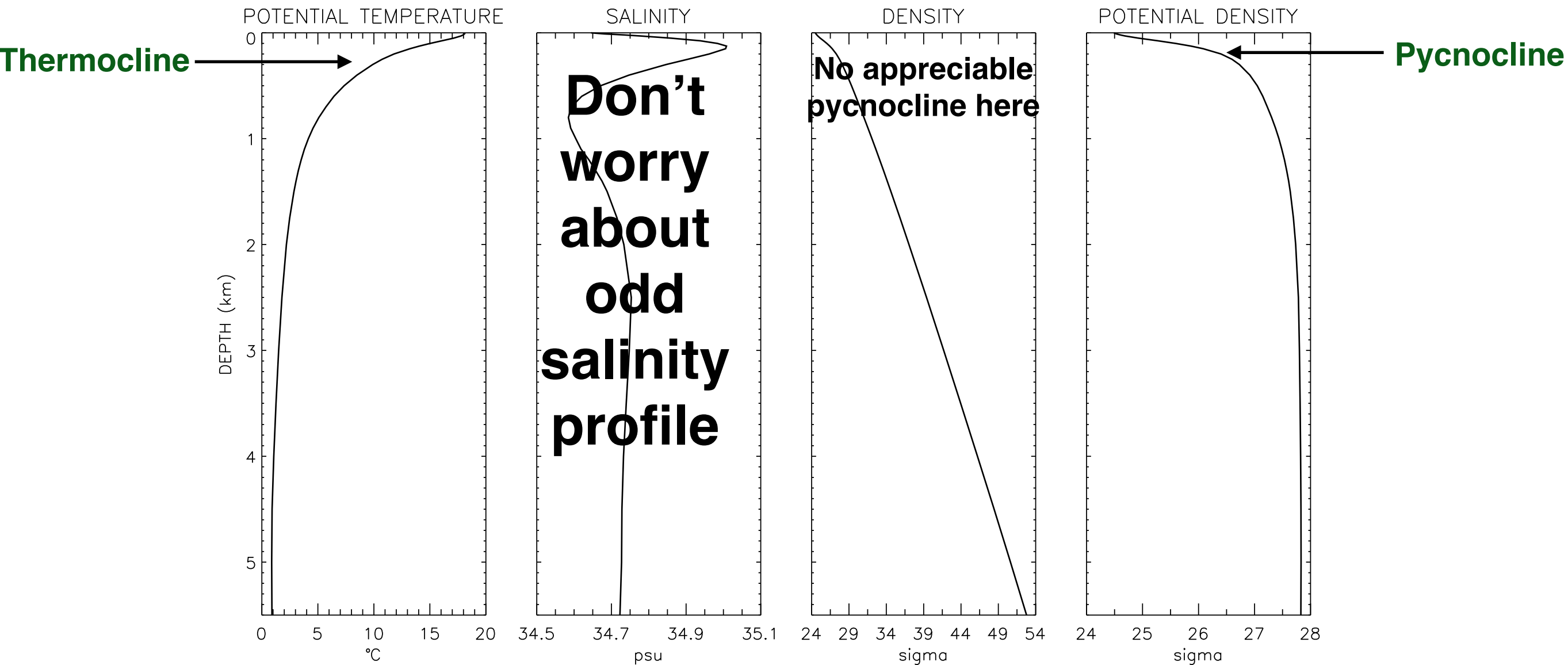
Some important stuff to know for the midterm...but not totally comprehensive (you should be mainly studying off lecture slides/HW/practice midterm)

USE THIS AS A REFERENCE FOR CONCEPTS YOU MAY NOT UNDERSTAND BY LOOKING AT THE LECTURE SLIDES/HW

DO NOT JUST TRY TO MEMORIZE THESE OR THE LECTURE SLIDES



What does a vertical temperature profile look like during the day (strong surface heating) and night (strong surface cooling)?



- Increase in density with depth at large depths is not present in potential density
- **Pycnocline** present in potential density profile and **thermocline** present in potential temperature profile
- **Potential temperature and potential density are CONSERVED (as they follow the flow)**, which also makes these good quantities to trace water masses...

With the realization that compressibility (pressure influence on density) is negligible for the ocean currents we care about, we can form a more dynamically meaningful measure of density and temperature:

Potential Density

- Removes effect of compressibility

$$\rho_{pot} = \rho(S, \Theta, p_{atmosphere}) - 1000\text{kg/m}^3$$

ρ_{pot} = potential density

S = salinity, Θ = potential temperature

$p_{atmosphere}$ = pressure of atmosphere

Potential Temperature

- **Compression by pressure** can change density —> changing the density changes the mass, which changes the internal energy, which can lead to a change in temperature

$$E = mC_pT$$

- Potential temperature removes effect of compressibility

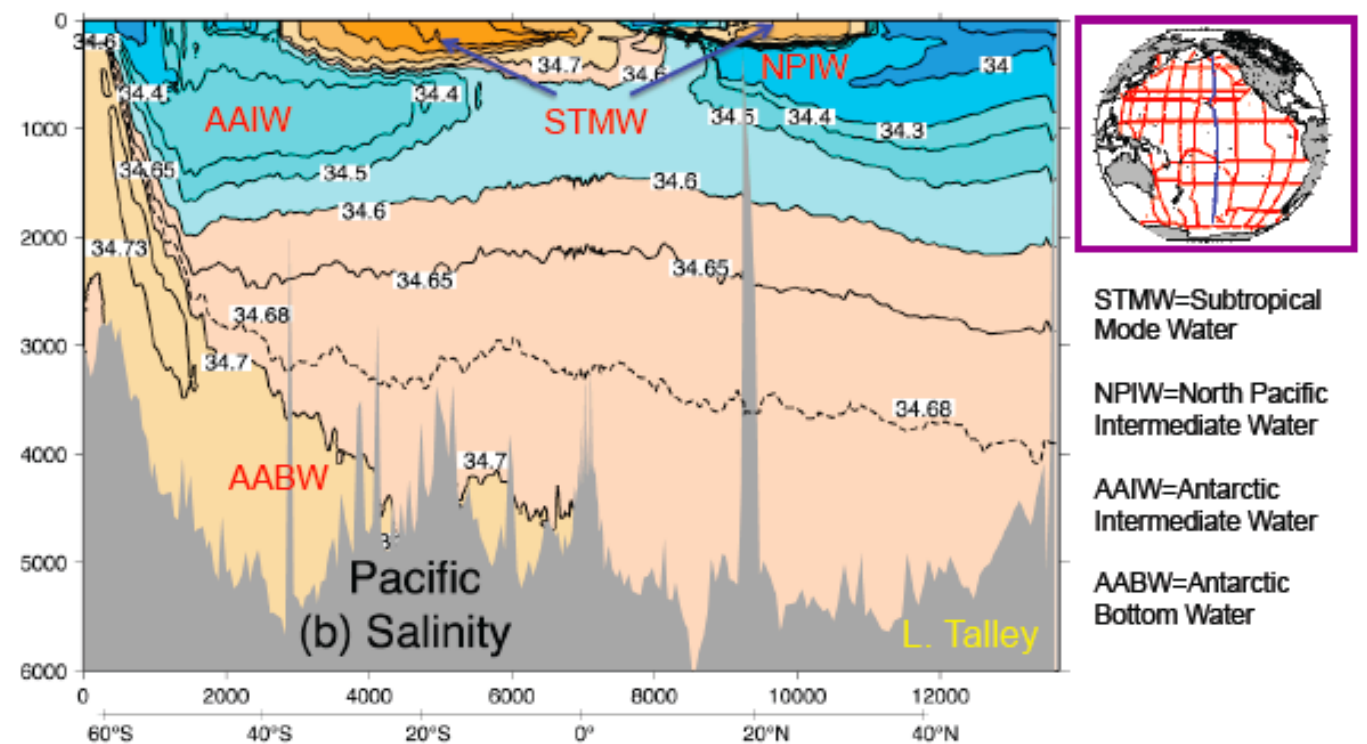
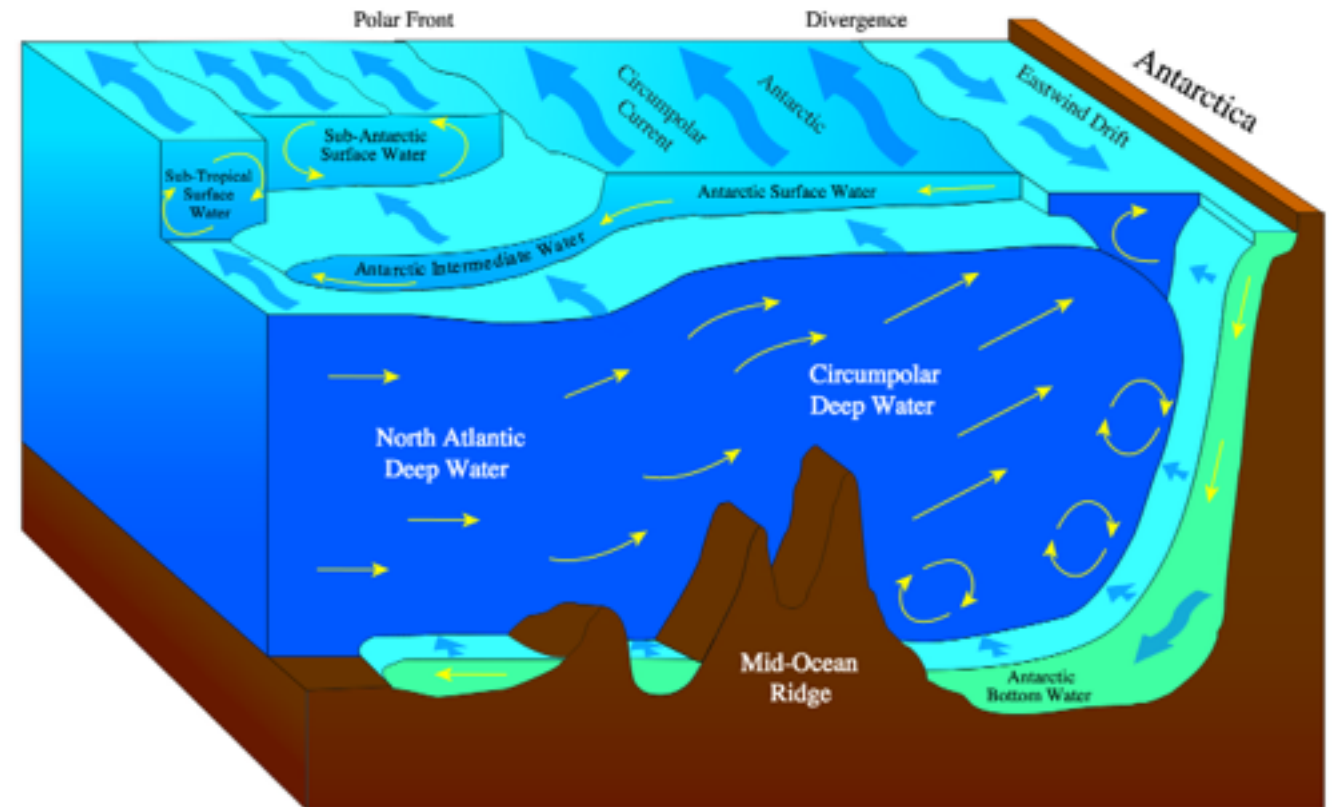
Both potential density and potential temperature lead to simpler measures of water column stability!

Water Masses

- Observing (taking measurements of) the entire ocean (all of its horizontal and vertical expanse) is essentially impossible.
- Water masses provide a way of geographically defining the ocean with limited observation

Things to care about for a water mass

- 1) Where it is formed (**origin**)
- 2) How it is formed (**mechanism**)
- 3) What properties are conserved as they flow with the water mass (**tracers**)
- 4) What depths does it travel through (**depth: surface, intermediate, bottom**)
- 5) Where does it end up (**destination**)



Planetary Geostrophic Momentum Equations

(**F** = **ma**) / **volume** $\rho_0 = \text{constant}$

$$\frac{\partial p}{\partial x} = \rho_0 f v$$

x-direction balance

Pressure gradients in **x-direction** and **Coriolis force** drive currents in **y-direction** (v)

$$\frac{\partial p}{\partial y} = -\rho_0 f u$$

y-direction balance

Pressure gradients in **y-direction** and **Coriolis force** drive currents in **x-direction** (u)

$$\frac{\partial p}{\partial z} = -\rho g$$

z-direction balance

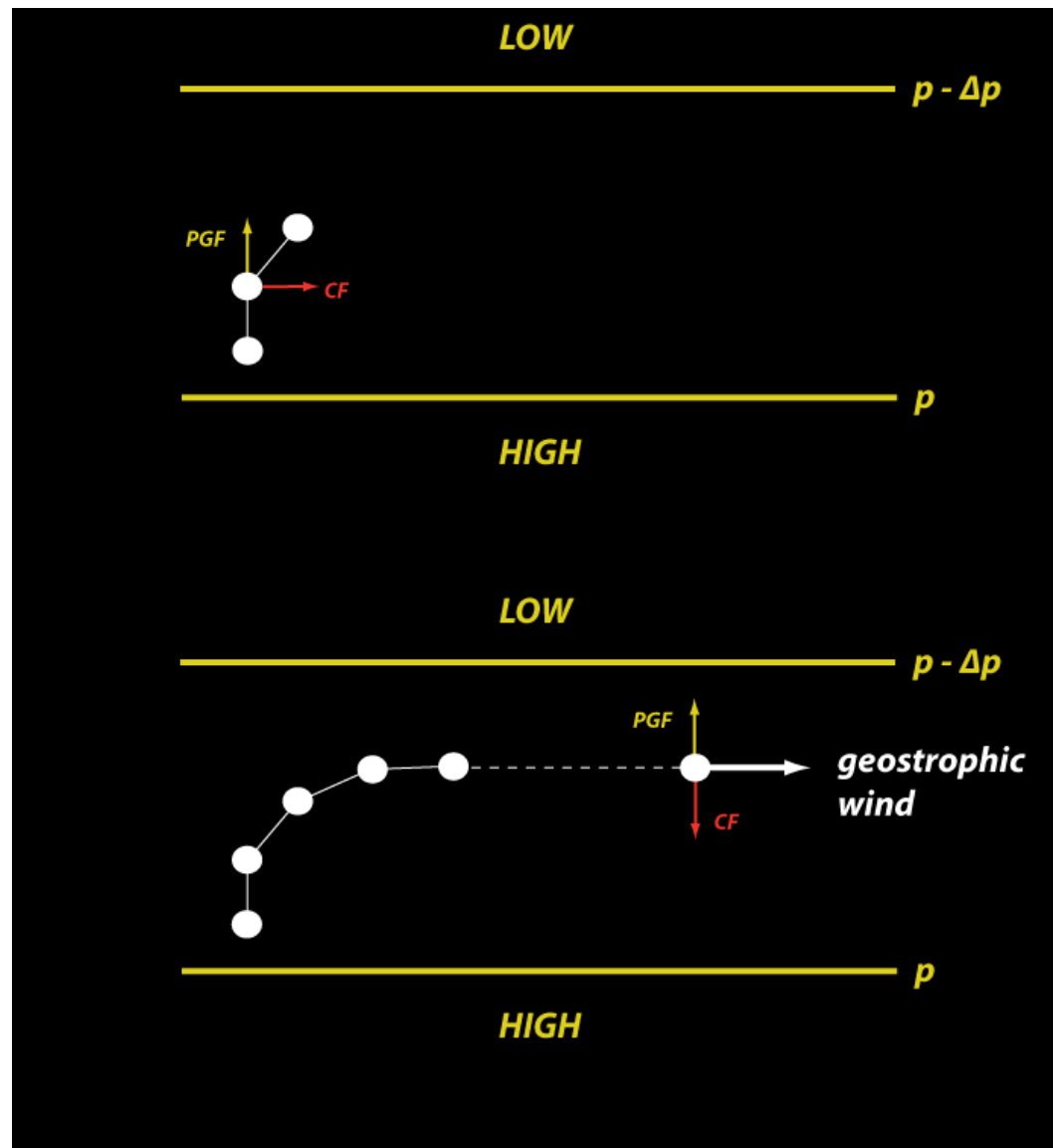
Pressure at any depth is equal to the weight of the fluid above it

“Hydrostatic”

“hydro” = water; “static” = not moving up or down

These equations mean nothing without a physical interpretation...

Geostrophic System (North Hemi)



An air (or water) parcel in this system is being acted on by 2 forces

- 1) Pressure gradient force (PGF)
- 2) Coriolis force (CF)

- The PGF wants to push the parcel from the HIGH to LOW pressure bars (bottom to top)
- The CF wants to make the air parcel deflect to the RIGHT (we're in the NH) of its flow at any point
- Once the parcel adjusts to both forces, it ends up flowing in between the lines of constant pressure (this is geostrophic flow)

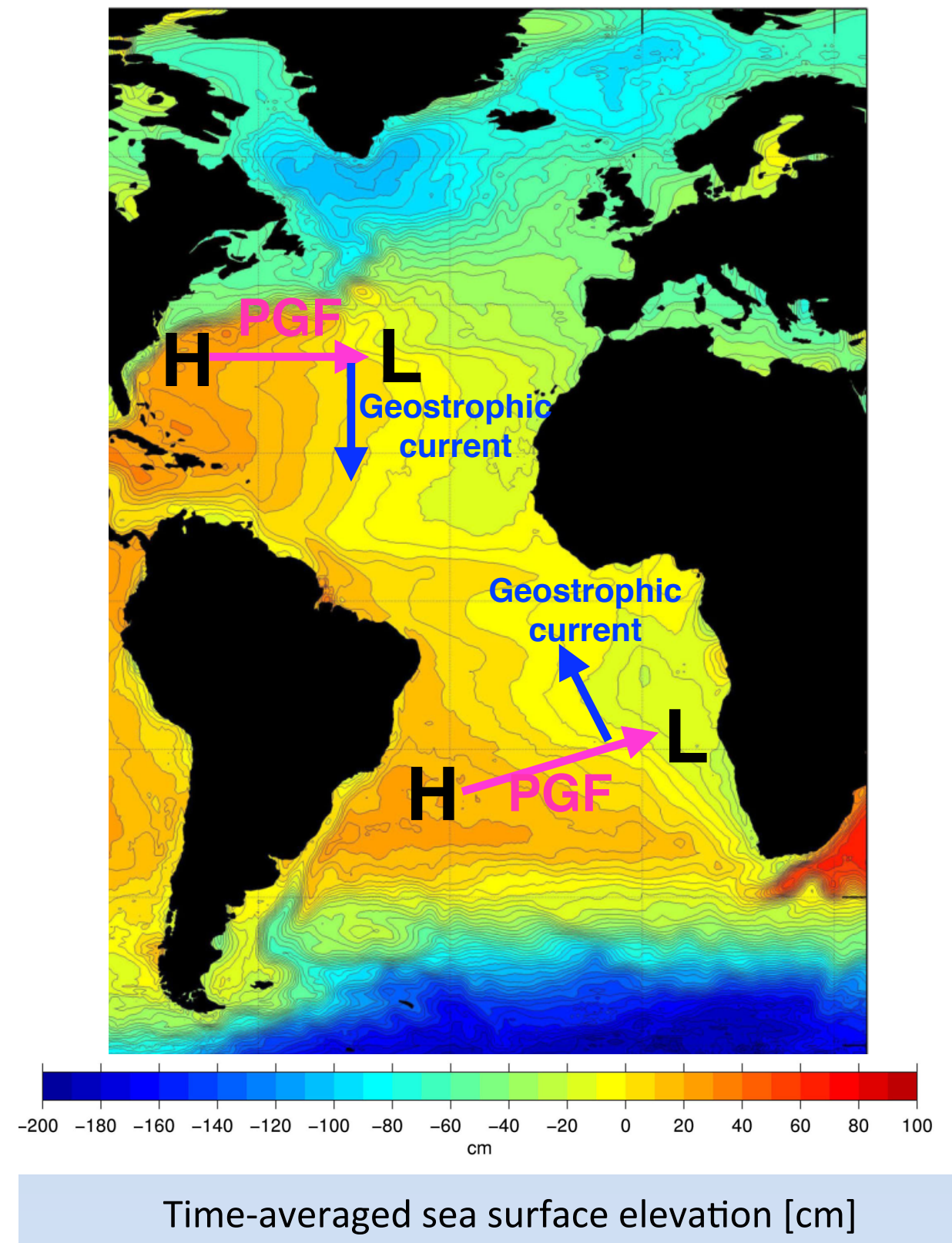
Geostrophy is a **BALANCE** ($PGF = CF$), so the air parcel ends up flowing **IN BETWEEN** the lines of constant pressure

*We say geostrophic flows run “parallel to isobars” (isobar = line of constant pressure)

***Geostrophic rule of thumb: in NH with geostrophic wind at your back, low pressure is to your LEFT (in SH, low pressure is to your right)**

Inferring geostrophic velocity

- 1) **Sea surface height map (x,y)**
 - Determine what direction the pressure gradients vectors are pointing (the PGF arrows point from HIGH pressure to LOW pressure = HIGH sea surface elevation to LOW sea surface elevation (red to blue))
 - Draw your PGF arrow that points from HIGH to LOW pressure
 - If you are in the NH, the geostrophic current is a vector directed to the RIGHT of your PGF arrow(vector)
 - If you are in the SH, the geostrophic current is a vector directed to the LEFT of your PGF arrow(vector)



Rules of Thumb

KNOW THESE FOR EXAM!

Northern Hemisphere

- Coriolis force deflects flows to the **RIGHT**
- Geostrophic flow is to the **RIGHT** of an arrow pointing from high to low pressure (i.e. the direction the water wants to flow due to the pressure gradient force)
- Ekman transport is to the **RIGHT** of surface wind stress

Southern Hemisphere

- Coriolis force deflects flows to the **LEFT**
- Geostrophic flow is to the **LEFT** of an arrow pointing from high to low pressure (i.e. the direction the water wants to flow due to the pressure gradient force)
- Ekman transport is to the **LEFT** of surface wind stress

Terminology

Pressure gradient: points from low to high pressure

Pressure gradient force: points from high to low pressure

Advice to not confuse yourself: when thinking of geostrophic velocities draw/think of arrows pointing in the direction of the pressure gradient FORCE (HIGH to LOW)...this will make it more intuitive to deduce geostrophic velocity from that

North Hemi: geostrophic velocity will flow to the RIGHT of the pressure gradient force (HIGH TO LOW arrow)

Thermal Wind: How do geostrophic velocities vary with depth??

$$\frac{\partial v}{\partial z} = \frac{-g}{\rho_0 f} \frac{\partial \rho}{\partial x}$$

Vertical variation
(shear) of
geostrophic velocity
in the y-direction

Horizontal
gradient of
density in
the x-
direction

x-component

**Thermal wind relates
horizontal density
gradients to vertical
changes in the
geostrophic velocity
(which is a horizontal
flow)**

$$\frac{\partial u}{\partial z} = \frac{g}{\rho_0 f} \frac{\partial \rho}{\partial y}$$

Vertical variation
(shear) of
geostrophic velocity
in the x-direction

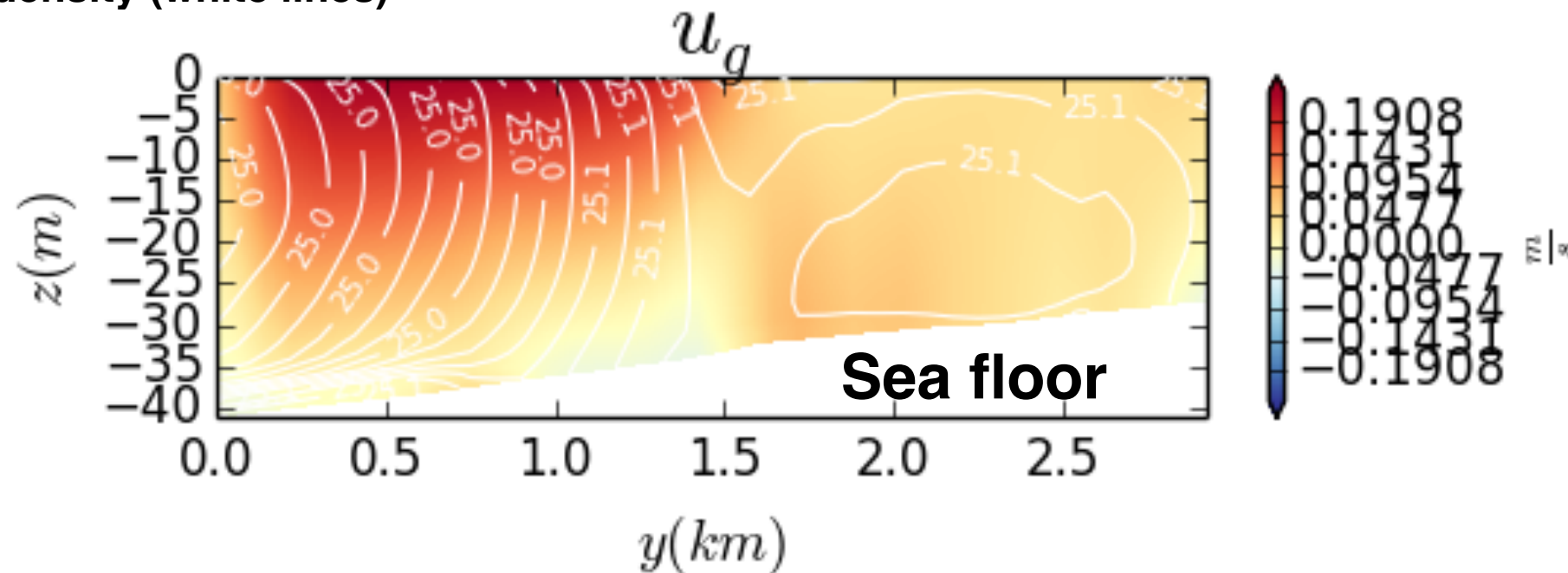
Horizontal
gradient of
density in
the y-
direction

y-component

**You DO need to
understand this
relationship for
the exam!!!**

Thermal Wind: Interpreting Data/Plots

This plot shows a 'cross-section' of geostrophic velocity (colors) and density (white lines)



****This is an example of a baroclinic current b/c it has vertical variation (i.e. the velocity is not constant with depth)

$$\frac{\partial u}{\partial z} = \frac{g}{\rho_0 f} \frac{\partial \rho}{\partial y}$$

u_g → geostrophic velocity in or out of page (**red means out of page, blue means into page**), its the same as “u” in the equation above

- Because there is a horizontal density gradient along the y-axis, the geostrophic velocity (that goes in or out of the page) has vertical shear (i.e. it increases or decreases with depth)
- In this case, the geostrophic velocity **DECREASES** with depth on the left half of the plotted domain

*****The reason why the velocity is stronger (and the shear more pronounced) on the left side of the plot is because the horizontal density gradient is stronger there...the white lines are vertical and close together, so if you were to step from left to right along the y-axis, density would be changing very rapidly from $y=0$ to $y=1.5\text{km}$; but from $y=1.5\text{km}$ to $y=3.0\text{km}$ (the right side), the density gradient is pretty small, so the geostrophic velocity and subsequent vertical shear of geostrophic velocity is relatively smaller and less noticeable there.

Gyres

There are 3 main types of dynamics going on in gyres that can all be traced back to the wind stress (in conjunction with Coriolis force):

- 1) Ekman dynamics (Ekman flow/transport and Ekman pumping/suction)**
- 2) Geostrophic currents due to sea surface height gradients**
- 3) North/south transport in center of gyre and at land boundaries due to potential vorticity conservation**

Large scale wind stress at surface

Ekman flow

**Ekman
pumping/
suction at
CENTER of gyre**

**Large scale curl
of wind stress
“injects”
vorticity into
ocean**

**Sets up large scale
sea surface height
gradients**

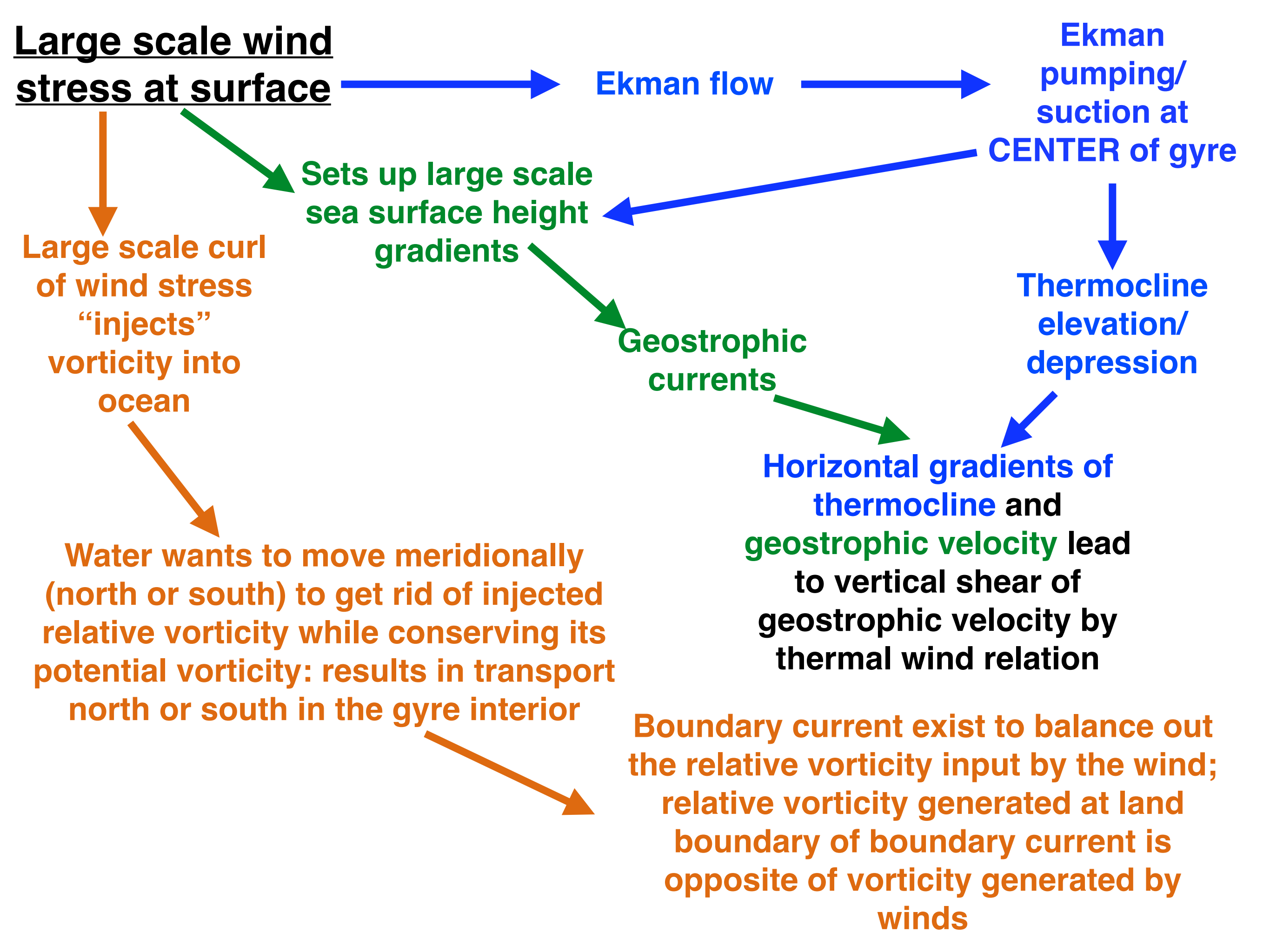
**Geostrophic
currents**

**Thermocline
elevation/
depression**

**Horizontal gradients of
thermocline and
geostrophic velocity lead
to vertical shear of
geostrophic velocity by
thermal wind relation**

**Water wants to move meridionally
(north or south) to get rid of injected
relative vorticity while conserving its
potential vorticity: results in transport
north or south in the gyre interior**

**Boundary current exist to balance out
the relative vorticity input by the wind;
relative vorticity generated at land
boundary of boundary current is
opposite of vorticity generated by
winds**



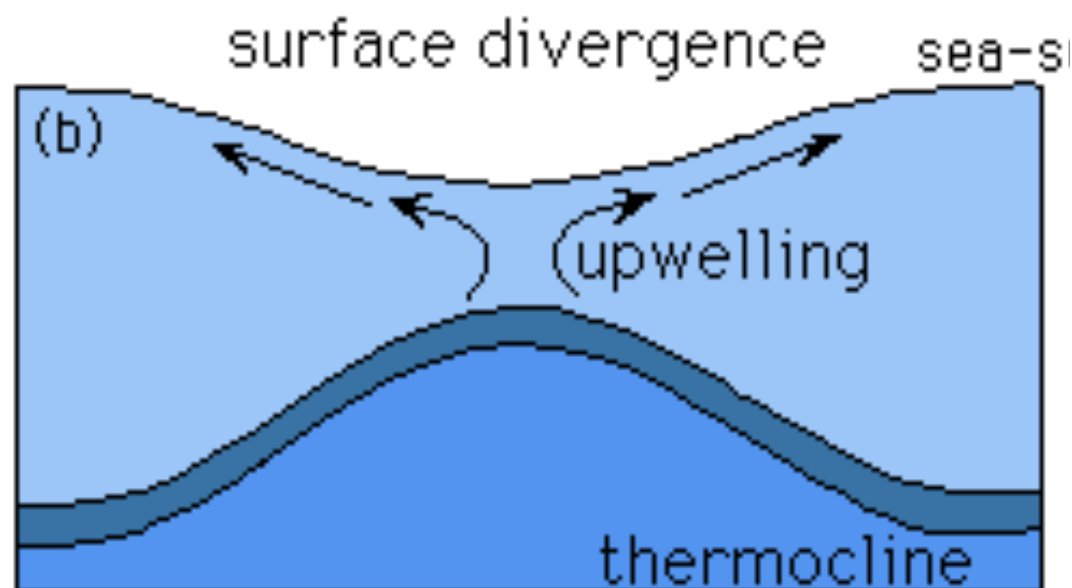
For any gyre:

- **Start with the large scale atmospheric pressure**
- **Deduce the geostrophic atmospheric wind from this pressure**
- **Use the wind pattern to infer Ekman transport (it will go into or out of the gyre radially)**
- **Use the direction of Ekman transport to infer whether there is upwelling (suction) or downwelling (pumping) at the center of the gyre**
 - **Essentially answer this question: is there convergence or divergence of water at the center of the gyre?**
- **Use this behavior at the center of the gyre to infer what the thermocline and sea surface height are doing**
- **Use the sea surface height to infer which way the surface pressure gradient forces want to push water**
- **Use these pressure gradient forces to infer a geostrophic current**
- **The horizontal variation of the thermocline combined with a geostrophic current will mean that the geostrophic current will have vertical shear (increase or decrease) with depth due to the thermal wind balance**
- **The final piece of the gyre story is the boundary current...figure out the sign of vorticity that the wind is inputting to the gyre and what type of boundary current would be needed to balance that out**

Thermocline and sea surface height response to Ekman pumping/suction

Upwelling (due to surface divergence) = SUCTION

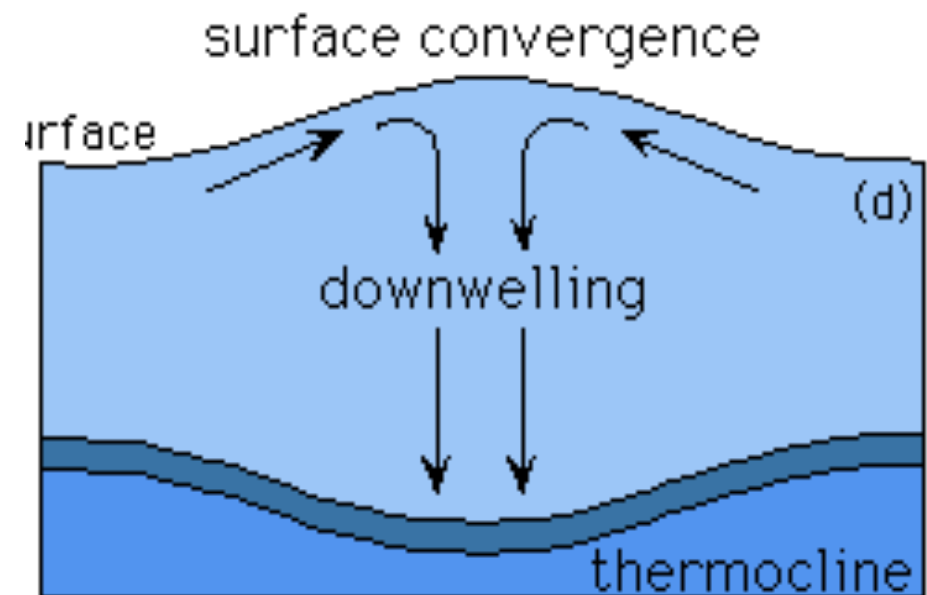
- thermocline raised up
- sea surface depressed



What's the geostrophic velocity?
(pick a hemisphere)

Downwelling (due to surface convergence) = PUMPING

- thermocline depressed
- sea surface raised up



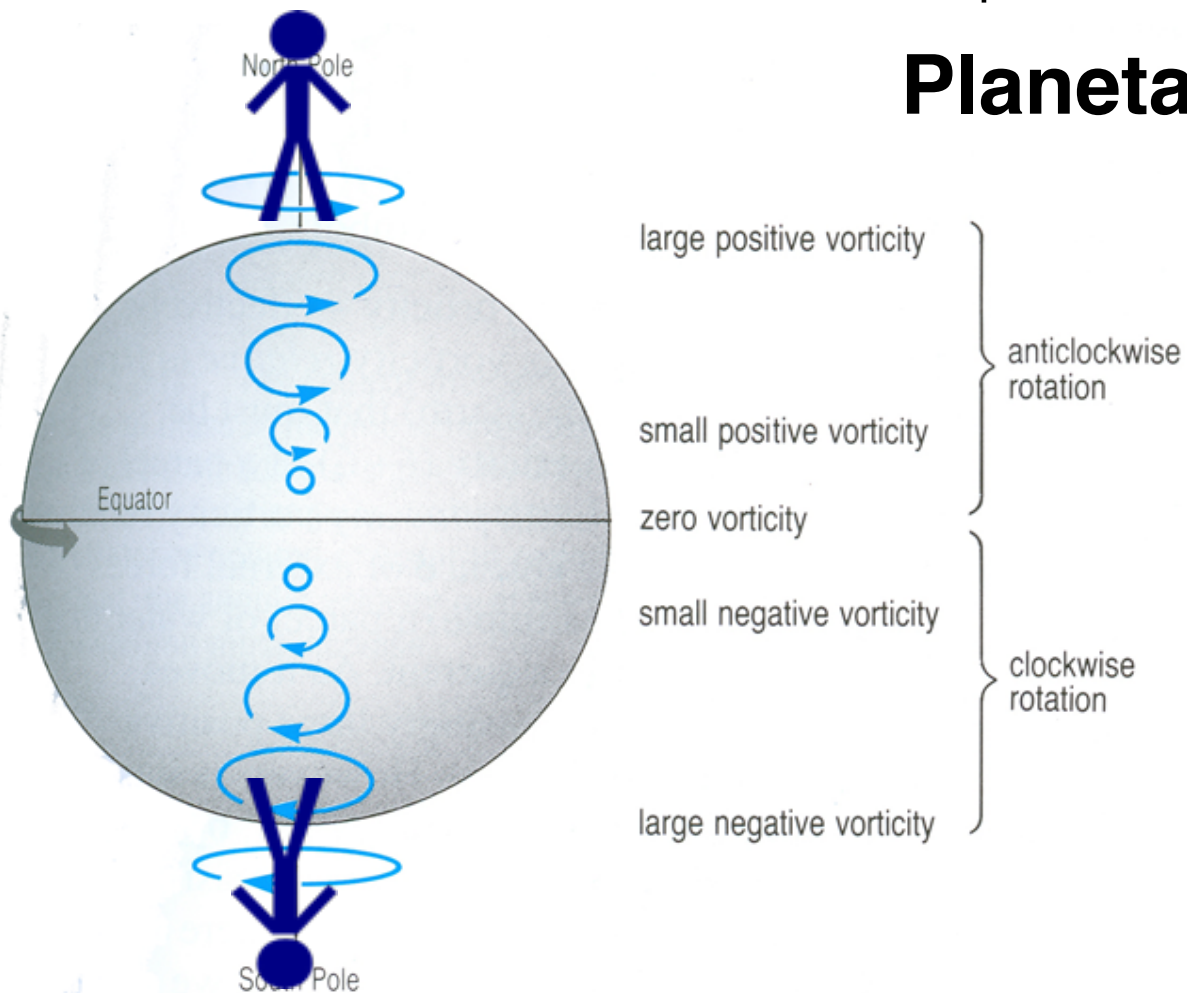
What's the geostrophic velocity?
(pick a hemisphere)

Planetary Vorticity

Because the Earth is always rotating, everything in it always has some vorticity. We call this vorticity “planetary vorticity”, simply b/c it is due to the planet’s own rotation.

$$\text{Planetary vorticity} = \text{Coriolis parameter} = f$$

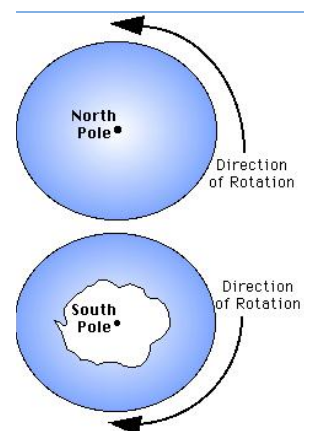
Units: [1/s]



NH - counterclockwise = cyclonic

SH - clockwise = cyclonic

“Cyclonic” always refers to having the same sense of planetary rotation. The “sense” of planetary rotation is opposite in each hemisphere. Thus cyclonic refers to counterclockwise rotation in the Northern Hemisphere (NH) and clockwise rotation in the Southern Hemisphere (SH)



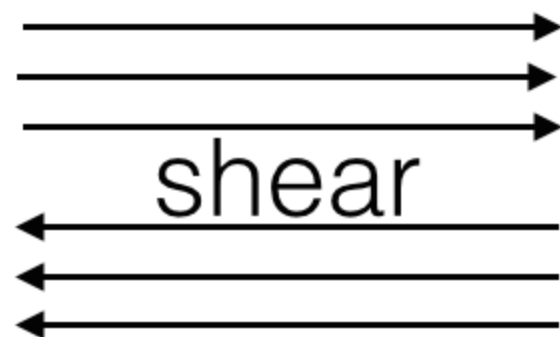
Relative Vorticity

Relative Vorticity $\zeta = (\nabla \times \mathbf{v}) \cdot \hat{\mathbf{k}} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ **Just tells us how much rotation or shear a fluid is exhibiting in its horizontal velocities**

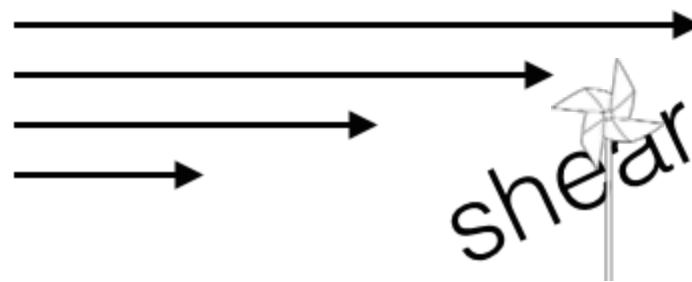
Vertical component of curl of 3D velocity

Units: [1/s]

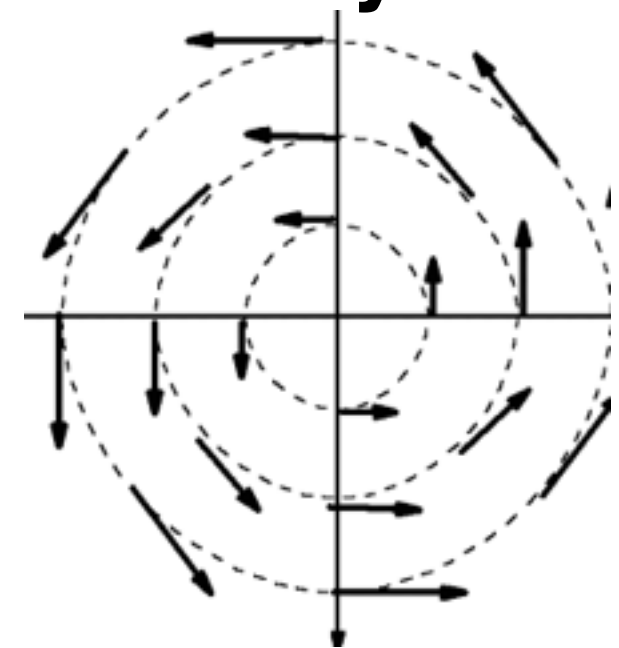
All 3 of these flow profiles have vorticity.



Clockwise



Clockwise



Counterclockwise



When we talk about vorticity we use the convention of saying whether or not the flow is “spinning” in a clockwise or counterclockwise manner...just imagine a pinwheel in the parts of the flow where there is shear and the way that pinwheel rotates.

For the North Hemi, what should the pressure in the center of this velocity field be if this were a geostrophic wind or current (high or low pressure)?

Absolute and Potential Vorticity (PV)

Absolute Vorticity: combine planetary and relative vorticity

$$\zeta_{relative} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$$

$$\zeta_{planetary} = 2\Omega \sin(\phi) = f$$

$$\zeta_{absolute} = \zeta_{planetary} + \zeta_{relative} = f + \zeta$$

Potential Vorticity (PV)

But in the ocean we like to think about “water columns” that have depth...

We also know that if we have a “fluid column”, we can change its vorticity by “stretching” it due to conservation of angular momentum...if we “stretch” out the column by making it taller, but more thin in diameter the vorticity will increase

So we need to incorporate this dependence of column height(depth) into our vorticity.

We include a depth-dependence with **potential vorticity**

Absolute vorticity



$$q = \frac{f + \zeta}{H}$$



Water column
thickness (depth)

Cyclonic and Anticyclonic

	North Hemi	South Hemi
Cyclonic	Counterclockwise	Clockwise
Anticyclonic	Clockwise	Counterclockwise

Easiest way to remember: just memorize that the NH's cyclonic rotation is counterclockwise....if you remember that, you can remember all the other ones by knowing that the SH is the opposite for cyclonic and anticyclonic is just the opposite of cyclonic in each hemisphere

YOU NEED TO KNOW THIS FOR THE EXAM!

Best way to get familiar with it is to give yourself multiple examples: put a high or low pressure center in a hemisphere and draw the geostrophic wind around it and then say whether its cyclonic or anticyclonic for that hemisphere

PV Thinking/Deductions

The way these questions *will probably be worded* (**ask Prof Stewart in Lecture tomorrow) is as follows:

Case 1:

The winds (or something else) tend to increase the vorticity of the ocean and thereby increase its potential vorticity ($q \sim f/H$). In which direction does this cause water columns to move (if H is constant)

Answer Case 1:

If q is being increased, then f must increase and this will cause northward movement

Case 2:

The winds (or something else) tend to decrease the vorticity of the ocean and thereby decrease its potential vorticity ($q \sim f/H$). In which direction does this cause water columns to move (if H is constant)

Answer Case 1:

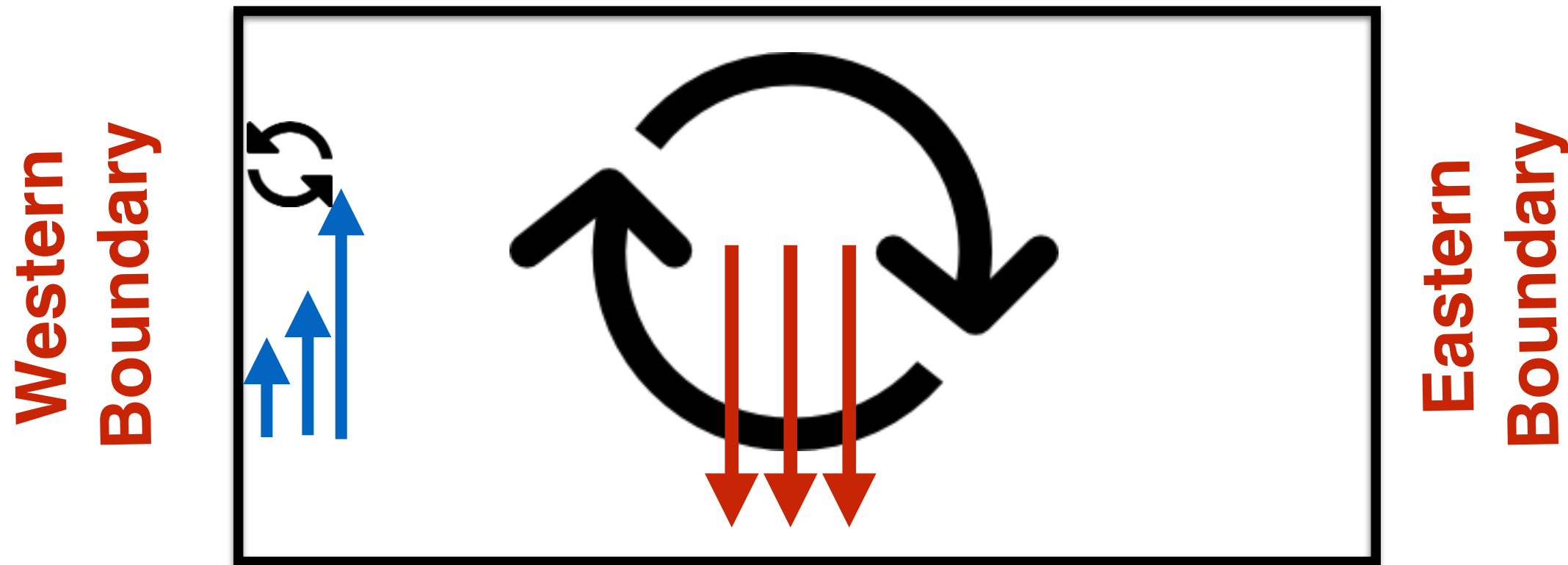
If q is being decreased, then f must decrease and this will cause southward movement

GETTING TO BOUNDARY CURRENTS FROM VORTICITY (RELATIVE AND POTENTIAL)

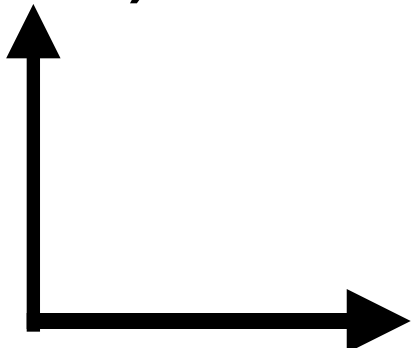
To balance out the negative vorticity input by the wind, the boundary current must create(supply) positive vorticity.

The western boundary current supplies this positive vorticity

It exists to 1) conserve mass and 2) balance out the vorticity input by the wind



y (north)

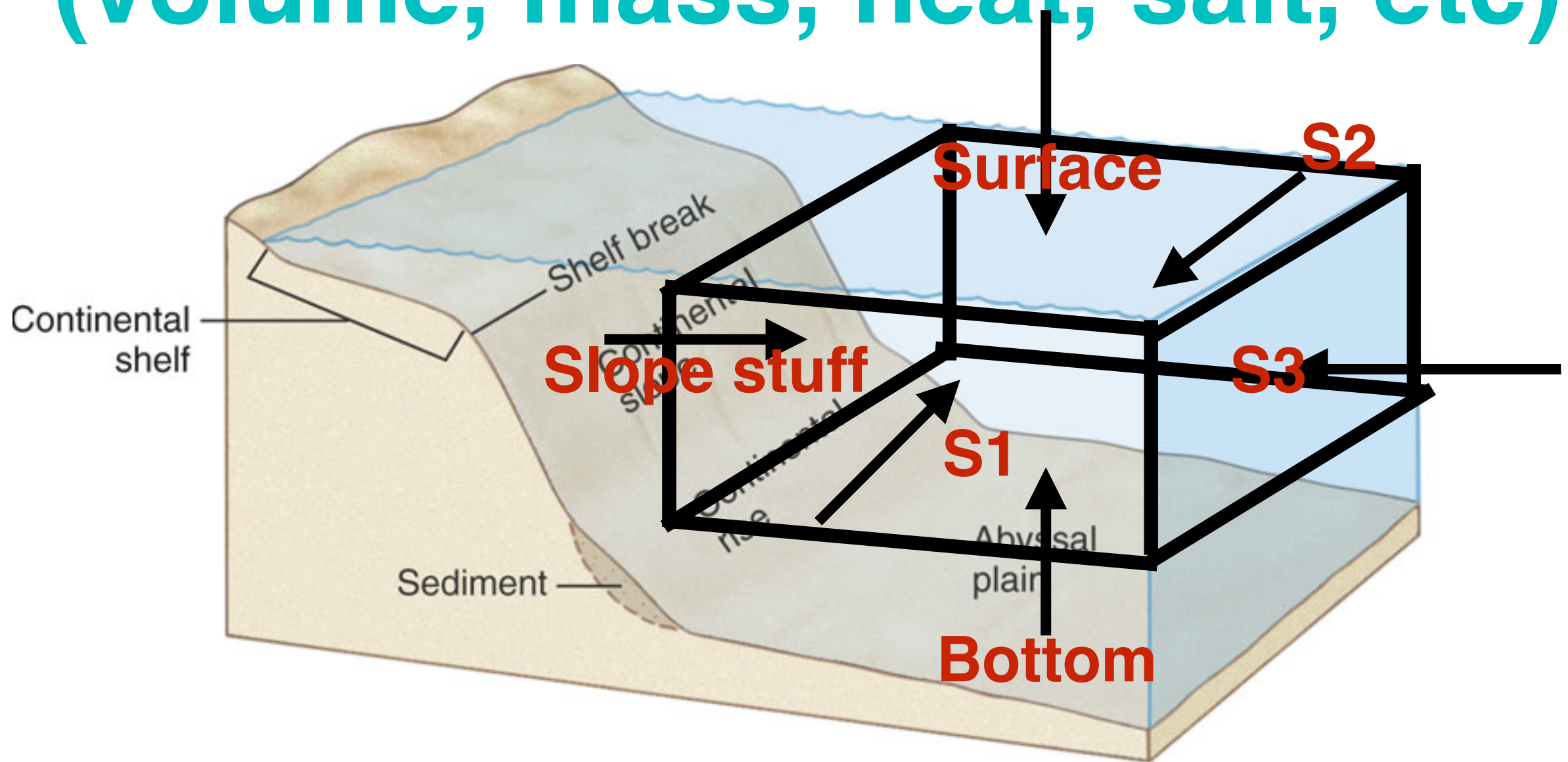


x (east)

→ (Interior transport)
→ (Boundary current)

Interior transport: caused by
Case 2 scenario on last slide
(winds act to *decrease* potential
vorticity)

Conservation of any Flux (volume, mass, heat, salt, etc)

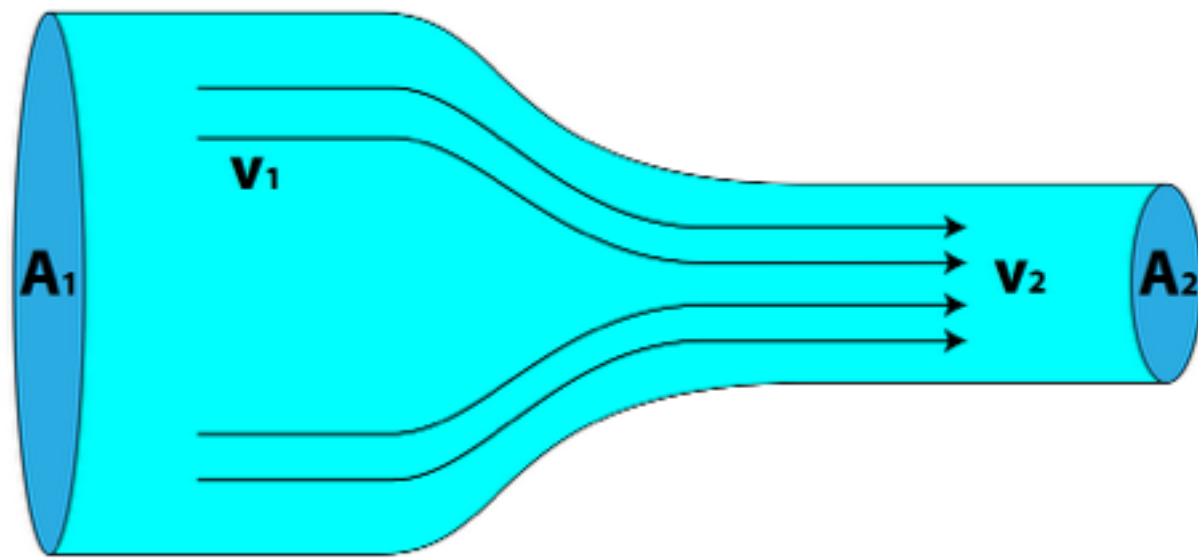


IN = OUT

ALL THE ARROWS NEED TO SUM TO ZERO

Conservation of Volume

We can use conservation of volume(or mass) to calculate the actual velocity of fluid going in and out of different areas (i.e. faces of a box), but you should **ALWAYS FIRST** be using this principle to give yourself an intuition of how a fluid flows in and out of different areas if it is conserving volume(mass)



IN = OUT

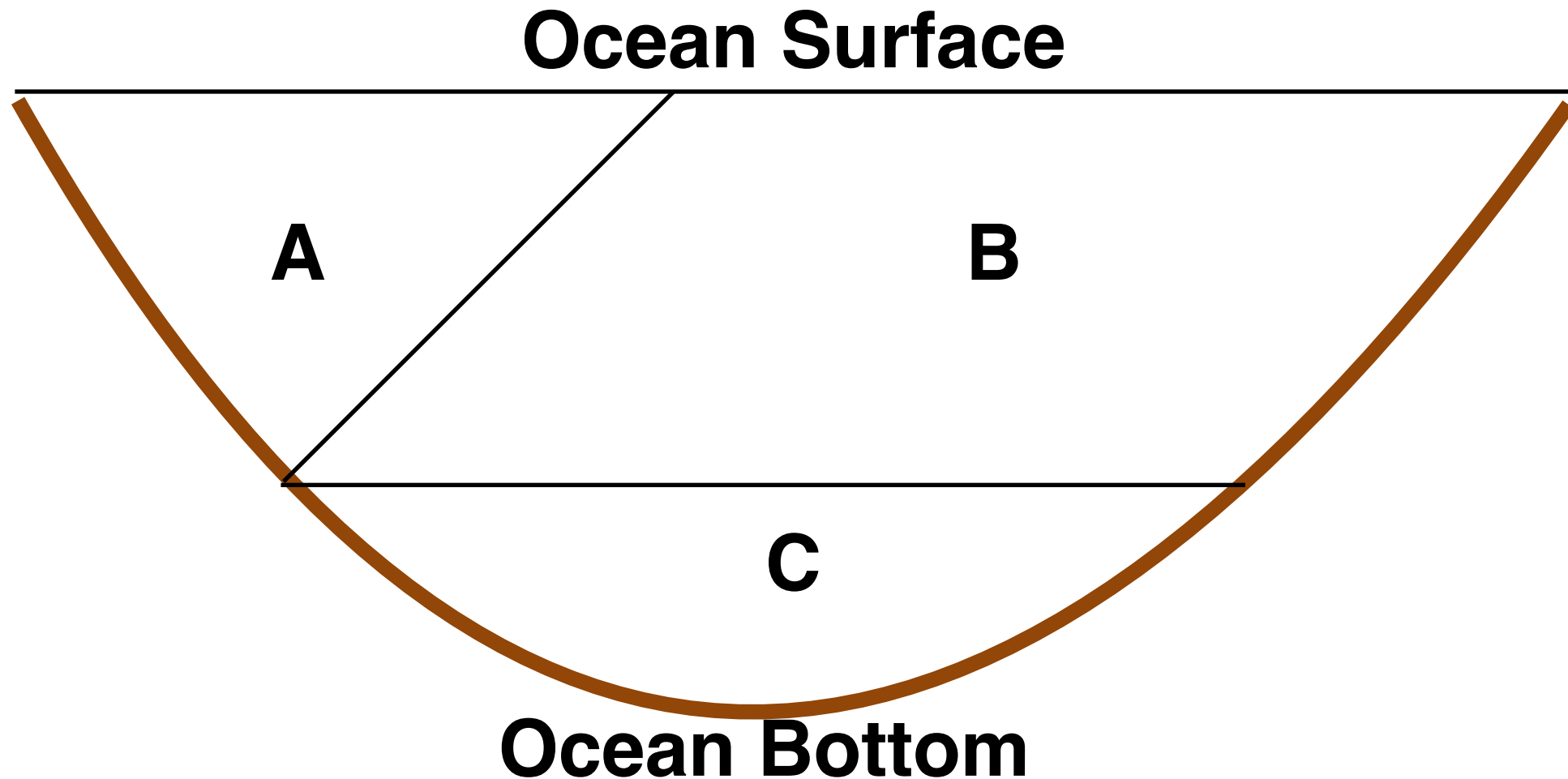
$$V_1 A_1 = V_2 A_2$$

- 1) Do you expect V_1 to be greater than or less than V_2
- 2) Show why mathematically using the conservation of volume

****So for any time you have a problem that has stuff (water, mass, etc) fluxing in and out of boxes, basins, any old shape...you will probably be able to figure out flow or transport using conservation of volume**

Conservation of Volume

Cross-section example to deduce direction of a flow



ψ Volume transport

$$\psi_A + \psi_B + \psi_C = 0 \quad \text{Conservation of Volume}$$

If we knew the direction of A and B, we could deduce the direction of C based on conservation of volume (e.g., if A and B were out of the page, C would have to be into the page).