analysis of magnetosonic waves instability using LANL-MPA data

1. introduction

In this report, we focus on instability of magnetosonic waves (at wave frequency $\omega = n\Omega_{H^+}$, wave normal angle $\theta = 89.5°$) due to $m = n$ harmonic resonance. Thus,

$$\omega - k_{\|} v_{\| res} = m\Omega_{H^+}$$

$$v_{\| res} = 0$$

The growth rate k_i due to such single harmonic resonance can be expressed as

$$k_i = \int_0^{+\infty} dv_{\perp} \left[W_{\perp} \frac{\partial f}{\partial v_{\perp}} + W_{\|} \frac{\partial f}{\partial v_{\|}} \right] \Big|_{v_{\|}=0}$$

, where $W_{\perp}$ and $W_{\|}$ are the weighting functions, dependent on particle velocity ($v_{\perp}$ and $v_{\|}$), wave parameters (ω and θ) and background plasma parameters (density, magnetic field strength and ion composition)

It can be shown that

$$W_{\|} << W_{\perp}$$

. Also $\frac{\partial f}{\partial v_{\|}}\Big|_{v_{\|}=0} = 0$ for phase space density with mirror symmetry about the plane of pitch angle = 90° (i.e., $v_{\|} = 0$).

So

$$k_i = \int_0^{+\infty} dv_{\perp} W_{\perp} \frac{\partial f}{\partial v_{\perp}} \Big|_{v_{\|}=0}$$

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Apply this to the real MPA data,

$$k_i = \int_{v_{\min}}^{v_{\max}} dv_\perp W_\perp \frac{\partial f}{\partial v_\perp}\Big|_{v_\parallel=0} = \sum_j \int_{v_j}^{v_{j+1}} dv_\perp W_\perp \frac{\partial f}{\partial v_\perp}\Big|_{v_\parallel=0} = \sum_j k_i^j$$

, where j is energy channel number.

By Analyzing the $W_\perp$ function, it can be shown that the $v_\perp$ of peak $W_\perp\Big|_{v_\parallel=0}$, termed as $v_{\perp peak}$ essentially scales as Alfvenic velocity V_A $(= \frac{B}{\sqrt{\mu_0 n_e m_{H+}}})$. Let $\Pi = v_{\perp peak}/V_A$. The relation between $v_{\perp peak}$ and m is plotted in lower pannel of Figure 1. When one consider the instability of magnetosonic waves at higher wave frequency, the more weight on the contribution of $\frac{\partial f}{\partial v_\perp}$ at the lower $v_\perp/V_A$, and verse vice. The growth rate of magnetosonic waves at $\omega \approx 20\Omega_{H^+}$ is driven mostly by the gradient at $v_\perp = V_A$. It should be noted that the actual contribution is the product of weighting $W_\perp$ and the gradient $\frac{\partial f}{\partial v_\perp}$.

Stongest growth rate occurs when the $v_\perp$ of peak weighting conincides with the $v_\perp$ of peak gradient:

$$v_\perp\Big|_{df_\perp/dv_\perp \text{ peak}} = v_\perp\Big|_{W_\perp \text{ peak}} = \Pi(m)V_A$$

It has been shown in previous studies that resonant particle energy is conserved in the wave frame moving with parallel phase speed $v_{p\parallel} = \omega/k_\parallel$ during wave particle cyclotron harmonic interaction $m \neq 0$. We can write the diffusion surface along which particles will move during the interaction in the differential equation:

$$v_\perp dv_\perp + (v_\parallel - v_{p\parallel})dv_\parallel = 0$$

.

Thus

$$\frac{dv_\perp}{dv_\parallel} = -\frac{v_\parallel - v_{p\parallel}}{v_\perp} = \frac{v_{p\parallel}}{v_\perp}$$

.

Since $v_{p\|}$ is generally much larger than $v_\perp$ for proton energy of interest, diffusion driven by wave particle interaction is along $v_\| = 0$ (energy diffusion), i.e., quasi-linear diffusion should smooth the gradient with respect to $v_\perp$ (e.g., ring-type distribution) at 90 degree pitch angle.

2. calculation of growth rate

- Select all the energy channel $E_j > 1$keV.
- Construct pitch angle distribution for protons at each energy channel
- Interpolate to get energy and pitch angle distribution function
- calculate $\frac{\partial f}{\partial v_\perp}$ and $\frac{\partial f}{\partial v_\|}$, given $\frac{\partial f}{\partial v}$ and $\frac{\partial f}{\partial \alpha}$, where α is pitch angle
- B field strength is obtained by dipole model or T96 model
- background plasma density is approximated by low energy proton density; All ions are assumed to be proton.
- small growth rate assumption allows us to work out growth rate explicitly.

The procedure has been applied to available LANL-MPA data. Analysis on one day LANL-MPA data for one sallite is shown in one corresponding figure with format like Figure 2:

- plot title includes information about satellite name, observation date, assumed B model (dip for dipole, T96 for T96 model), chosen E_{min} (1 keV).
- first panel shows $f_\perp$ as a function of E, x-axis represents UT. Also E_A $(=\frac{1}{2}m_{H^+}V_A^2)$ is plotted as black circles.
- fourth panel shows $\frac{d\log f_\perp}{d\log E}$: yellow represent positive gradient; dark blue negative gradient; light blue for relative flat gradient. Also local time is denoted by triangles along

the x-axis. light blue right-pointing triangle for dawn, white upward pointing triangle for noon, dark blue left-pointing triangle for dusk, and black downward pointing triangle for midnight.

• second panel shows $\log_{10}(k_i)$ as a function of normalized wave frequency ω/Ω_{H^+} and UT. Color regions represent positive growth rate while white regions are for damping.

• third panel shows the energy channel of dominate contribution for wave growth, i.e., the channel where k_i^j maximizes over all j.

A few points should be noted as follows.

• $f_\perp$ shows some degree of noisy fluctuation at low energy channel, e.g., positive gradient associated with fluctuation at energy just above 1 keV, which may falsely contribute to wave growth just below ω_{LHR} ($\sim 40\Omega_{H^+}$).

• The way to identify whether growth rate is real or false: 1) see whether there is consistent positive gradient feature (ring-type feature) over a period of time, by looking at first and third panels, and then 2) check whether the dominant energy channel corresponds to the energy just below the ring energy with positive gradient, by looking at third and fourth panels.

3. comment on growth rate

The following comment is made for figures using dipole field, which should be appliable to figures using T96 model.

• 2001/04/20-a1 (YYYY/MM/DD; satellite name "a1"): two nice rings (duskside and nightside); E_A is above the regions of positive gradient associated with the rings; instability occurs at high wave frequencies $> 30\omega_{H^+}$; unstable wave frequency band shifts toward higher frequencies as UT goes on, because the ring energy goes down as UT goes while E_A varies little for this time period (consequently, the ratio $v_\perp\Big|_{df_\perp/dv_\perp \text{ peak}}/V_A$ decreases).

- 2001/04/21-a1: ring near the plume. Inside the high density plume, instability tends to occur at lower harmonic frequencies while outside the plume, instability tends to occur at higher harmonics. density fluctuations inside and at the edge of plume also make the frequency band of instability fluctuate.

- 2001/04/22-a1: a nice ring on pre-noon (8.5¡UT¡9.5) with ring energy extending up to a few 10 keV on early MLT. At such early MLT, ring energy is above E_A, instability occurs at lower harmonics frequency down to 5 Ω_{H^+}. As MLT goes up, the ring energy decreases and then drop below E_A. Therefore, the instability shifts toward the higher harmonic frequencies.

- 2001/04/23-a1: one ring at noon and another ring at night. upward shifting wave frequency band of instablity is also observed at $10.5 < UT < 11.0$. As for the seond ring at night, there is a sudden drop in plasma density around UT=2000, during which the E_A experiences transition from below ring energy to above. As a consequence, the wave frequency band of instability jumps toward 30-40 Ω_{H^+} from 15-25 Ω_{H^+}.

- 2005/08/31-a1: one pronounced ring at afternoon sector with ring energy down to 1 keV. This ring coexists with plasmaspheric plume with density (thus E_A) fluctuation inside. wave frequency band of instability fluctuates throughout the plume. Similar rings have also been observed, including
  - 2005/08/31-a2: a ring at the dusk sector observed (similar to the one above).
  - 2005/08/30-l0: ring observed between dusk and midnight.
  - 2001/04/22-l4: ring observed between noon and dusk.

- 2005/08/31-l4: a nice ring observed in the afternoon ($0 < UT < 5$) inside the plume with less density fluctuation. However, no instability occurs because the ring energy is well above E_A. The growth rate seen in this time period is not real because it is falsely driven by noisy fluctuation in ring current proton PSD at energy below 3 keV (see third panel) while the ring energy is above 5 keV.

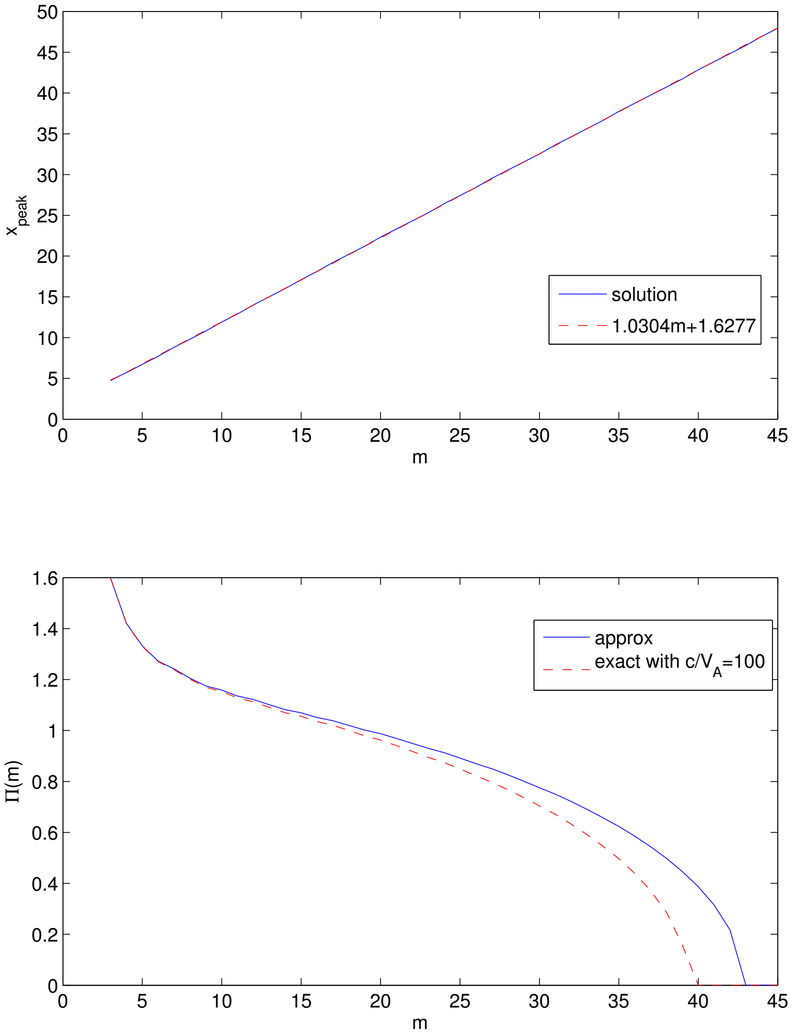

Figure 1. Ignore upper panel. lower panel: x-axis harmonic number, y-axis is the ratio $\Pi = v_{\perp peak}/V_A$, where $v_{\perp peak}$ is the $v_\perp$ of the $W_\perp\Big|_{v_\parallel=0}$ peak. Blue solid line is the approximate relation between m and $v_{\perp peak}$, which does not depend on wave parameters and background parameters. Red dashed line is for the exact relation with $c/V_A = 100$.

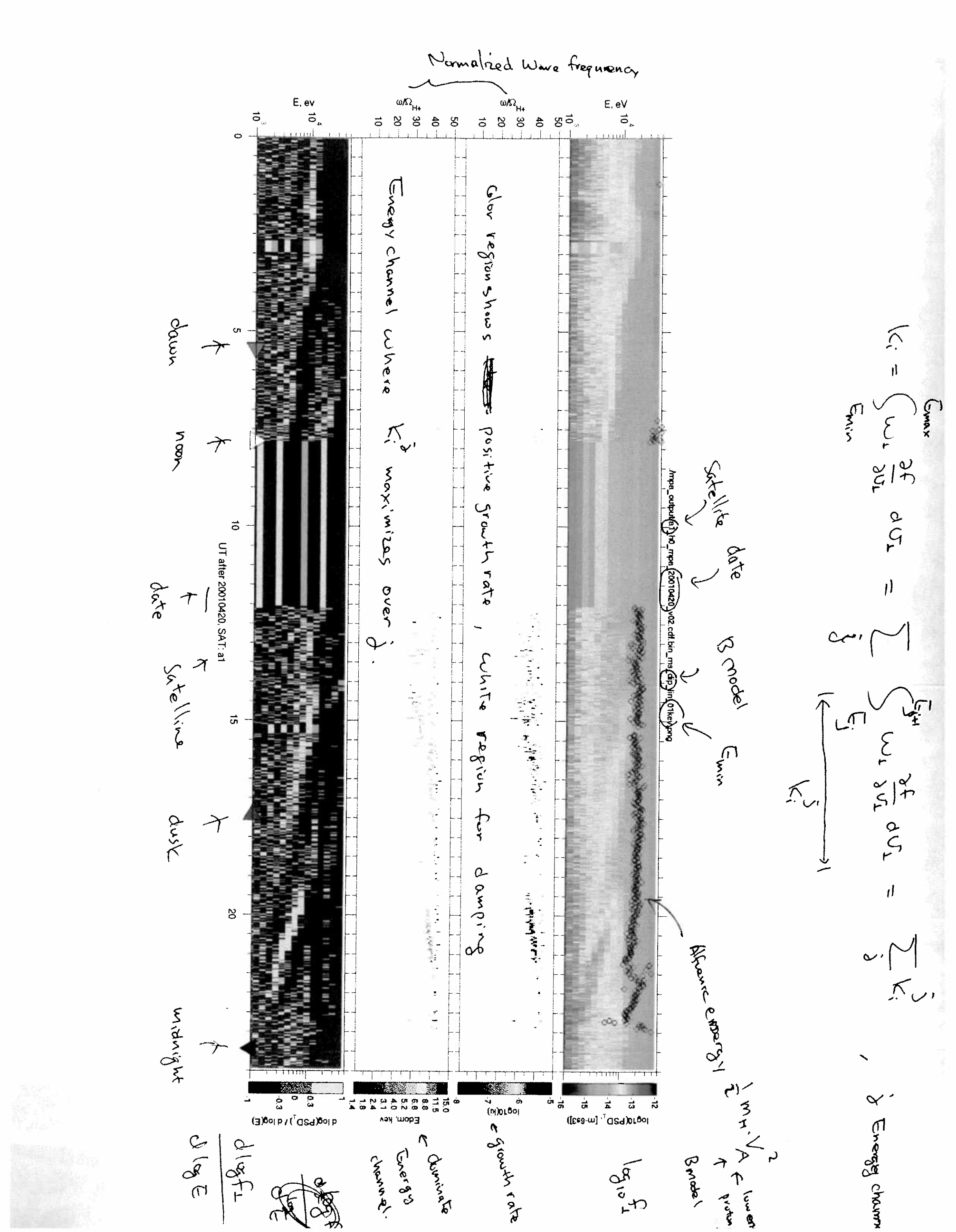

Figure 2. Description of figure format