Numerically Delicate Aspects of Splitting Decisions Made in Ocean Modeling Codes

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Motivation

- Design a code which is guaranteed to produce an accurate solution even when time step sizes are set close to the limits of computational stability:
  - **leave no unexplained time-step restrictions;**
- Mode splitting is motivated by time-scale separation between the external gravity and the fastest internal wave, Simons (1974); Madala & Piacsek (1977); Berntsen, Kowalik, Sælid,& Sørli (1981); Blumberg & Mellor (1987); Bleck & Smith (1990); Killworth et al (1991); Dukowicz & Smith (1994).
- Stability of implicit free-surface relies on some degree of backward-Euler stepping for fast variables.
- Theoretical studies of stability of split-explicit codes are performed in the framework of linearized system over flat bottom, Higdon & Bennett (1996); Higdon & de Szoeke (1997); Hallberg (1997); Higdon (1999, 2002, 2005, 2008); Kamenkovich & Nechaev (2009).
- Virtually all literature dealing with mode-splitting in ocean modeling concentrates on splitting pressure gradient terms.
- Well understood at the present time.
- Mode splitting brings the necessity of making **secondary splitting decisions** – what to do about vertically integrated advection, Coriolis, lateral viscosity, bottom drag, necessity to linearize the stiff part of the barotropic mode in the case of implicit free surface (possible loss of conservation). These are dealt with on case-by-case basis individually for each model, sometimes *ad hoc*.

Motivation examples

- Rueda, Sanmiguel-Rojas, & Hodges (2007) examined TRIM core of Casulli & Cheng (1992) and found it *unconditionally unstable* with respect to baroclinic internal waves. The code is 2-time-level using upstream-biased advection for everything. Stability of advection *alone* does not guarantee stability of the whole model. Rueda et al propose to redesign time stepping.

- Morel, Baraille, & Pichon (2008) examined HYCOM code and found it *unconditionally unstable* with respect to the way how the vertically integrated advection and Coriolis are dealt with – essentially due to effectively forward-in-time, centered-in-space treatment resulting from the particular splitting algorithm (LF main stepping, linearized BM going from n to $n+1$). Their proposed remedy is to keep BM linear, while running it from $n-1$ to $n+1$, thus mimicking the main LF stepping, but doubling the cost of BM.

- Marsaleix, Auclair, Floor, Herrmann, Estournel, Pairaud, & Ulses (2008) proposed doing the same in their *Symphonie* model (POM-based) on theoretical grounds arguing for energetic consistency. ...Seem to abandon the idea since then.

- Mainstream HYCOM ? [Morel et al (2008) is cited only 4 times to date]

- reliance on smallness $\overline{(\overline{u}+u')\,(\overline{v}+v')} = \overline{u}\,\overline{v} + \overline{u'v'}$ where $\|\overline{u}\,\overline{v}\| \ll \left\|\overline{u'v'}\right\|$

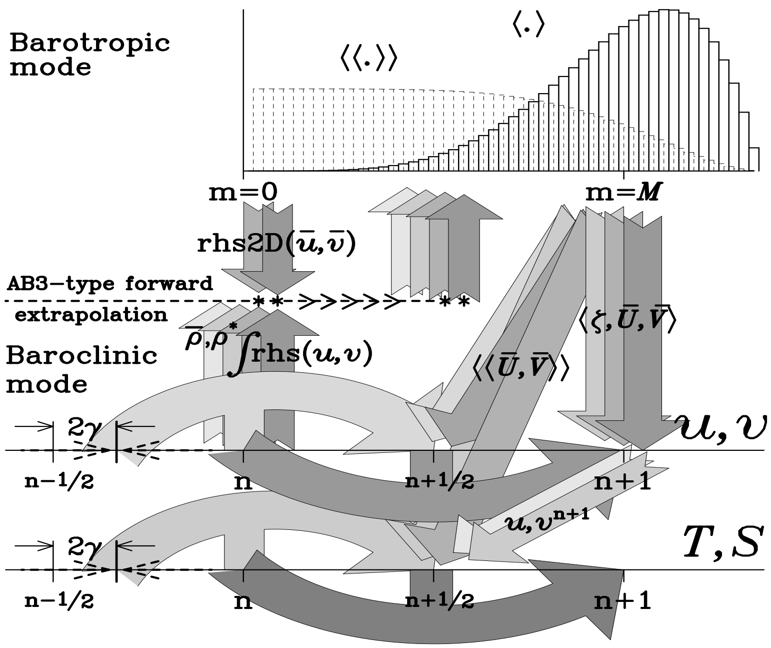
- Same applies to split-explicit versions of MOM with LF main stepping (i.e., prior to MOM 4p1)

Motivation, continued...

- POM and ROMS (all variants) are stable with respect to this matter because of recomputing vertically-integrated Coriolis and (at least partially) advection terms at every time step.
- *Partially* because ROMS uses a third-order upstream-biased, QUICK-type (but not QUICKEST) advection for 3D momentum equations, which is not practical to recompute at every barotropic step, so a centered scheme is used in BM. QUICK (unlike QUICKEST) is not compatible with forward stepping, and is subject to *flux-splitting instability*, Leonard, McVean, Lock (1996) – the time-stepping requirements are similar to that for a centered scheme.
- This, however, is not the most efficient way.
- Need a test problem with non-trivial barotropic contribution to illustrate whether it matters

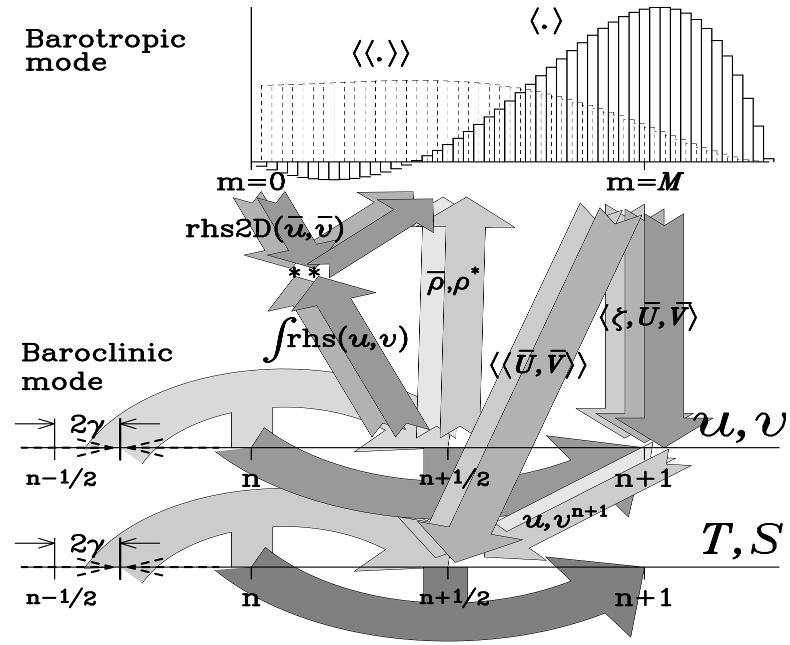
Two variants ROMS mode coupling in ROMS

old UCLA AGRIF Rutgers (uses same coupling, but different time stepping)



- BM during predictor stage of main step
- AB3 forward extrapolation for 3D→2D forcing r.h.s. terms
- **flexible:** can be modified to skip Coriolis, advection, and viscous terms in BM
- **hard to implement implicit bottom drag**

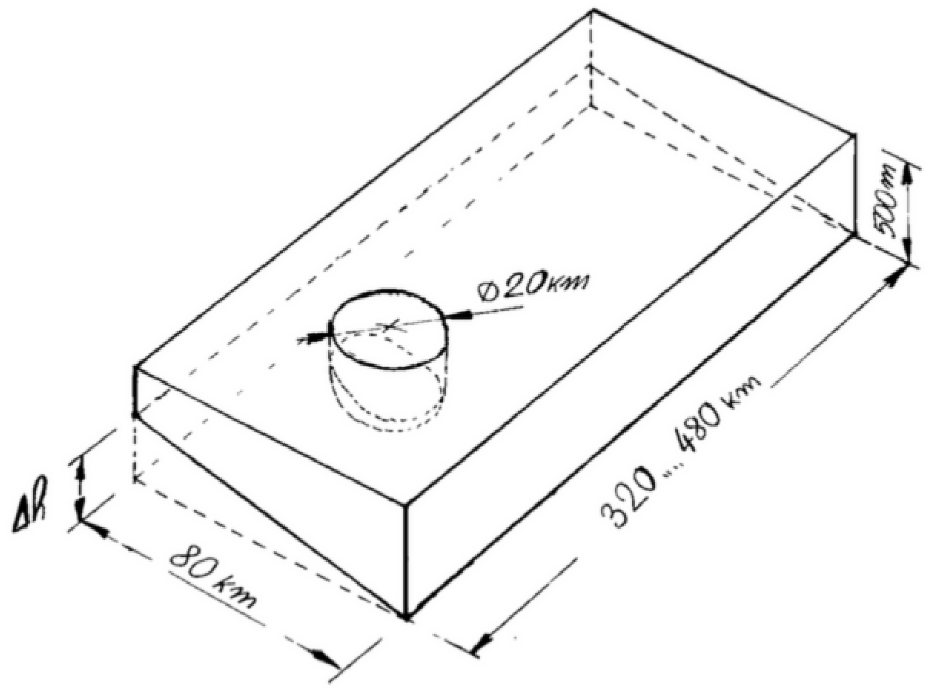
UCLA (current, also 2005 paper)



- BM during corrector stage
- 3D→2D forcing r.h.s. terms already centered at $n + 1/2$
- **not so flexible, but leaves possibilities**
- **compatible with implicit bottom drag**

Test problem configuration: Barotropic island wake over topographic slope

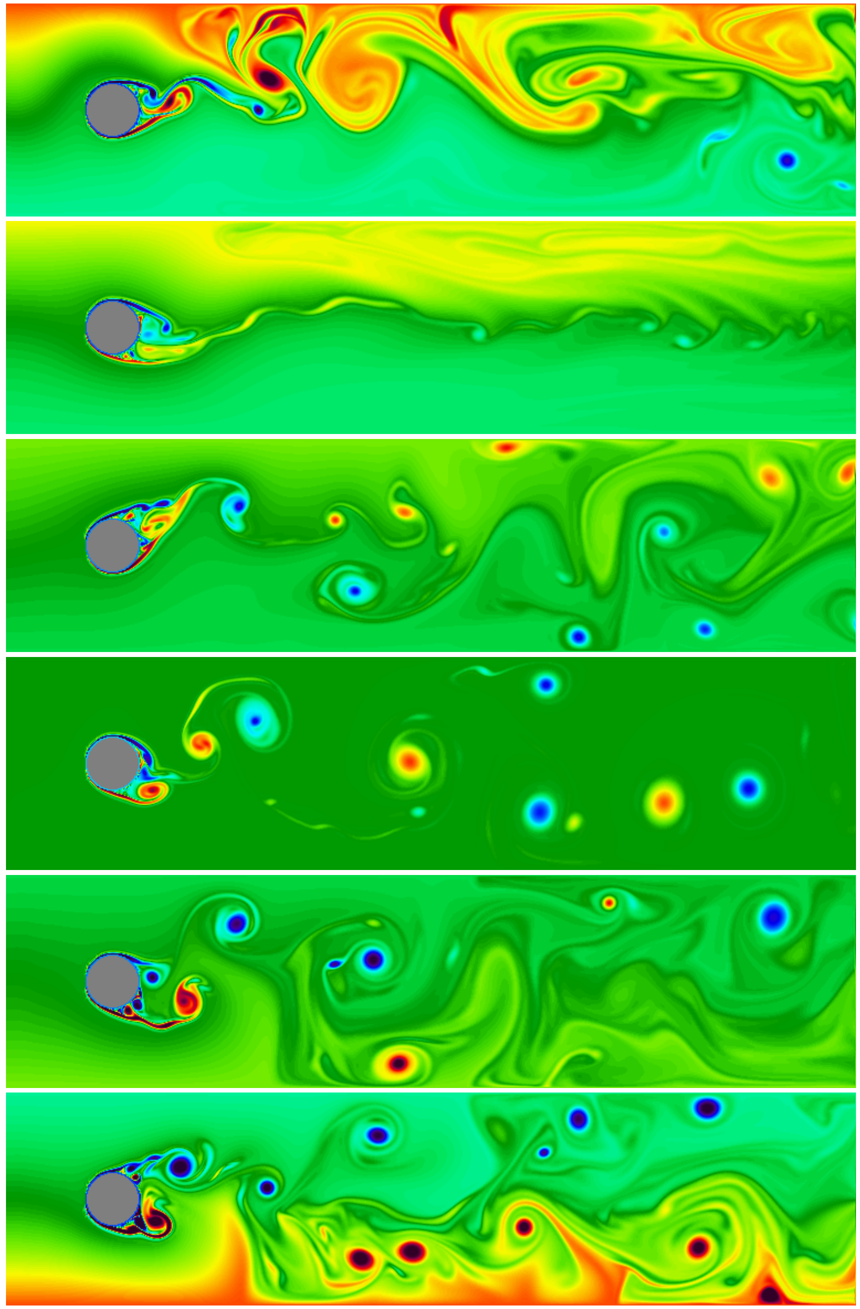
- Barotropic (non-stratified)
- **Solved as a 3D problem in ROMS**
- domain width $80km$
  length 320...$480km$
  $h_{\text{max}} = 500m$,
  variable $\Delta h = 150...350m$
- circular island $20km$
- f-plane, $f = 10^{-4}s^{-1}$
- uniform inflow, $u_{\text{in}} = 15cm/s$
- open at inflow and outflow sides
  Flather(characteristic) for $\overline{u}$ and ζ;
  radiation(advective) b.c. $\overline{v}$
- free-slip at side walls
  (channel configuration)
- no-slip b.c. at the island
- free-slip bottom
- no explicit dissipation
- grid resolution $\Delta x = \Delta y = 416m$,
  most cases, down to $208m$ in few



Topographic beta-effect $\beta = \dfrac{f}{h} \cdot \dfrac{\partial h}{\partial y} \sim 10^{-9}m^{-1}s^{-1}$ vs. planetary $\beta = \sim 10^{-11}m^{-1}s^{-1}$

Estimate $\dfrac{1}{Fr} = \dfrac{\sqrt{gh}}{u_{\text{in}}} \sim 450 \quad \rightarrow$ max splitting ratio $M \sim \dfrac{1}{2\ldots 3} \cdot \dfrac{0.71}{0.87} \cdot 450 \sim 150$

Barotropic wake over slopping bottom, showing $BPV = \frac{f + \nabla \times \mathbf{u}}{h + \zeta}$



$\Delta h = +335m$
slope=0.42%
(positive β-effect)

$\Delta h = +275m$
slope=0.34%

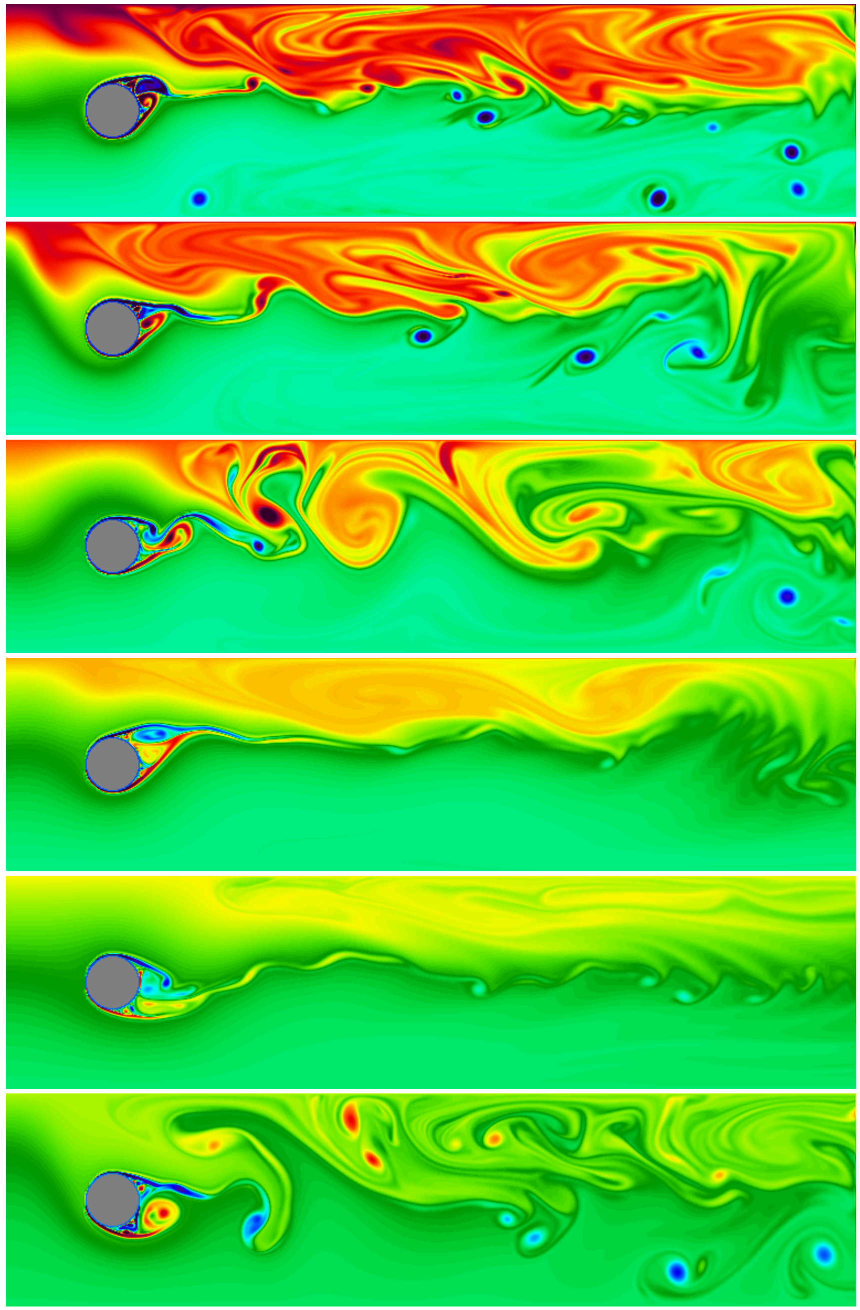
$\Delta h = +200m$
slope=0.25%

flat bottom

$\Delta h = -200m$
slope=-0.25%

$\Delta h = -335m$
slope=-0.42%
(negative β-effect)

positive β-effect with finer increment of topographic slope



$\Delta h = 386m$
slope=0.48%

$\Delta h = 361m$
slope=0.450%

$\Delta h = 335m$
slope=0.416%

$\Delta h = 306m$
slope=0.380%

$\Delta h = 274m$
slope=0.341%

$\Delta h = 239m$
slope=0.293%

$\Delta h = +335m$, **positive β-effect**, showing BPV, time in days

slope=0.416%, **the most unstable flow regime**

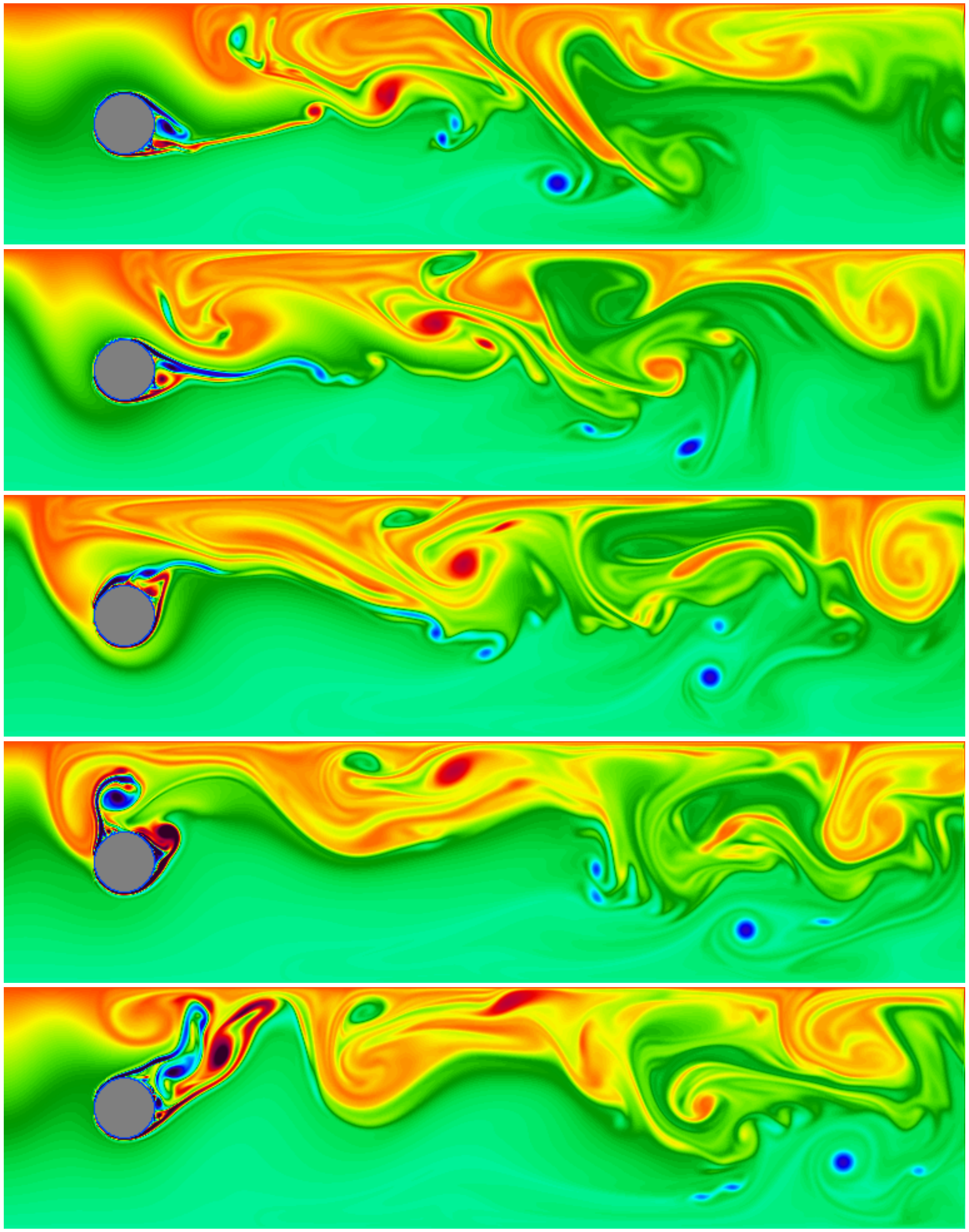
$t = 80.5$

$t = 82.5$

$t = 84.5$

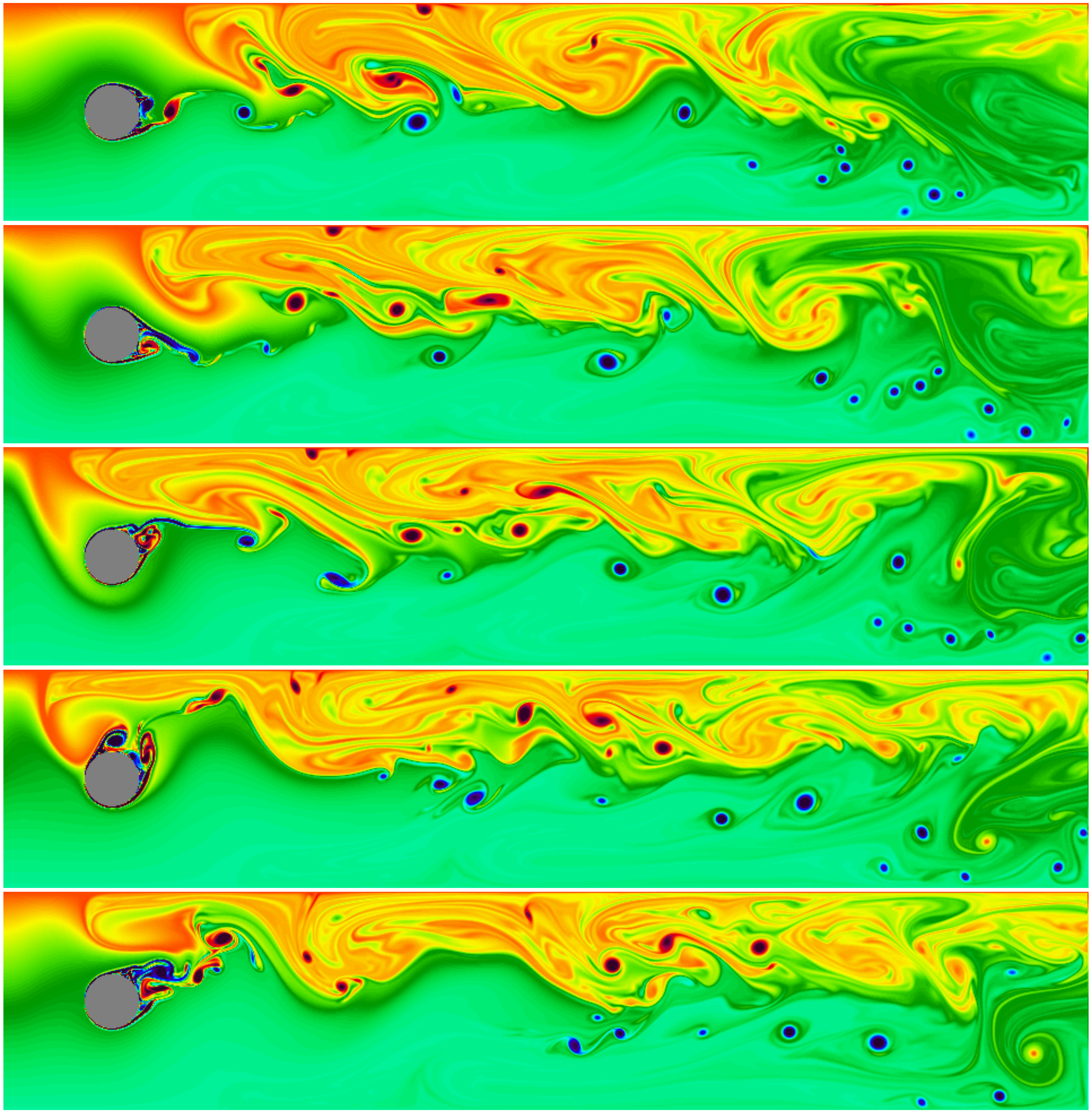
$t = 86.5$

$t = 88.5$



$\Delta h = +335m$, **positive β-effect**, high resolution, time in days

slope=0.416%, **the most unstable flow regime**



$t = 90.0$

$t = 92.5$

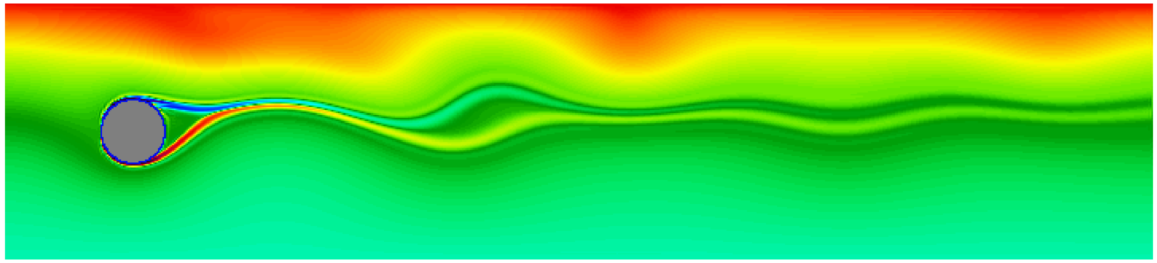
$t = 94.875$

$t = 96.750$

$t = 99.125$

low res.
slope=0.40%

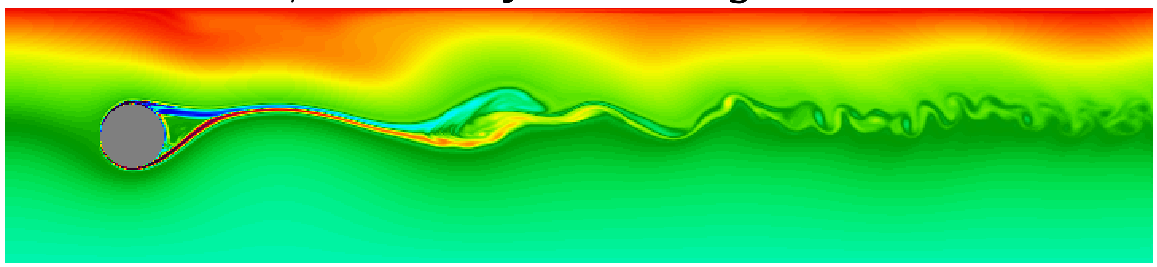
stationary and stable



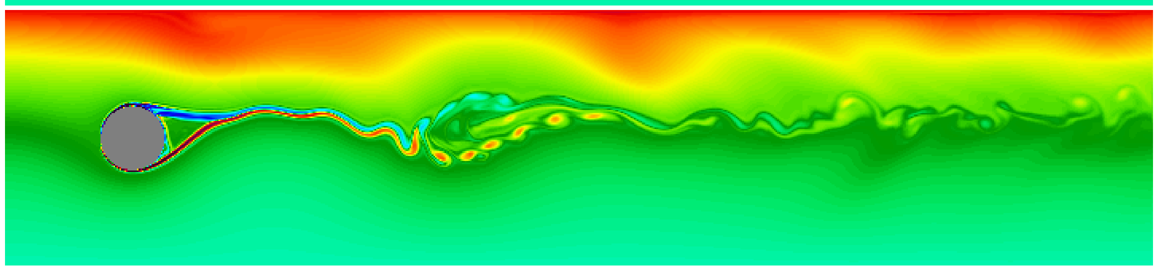
high res.: chaotic behavior, randomly switching between two states

same slope

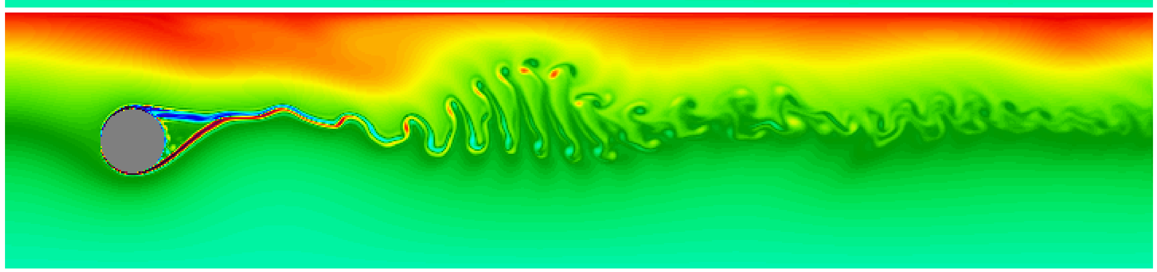
max stable state



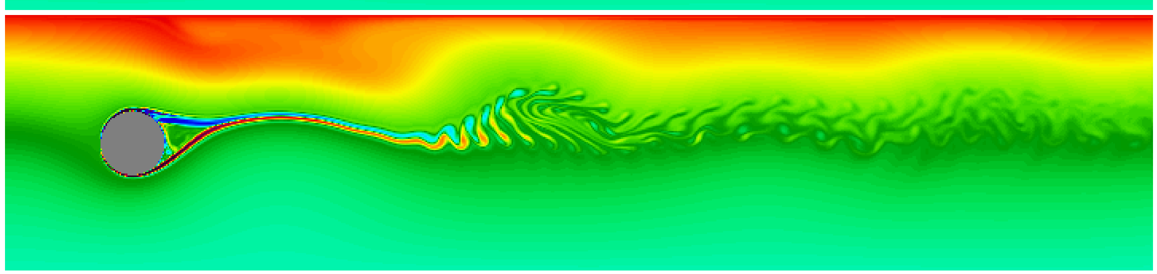
transient toward unstable



max unstable



transient toward stable



need proper amount of dissipation to achieve this regime

Illustration of consequences of inaccurate splitting

- examining different failure modes under different flow regimes

flat bottom, **stable splitting**, showing BPV

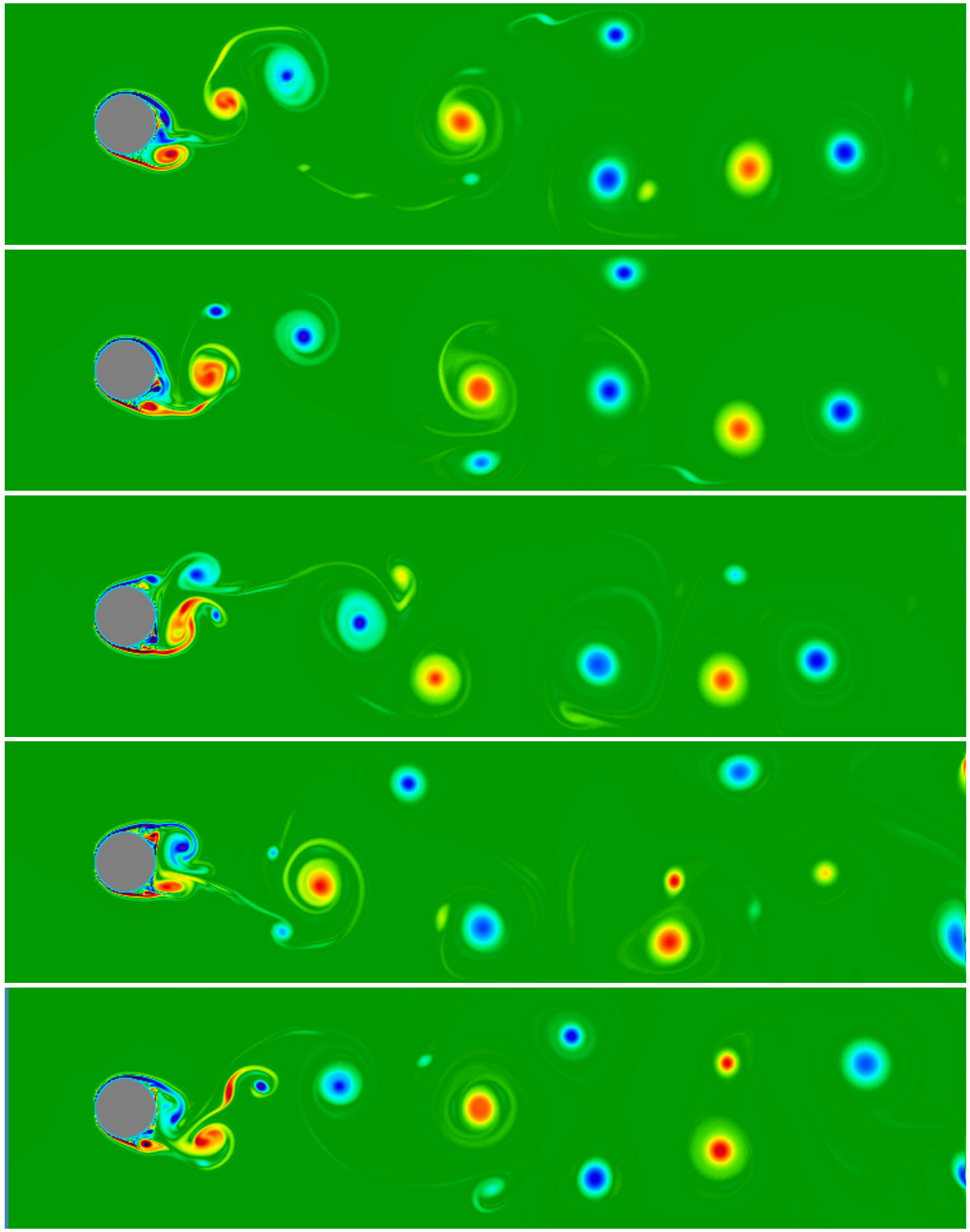
$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 240s$
$M = 66$

$\Delta t = 360s$
$M = 99$

$\Delta t = 432s$
$M = 119$



decorrelation after 30-day run starting from common initial

flat bottom, **unstable splitting**, showing BPV

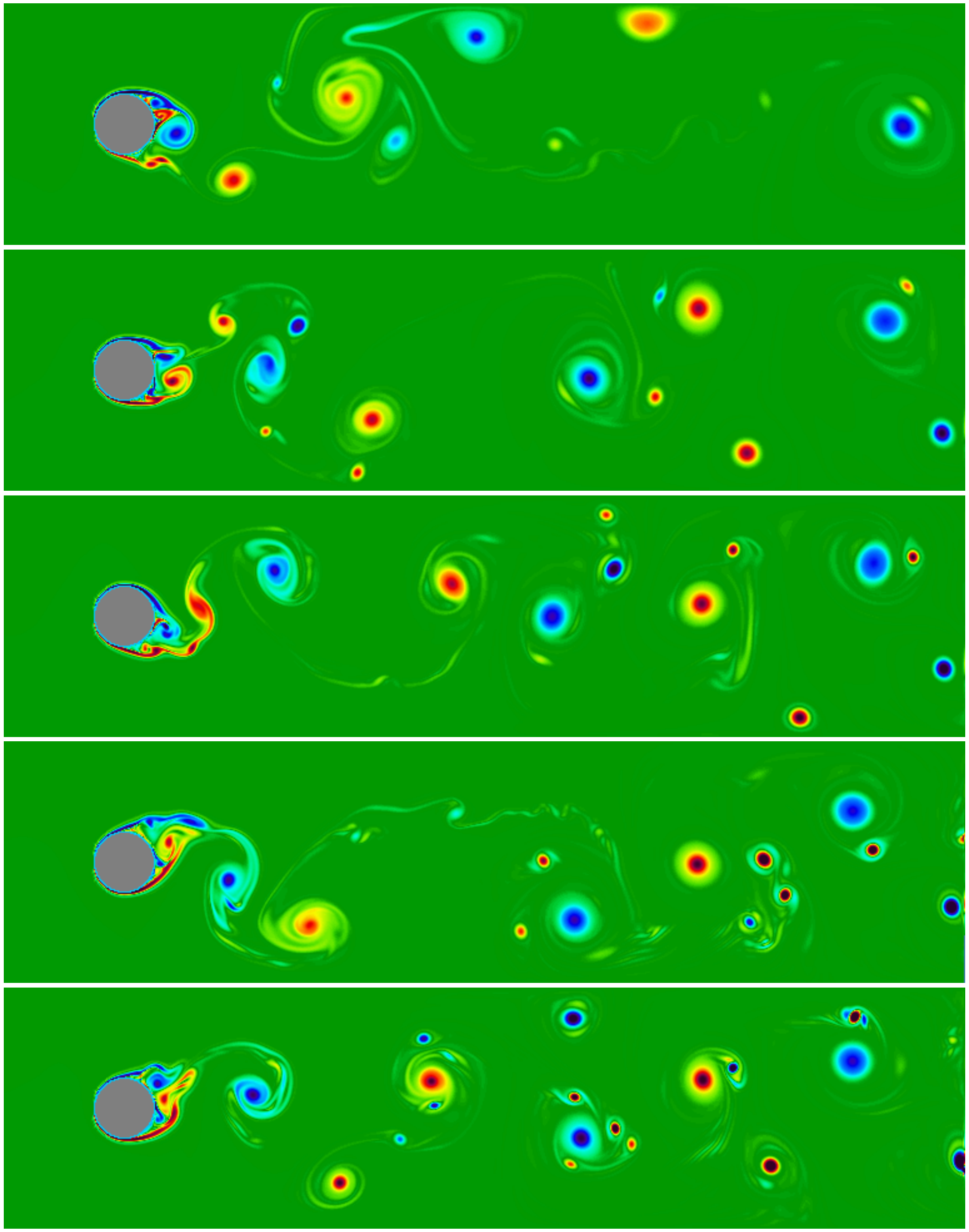
$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 150s$
$M = 41$

$\Delta t = 180s$
$M = 50$

$\Delta t = 200s$
$M = 55$



non-physical growth of existing extrema, but *little* dispersive errors

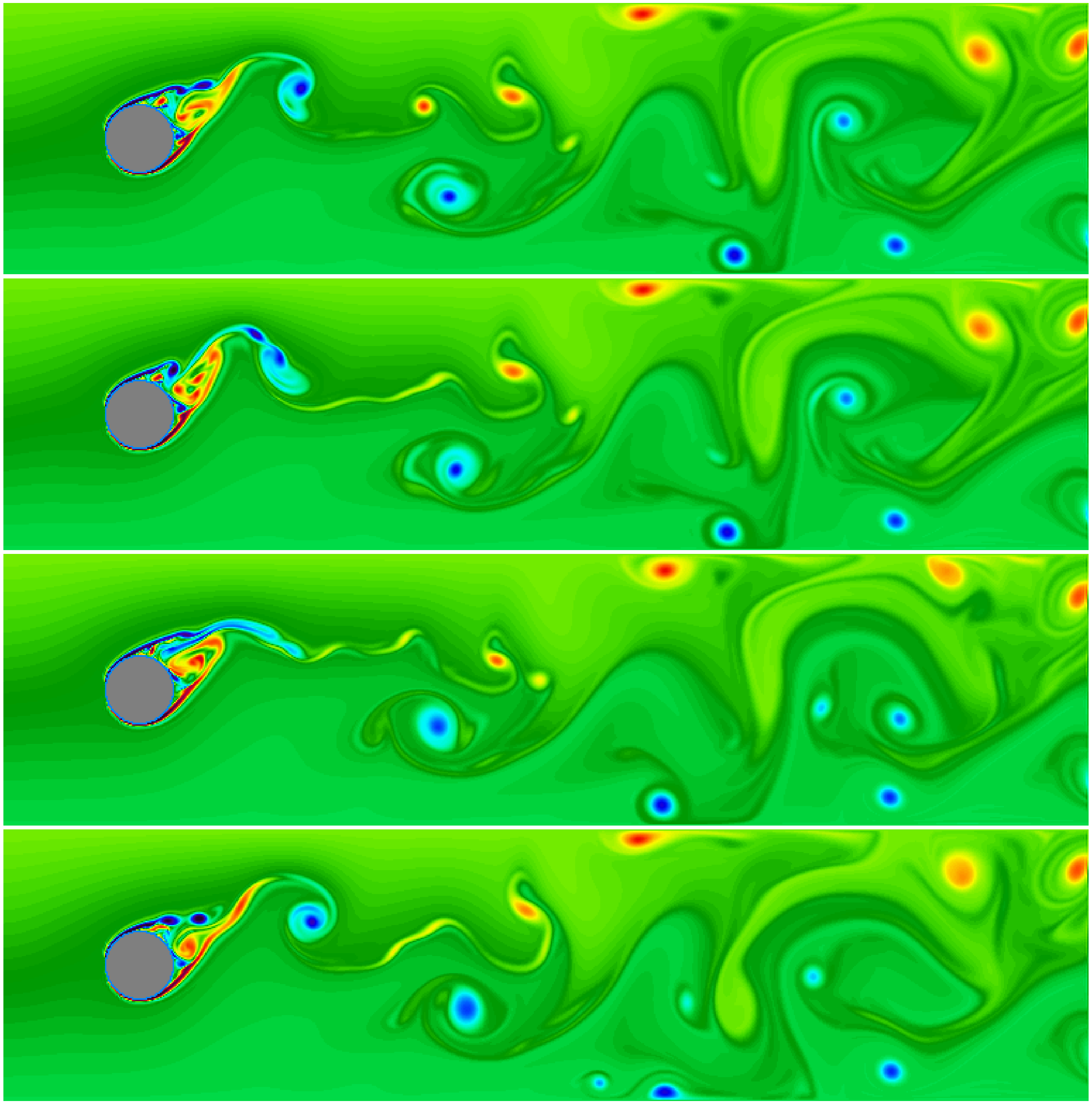
$\Delta h = 200$ (positive β-effect), **stable splitting,** showing BPV

$\Delta t = 60s$
$M = 17$

$\Delta t = 240s$
$M = 66$

$\Delta t = 432s$
$M = 119$

$\Delta t = 491s$
$M = 135$



$\Delta t = 540s$ is computationally unstable

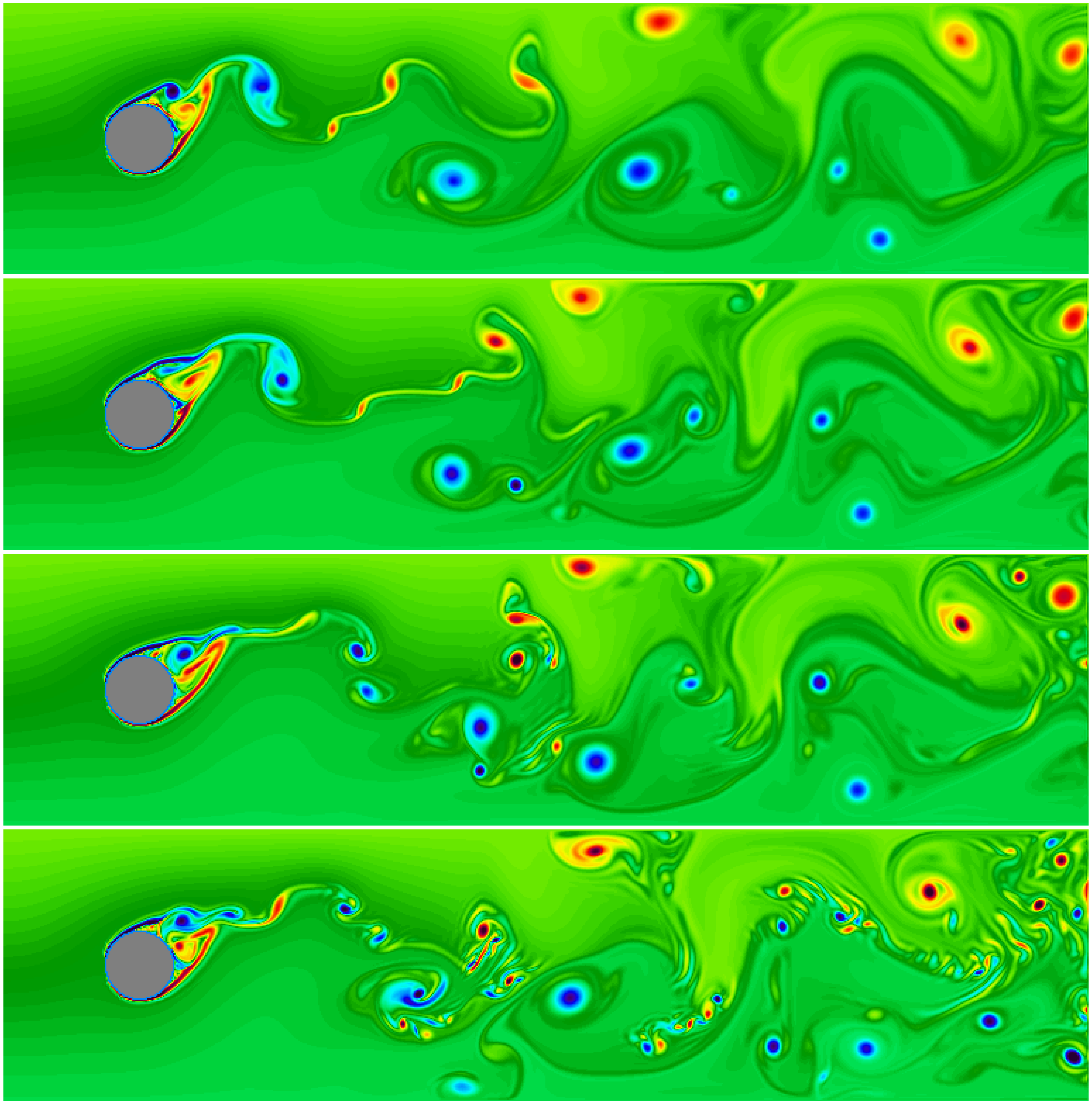
$\Delta h = 200$ (positive β-effect), **unstable splitting,** showing BPV

$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

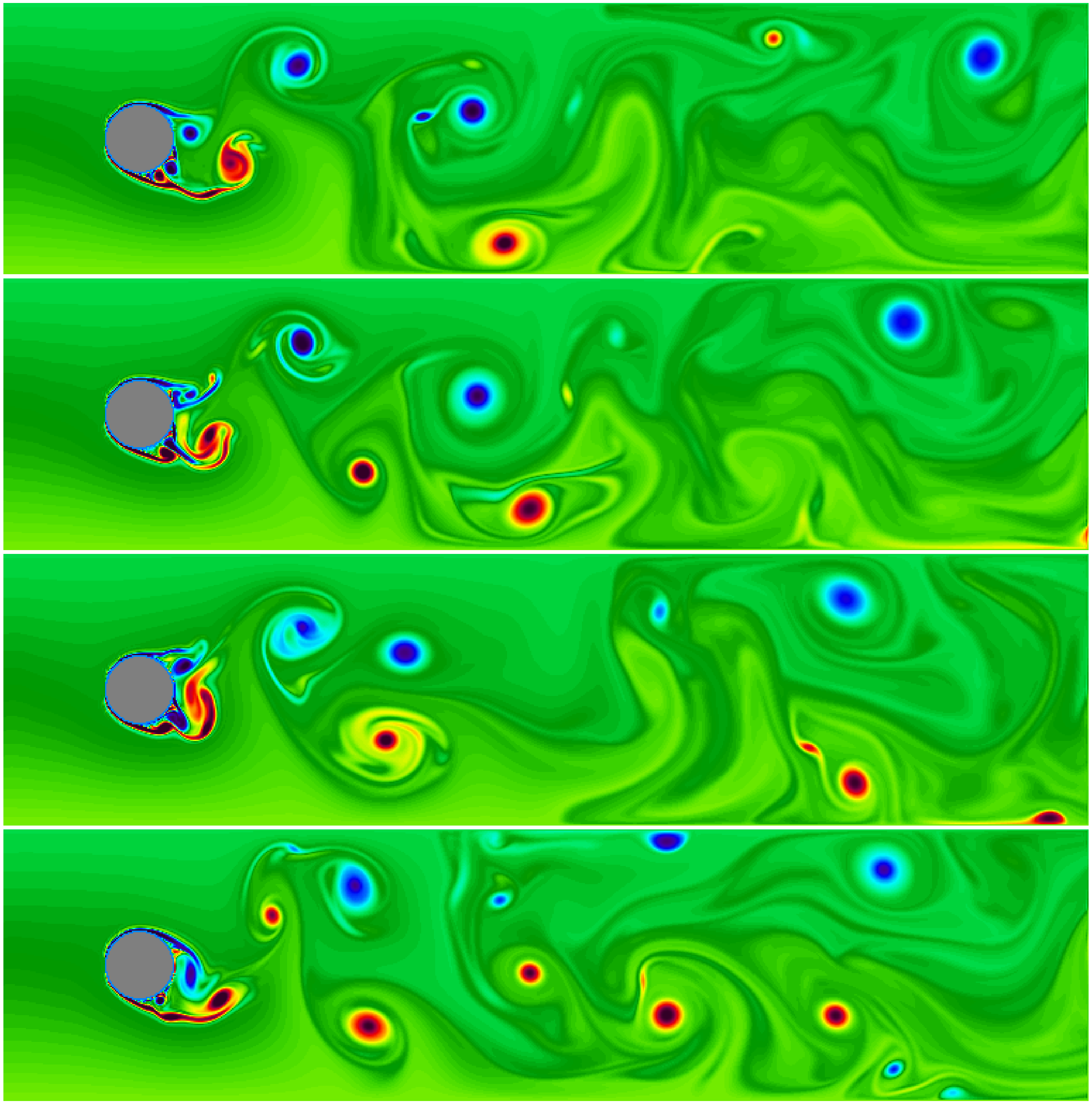
$\Delta t = 180s$
$M = 50$

$\Delta t = 200s$
$M = 55$



$\Delta t = 240s$ is computationally unstable

$\Delta h = -200$ (negative β-effect), **stable splitting,** showing BPV



$\Delta t = 60s$
$M = 17$

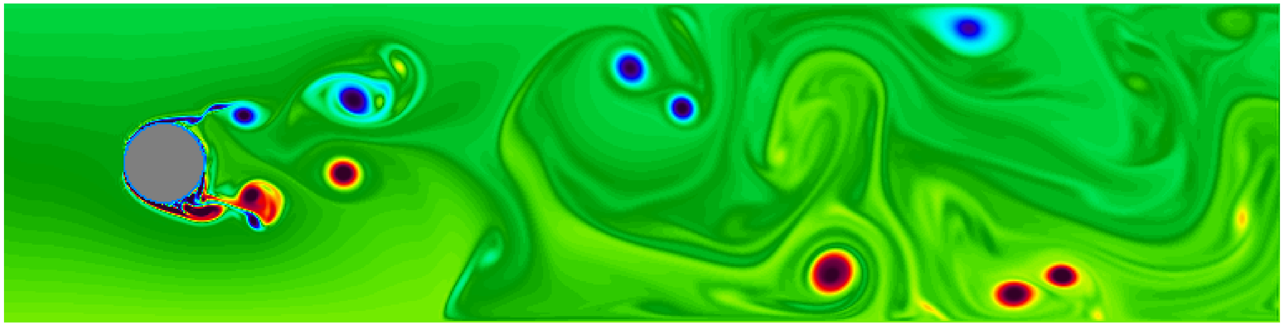
$\Delta t = 120s$
$M = 33$

$\Delta t = 240s$
$M = 66$

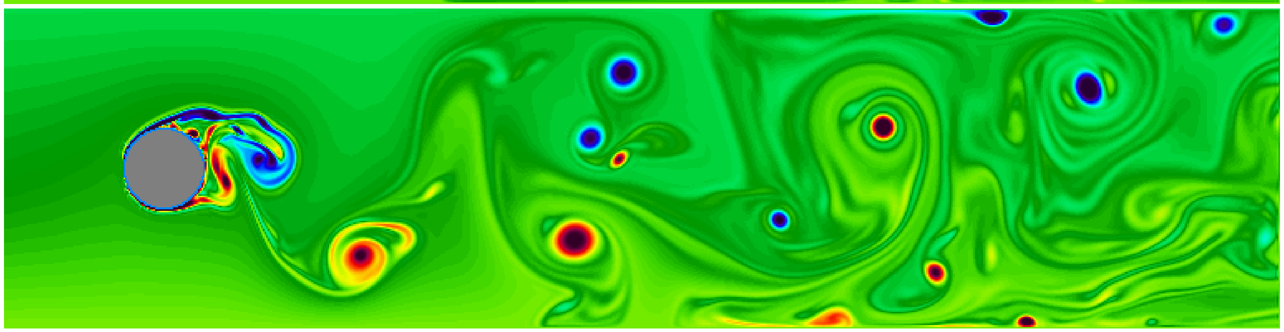
$\Delta t = 360s$
$M = 99$

$\Delta h = -200$ (negative β-effect), **unstable splitting,** showing BPV

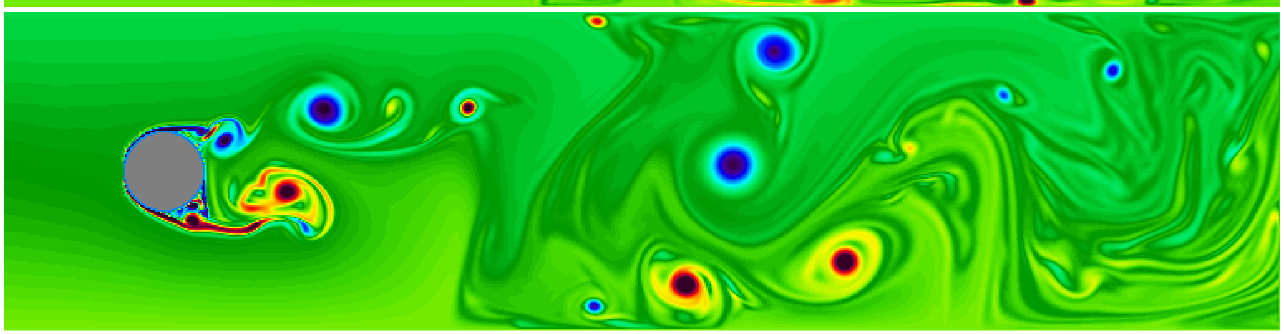
$\Delta t = 60s$
$M = 17$



$\Delta t = 120s$
$M = 33$



$\Delta t = 150s$
$M = 41$



mostly numerical dispersion $\Rightarrow$ accumulation of small-scales

$\Delta t = 180s$ is computationally unstable

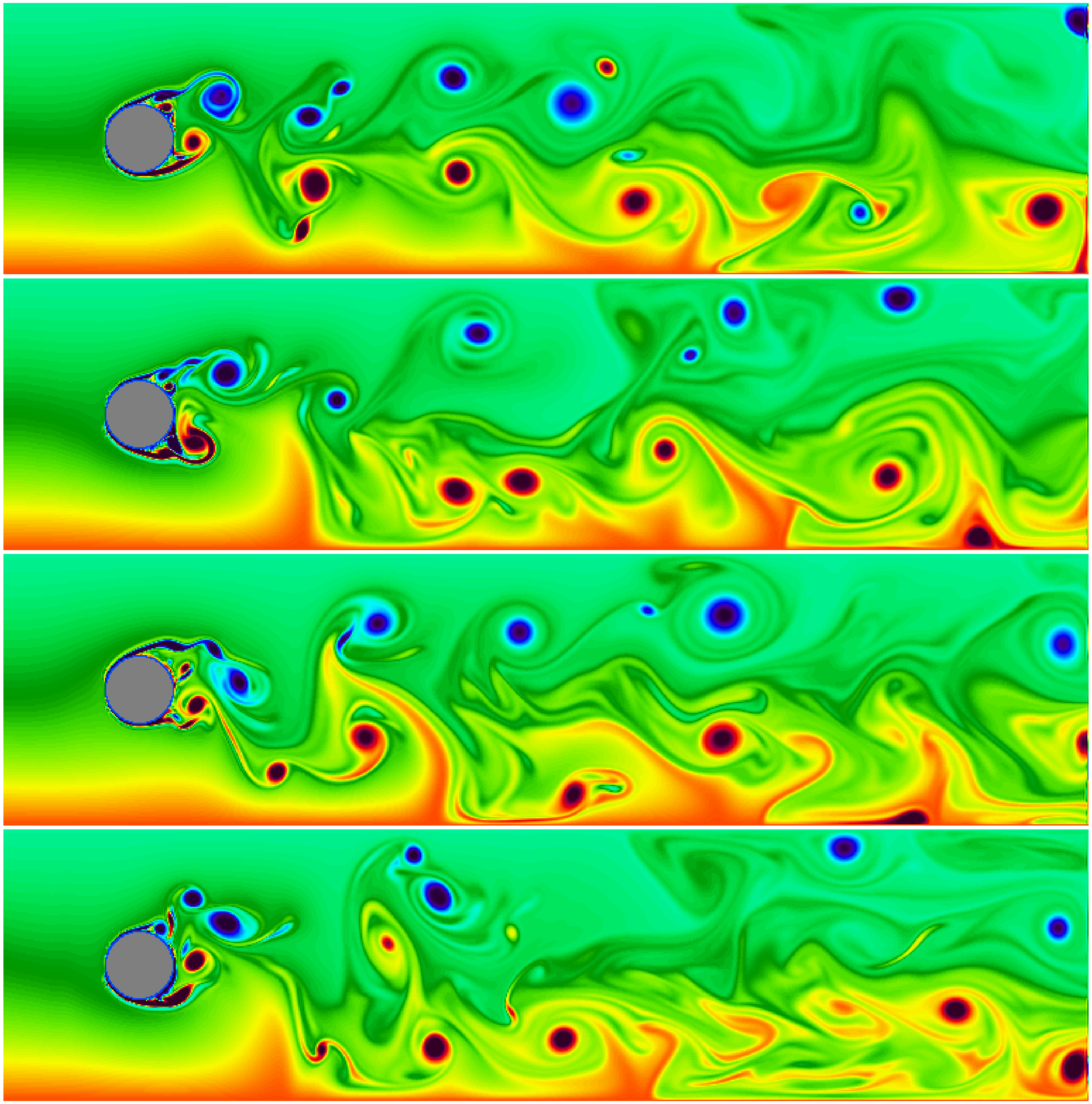
$\Delta h = -335m$ (negative β-effect), **stable splitting,** showing BPV

$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 240s$
$M = 66$

$\Delta t = 300s$
$M = 82$

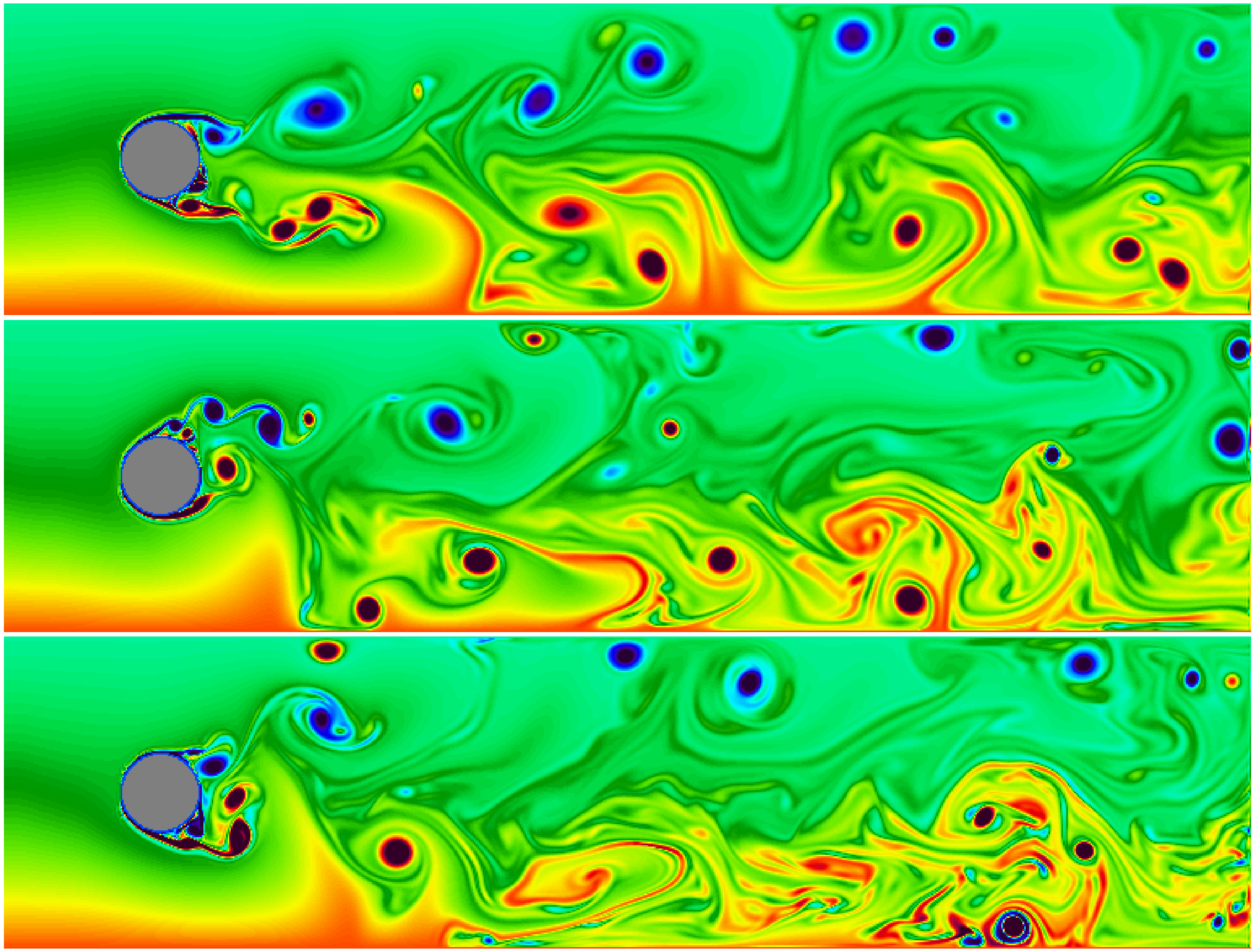


$\Delta h = -335m$ (negative β-effect), **unstable splitting,** showing BPV

$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

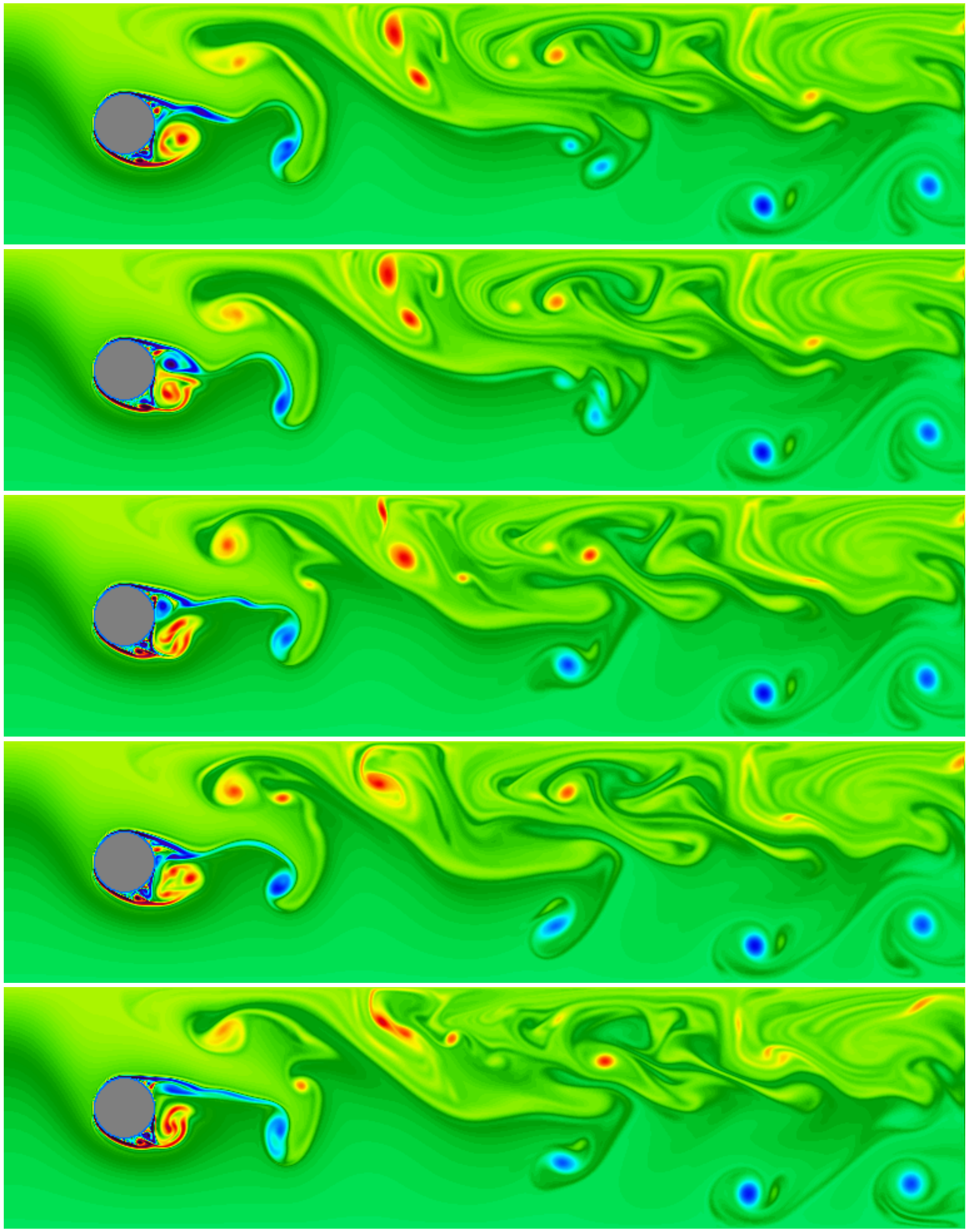
$\Delta t = 150s$
$M = 41$



$\Delta t = 150s$ is already on the limit of computational stability

More restrictive than $\Delta h = -200m$

$\Delta h = +239m$ (positive β-effect), **stable splitting**



$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 240s$
$M = 66$

$\Delta t = 360s$
$M = 99$

$\Delta t = 432s$
$M = 119$

decorrelation of small scales only

$\Delta h = +239m$ (positive β-effect), **unstable splitting**

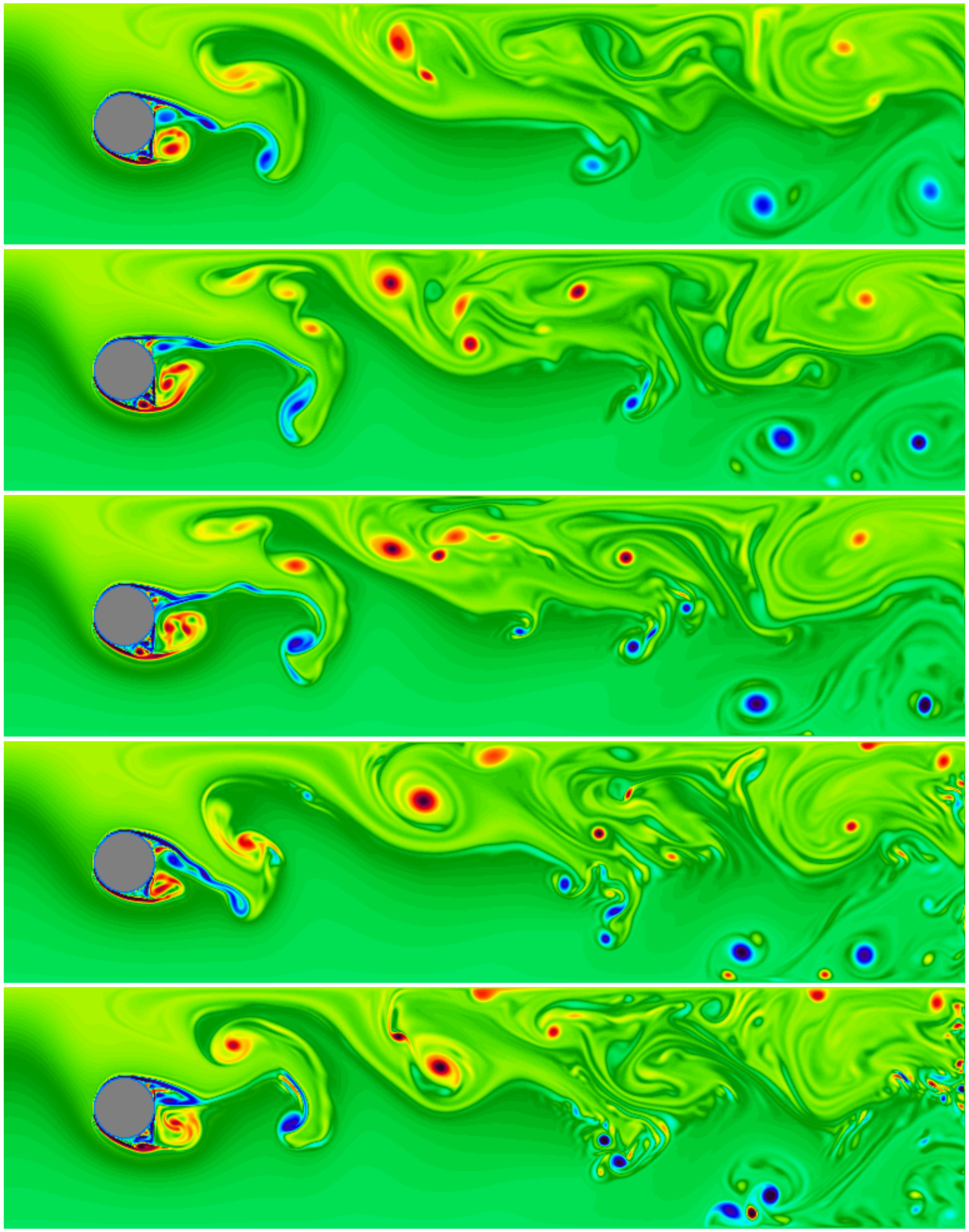
$\Delta t = 30s$
$M = 9$

$\Delta t = 120s$
$M = 33$

$\Delta t = 150s$
$M = 41$

$\Delta t = 180s$
$M = 50$

$\Delta t = 200s$
$M = 55$



amplification of existing vortices and sharpening of fronts

$\Delta h = +275$ (positive β-effect), **stable splitting**, showing BPV

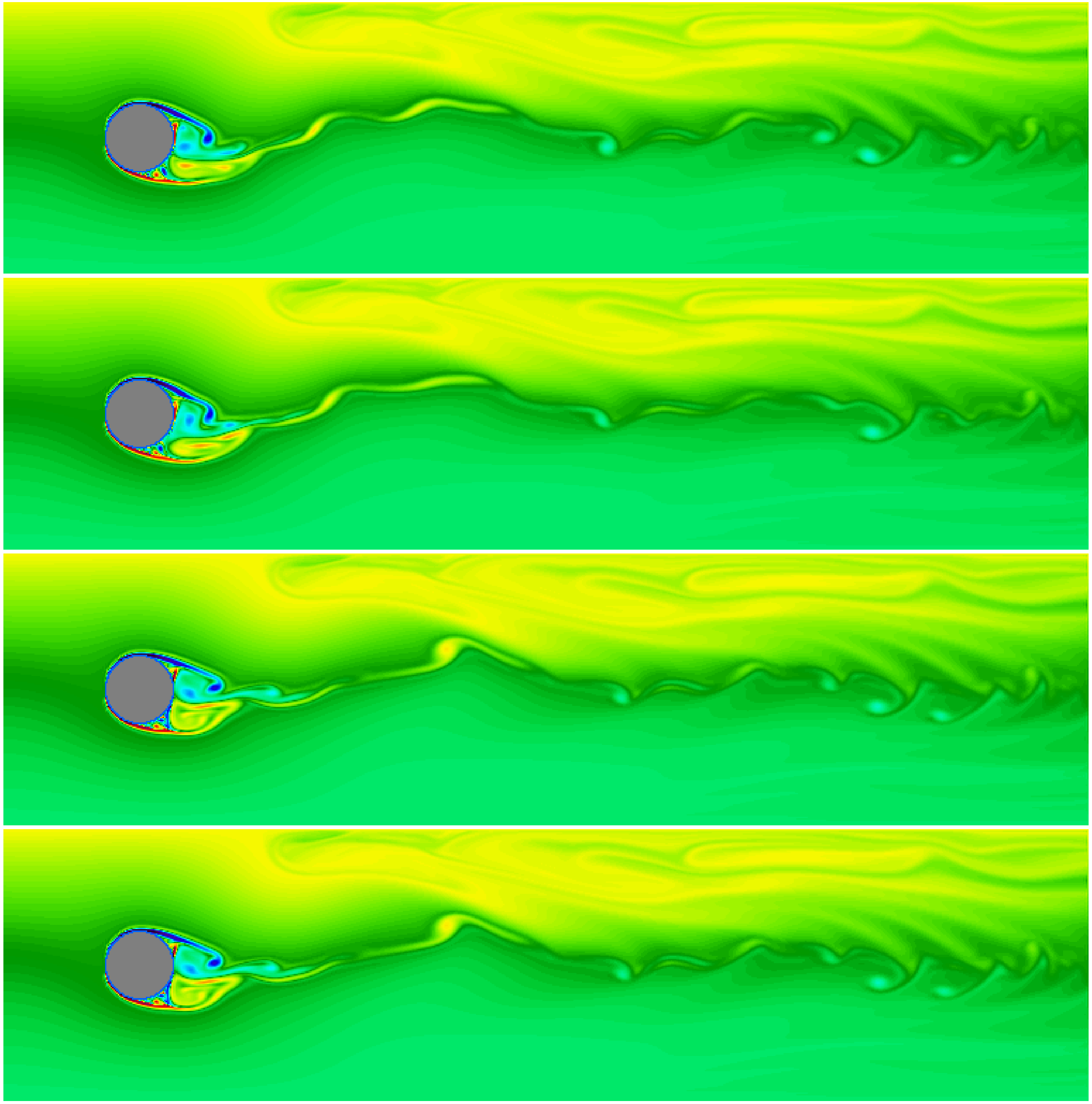
$\Delta t = 60s$
$M = 17$

$\Delta t = 432s$
$M = 119$

$\Delta t = 491s$
$M = 135$

$\Delta t = 540s$
$M = 148$

$Cu_{\mathrm{adv}} \approx 0.6$



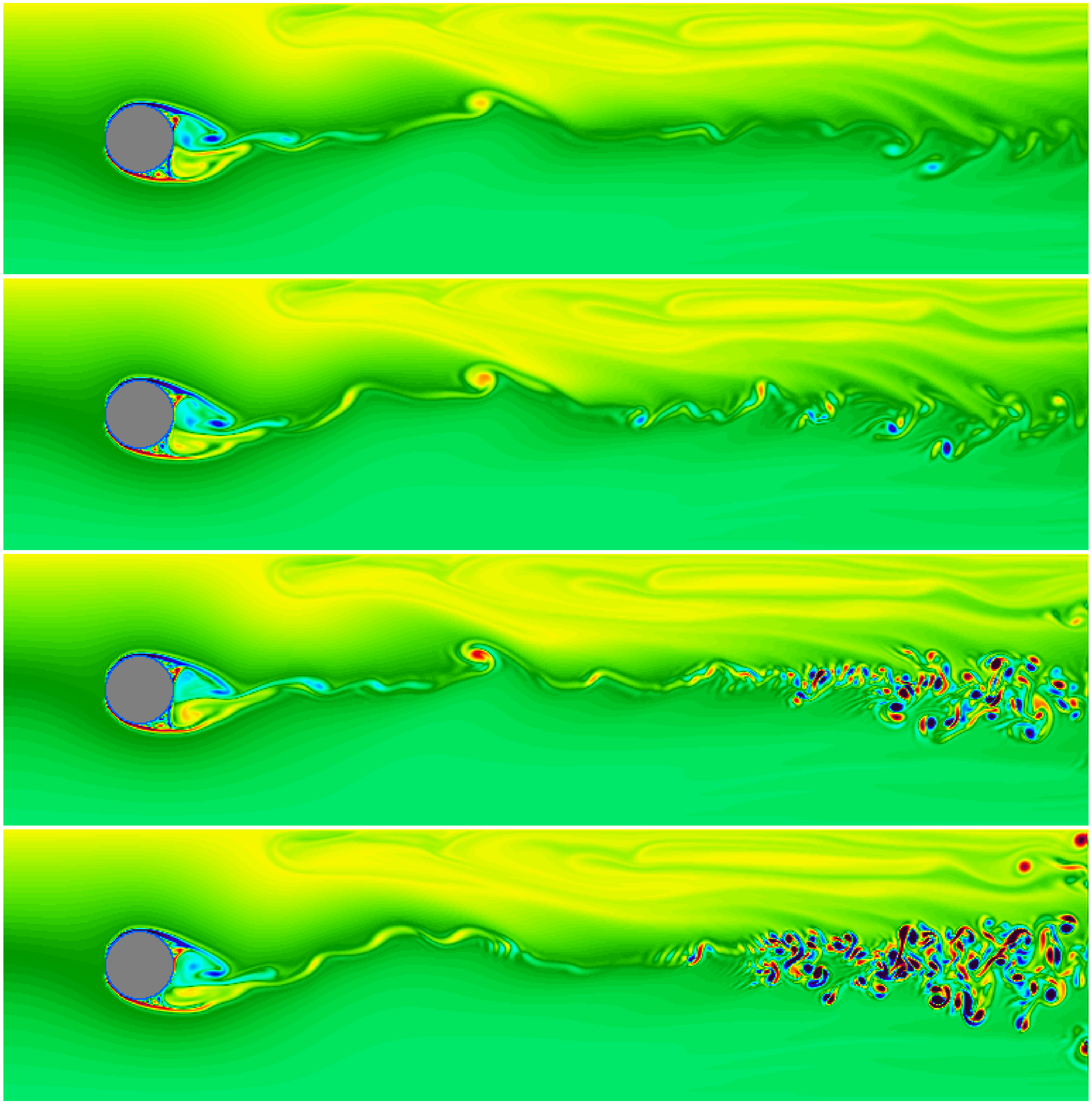
$\Delta h = +275$ (positive β-effect), **unstable splitting,** showing BPV

$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 180s$
$M = 50$

$\Delta t = 200s$
$M = 55$



$\Delta h = +335m$ (positive β-effect) **stable splitting**

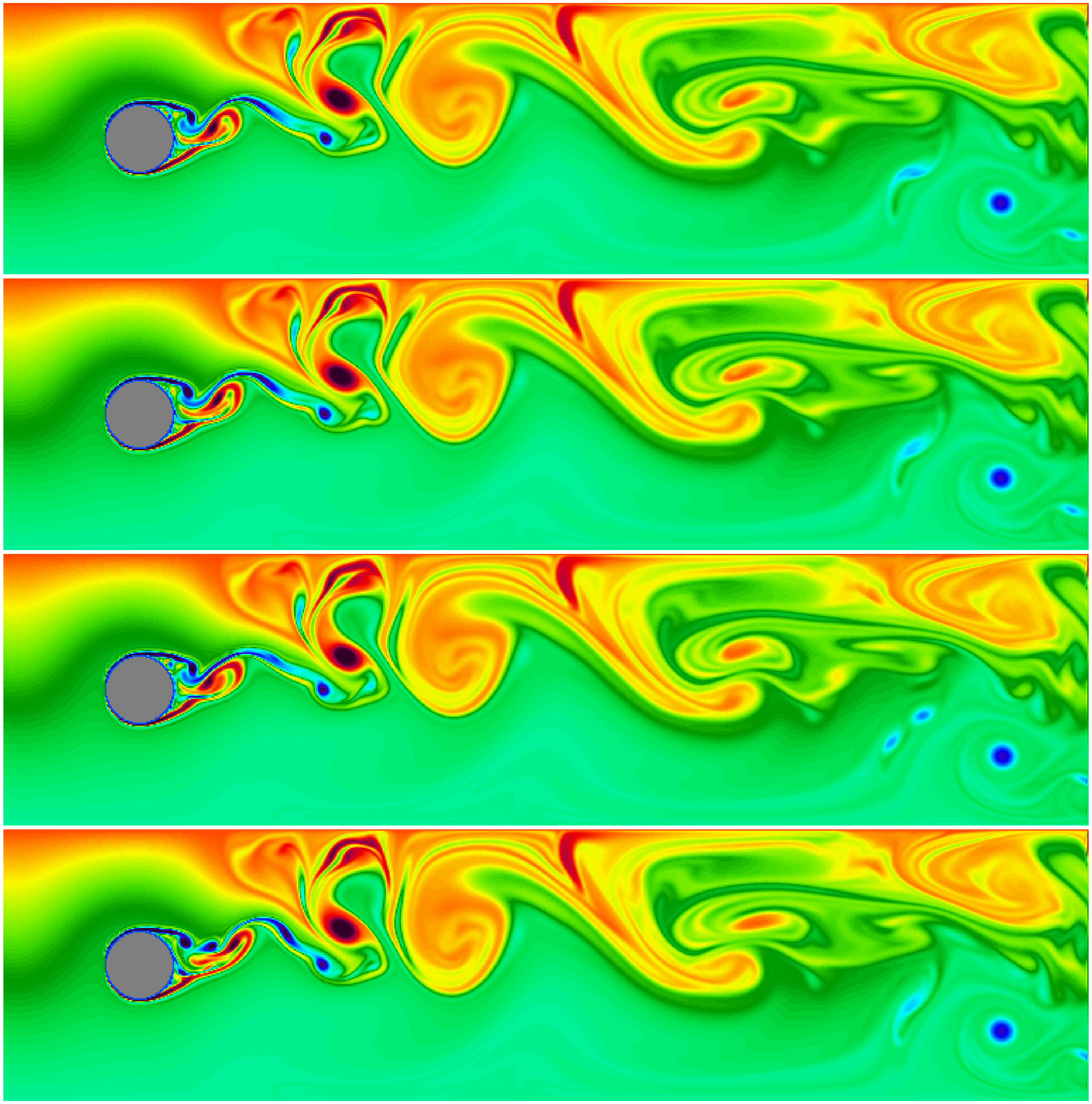
$\Delta t = 60s$
$M = 17$

$\Delta t = 120s$
$M = 33$

$\Delta t = 240s$
$M = 66$

$\Delta t = 300s$
$M = 82$

$C_{\text{adv}} \approx 0.6$

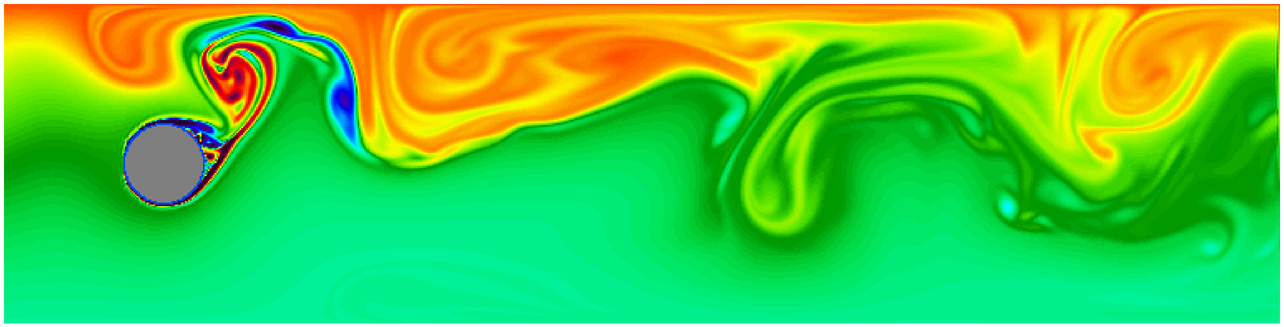


decorrelation of small scales only

$\Delta h = +335$ (positive β-effect) **stable splitting**

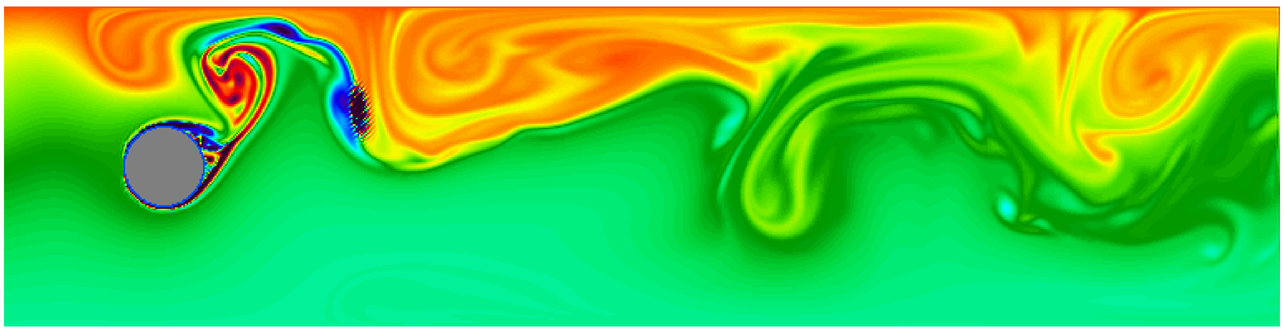
$\Delta t = 300s$
$M = 82$

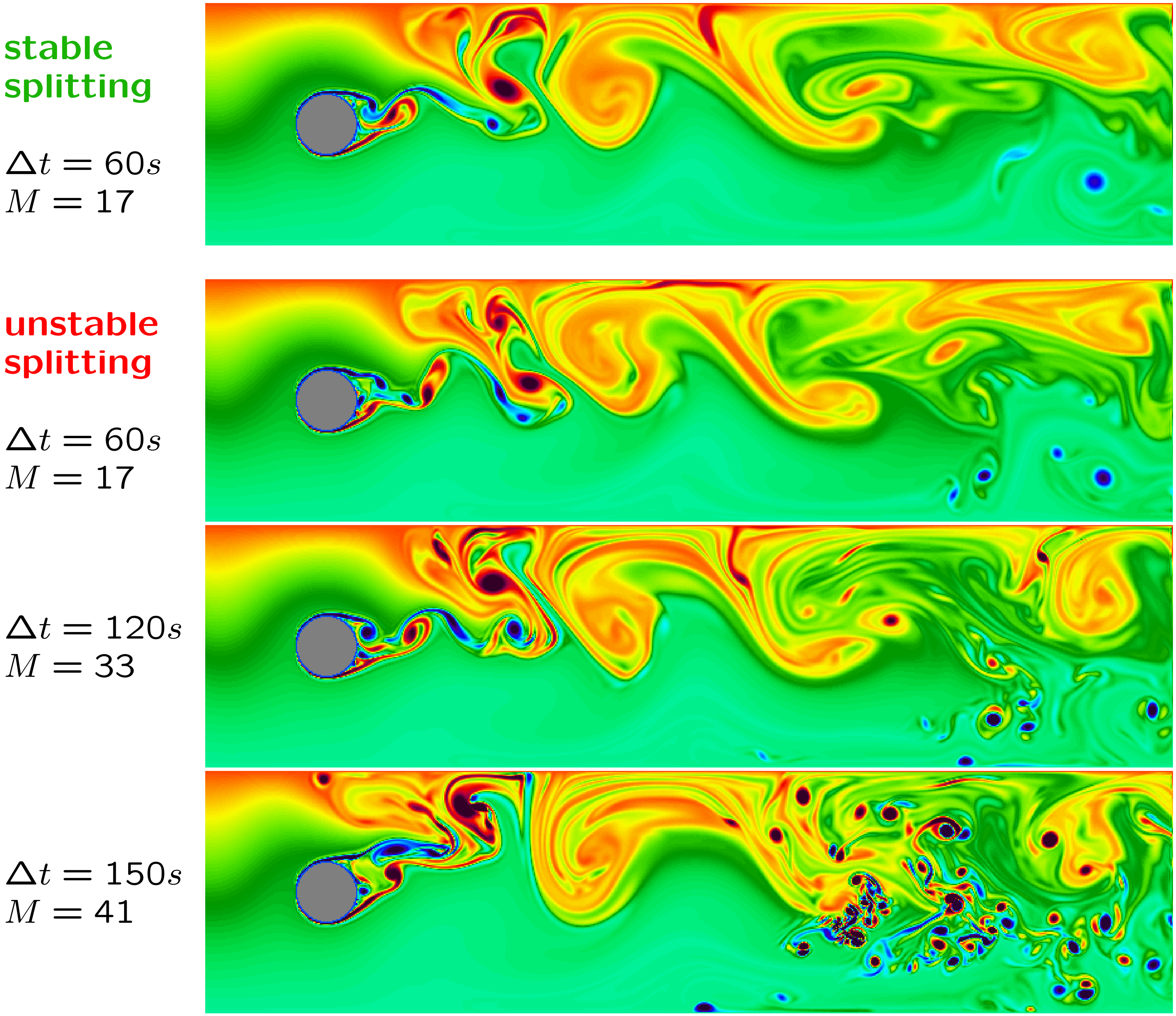
rec=130



$\Delta t = 360s$
$M = 99$
same rec

$C_{\text{adv}} \approx 0.75$




stable splitting
$\Delta t = 60s$
$M = 17$
unstable splitting
$\Delta t = 60s$
$M = 17$
$\Delta t = 120s$
$M = 33$
$\Delta t = 150s$
$M = 41$

Conclusion

- Splitting with Coriolis, advection, and lateral viscosity terms computed **solely within the 3D part** is possible and is accurate if properly done. This is the prefered way to go.

- Somewhat contradicts the long-established ROMS (also POM) practices.

- Under proper conditions unstable splitting causes non-physical behavior **long before the limit of computational stablity has been reached** resulting in the appearace of "gray zone" where the model is not accurate, but is still stable.