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Radiative Properties of Cirrus Clouds in NOAA 4 VTPR Channels: Some Explorations of Cloud Scenes from Satellites

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Abstract – A one-dimensional spectral infrared radiative transfer model has been developed for atmospheres containing cirrus clouds and absorbing gases above, below and within the cloud. The transfer model takes into consideration the inhomogeneity of the cloudy atmosphere, the gaseous absorption in scattering cloud layers and the wavenumber dependence of radiative transfer. In addition, the cirrus cloud is further divided into a number of sub-layers to account for the non-isothermal and inhomogeneous cloud characteristics. Single-scattering properties for ice crystals are calculated assuming ice cylinders 200 and 60 μm in length and width, respectively, randomly oriented in a horizontal plane. The spectral infrared transfer program is applied to VTPR channels of the NOAA 4 satellite to simulate upward radiances in cirrus cloud conditions.

Comparisons between satellite observed and theoretically simulated upward radiances are carried out for selected cirrus cloud cases. Incorporating atmospheric profiles obtained from radiosonde and the observed cloud information into the spectral transfer program, we show a systematic agreement between observed and computed upward radiances. Systematic reduction patterns of the upward radiance caused by the increase of the cloud ice content are clearly demonstrated for VTPR channels employing tropical and midlatitude atmospheric profiles. Having the quantitative relationships between upward radiances and ice contents, procedures are described for the inference of the cloud ice content and cloud amount. The proposed method has been successfully applied to the three cirrus cloud cases.

Key words: Satellite meteorology; Cirrus clouds: Radiative transfer through cirrus.

1. Introduction

There has been scarcely any study focusing on the evaluation of cloud properties from passive satellite sensing. Perhaps, it is because of the complexity of the cloud interaction with the radiation field of the atmosphere. This is especially evident for the high, semi-transparent cirrus clouds, consisting of non-spherical ice crystals possibly having a preferred orientation in the atmosphere. Cirrus clouds are good indicators of the weather to come and play significant roles on the climate of the earth-atmospheric system. More importantly, from the passive remote sensing point of view, semi-transparent cirrus clouds produce significant interference effects which jeopardize the information content of upwelling radiances with respect to the thermodynamic state of the atmosphere. Cirrus clouds are global in nature. Thus,

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for the purpose of either atmospheric sounding or climatic investigation, it seems extremely important to explore means for the determination of the composition and structure of cirrus clouds over the global scale. HOUGHTON and HUNT (1969) were pioneers proposing passive remote sensing of ice clouds using two far infrared wavelengths. A number of studies were also addressed to questions associated with the radiative properties of cirrus clouds based on field observations (KUHN and WEICKMANN, 1969; PLATT and GAMBLING, 1971), and theoretical calculations (LIOU, 1973, 1974). Recently, LIOU (1977) proposed a retrieval technique for recovering the thickness and ice content of cirrus clouds employing four spectral regions in the 10 μm windows.

Since upwelling radiances from the carbon dioxide or water vapor bands contain simultaneous information of the cloud and the thermodynamic state of the atmosphere, it was thought that perhaps certain cirrus cloud information may be derived from a combination of observing channels. This leads us to investigate radiative properties of cirrus clouds in VTPR (Vertical Temperature Profile Radiometer) channels of the NOAA 4 satellite and explore the possible cirrus cloud scenes utilizing upwelling radiance observations from these channels.

Section 2 presents the theoretical foundation for the calculation of the transfer of spectral infrared radiation in cirrus cloudy atmospheres. Section 3 consists of a brief description on the numerical procedures employed. Comparisons between satellite observed and theoretically simulated upward radiances for selected cirrus cloud cases are given in Section 4. In the subsequent section we discuss the radiative properties of cirrus clouds and cirrus cloudy atmospheres, and outline procedures that may be used for the determination of the vertical ice content of the cirrus cloud.

2. *Infrared radiation program for spectral channels*

A plane-parallel atmospheric model similar to the one proposed by LIOU (1974) is employed to evaluate the transfer of spectral infrared radiation in cloudy atmospheres. The model consists of three layers with CO_2 and H_2O gases above and below the cirrus cloud. It is assumed that cirrus clouds are composed of long circular cylinders randomly oriented in a horizontal plane (LIOU, 1972) so that the phase function, single-scattering albedo and extinction cross-section can be calculated for the wavenumbers considered employing the recent refractive index data of ice obtained by SHAAF and WILLIAMS (1973). Based on observations of WEICKMANN (1949) and HEYMSFIELD and KNOLLENBERG (1972), a mean length of 200 μm and a mean radius of 30 μm are used in all the transfer calculations. For the model calculations, we have used the temperature, pressure and water vapor profiles taken from climatological tables compiled by MCCLATCHEY *et al.* (1971). In the comparisons between theoretical calculations and satellite observations for selected cirrus cloudy cases, however, actual atmospheric soundings with respect to the temperature, humidity and cloud parameters are used as to be described in Section 4. All of the calculations reported in this paper assume that cirrus clouds are semi-transparent.

The NOAA 4 VTPR instruments consist of six channels in the 15 μm band of CO_2, one channel in the window region at 12 μm and one channel in the rotational band of water vapor. Table 1 shows the nominal characteristics of the eight filters

Table 1

Nominal spectral intervals for VTPR channels (after McMillin *et al.* (1973))

Channel	Center wavelength (μm)	(cm^{-1})	Half-width (cm^{-1})
1	14.96	668.5	3.5
2	14.77	677.5	10
3	14.38	695.0	10
4	14.12	708.0	10
5	13.79	725.0	10
6	13.38	747.0	10
7	18.69	535.0	18
8	11.97	833.0	10

in the VTPR instruments. The center wavenumbers are employed to calculate the single scattering properties of ice crystals which are presumably valid for the entire spectral regions. Atmospheric transmittances for the eight spectral intervals are available for the zenith angles of 0° and 23°47′ (McMillin *et al.* 1973). According to their report, transmittances for CO_2 are based upon calculations made by Drayson (1971) employing the point-by-point method for a number of temperature profiles. From these transmittances, transmittances for a given temperature profile may be obtained by means of interpolation. Ozone transmittances, which are a minor correction, are calculated from a line-by-line technique for the lower atmosphere using climatological ozone profiles (McClatchey *et al.*, 1971). As for water vapor, a procedure proposed by Weinreb and Neuendorffer (1973) is utilized to obtain the transmittances of selective absorption in inhomogeneous atmospheres. The effect of the water vapor continuum is also included based on laboratory measurements (Bignell, 1970). Thus, the transmittance of a VTPR channel for a given height z is given by

$$T_{\Delta\nu}(z) = T_{\Delta\nu}^{CO_2}(z)T_{\Delta\nu}^{H_2O}(z)T_{\Delta\nu}^{O_3}(z). \qquad (1)$$

Figure 1 depicts the weighting functions of the VTPR channels. Since clouds are normally tropospheric in origin and are located below about 200 mb or so, we would anticipate that their effects on upward radiances observed in channels 1, 2 and 3 of the CO_2 band are relatively insignificant.

The infrared radiation program begins with solving the transfer equation for a plane-parallel cloud layer consisting of absorbing gases in local thermodynamic

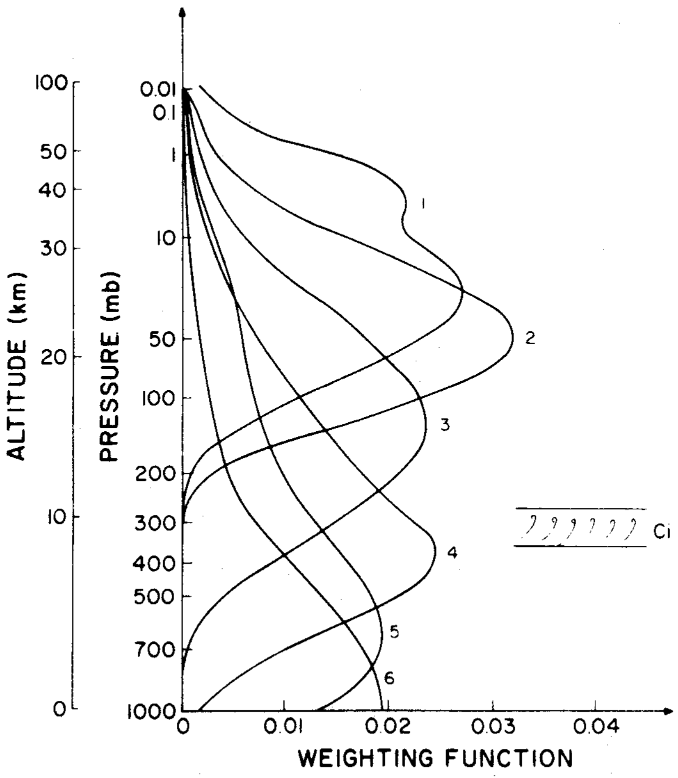

Figure 1
Weighting functions of the VTPR CO_2 channels (after McMillin *et al.* 1973).

equilibrium. The basic equation describing the monochromatic infrared radiation field is given by

$$\mu \frac{dI_\nu(\tau, \mu)}{d\tau} = I_\nu(\tau, \mu) - \frac{\tilde{\omega}_\nu}{2} \int_{-1}^{+1} P_\nu(\mu, \mu') I_\nu(\tau, \mu')\, d\mu' - (1 - \tilde{\omega}_\nu) B_\nu[T(\tau)], \quad (2)$$

where the single scattering albedo is given by

$$\tilde{\omega}_\nu = \beta_{s,\nu}/(\beta_{s,\nu} + \beta_{a,\nu} + nk_\nu). \quad (3)$$

I_ν represents the monochromatic radiance of wavenumber ν, μ the cosine of the emergent angle with respect to the zenith, τ the optical depth, P_ν the normalized axially symmetrical phase function, T the cloud temperature which is a function of height or optical depth, $B_\nu(T)$ the Planck function, h and k the Planck's and Boltzmann's constants, respectively, c the velocity of light, $\beta_{s,\nu}$ and $\beta_{a,\nu}$ the volume scattering and absorption cross sections for cloud particles of wavenumber ν, n the number density of the absorbing gases within the cloud layer, and k_ν the absorption coefficient of the gases.

The normalized phase function may be expanded into Legendre polynomials consisting of a finite number of terms. Upon replacing the integration in equation (1) by summation according to the Gauss' quadrature formula, a set of first-order inhomogeneous differential equations are derived. By seeking the homogeneous and particular solutions of the differential equations as described by CHANDRASEKHAR (1950), the complete solutions of the scattered radiance for a given discrete-stream j assuming an isothermal cloud temperature T_c may be written (LIOU, 1973 a, b)

$$I_\nu(\tau, \mu_i) = \sum_m L_m \phi_m(\mu_i) \exp(-k_m \tau) + B_\nu(T_c), \tag{4}$$

where $\sum_m$ denotes summation over the $2n$ discrete streams employed, ϕ_m and k_m are the eigenfunction and eigenvalue of the differential equations, whose values depend upon the phase function and single-scattering albedo, and L_m are a set of constants of proportionality to be determined from the radiation boundary conditions above and below the cloud layer.

We assume that the variation of the Planck function with respect to the wavenumber is much smaller than that of the transmission function T_ν. Thus, the spectral downward and upward radiances arising from the molecular absorption and emission reaching the cloud bottom and top, respectively, are given by

$$I_{\Delta\nu}^{\downarrow}(z_t, -\mu_i) = \int_{z_t}^{\infty} B_{\Delta\nu}[T(z)]\, dT_{\Delta\nu}(z, z_t; -\mu_i), \tag{5}$$

$$I_{\Delta\nu}^{\uparrow}(z_b, \mu_i) = B_{\Delta\nu}(T_s) T_{\Delta\nu}(z_b, 0; \mu_i) + \int_0^{z_b} B_{\Delta\nu}[T(z)]\, dT_{\Delta\nu}(z_b, z; \mu_i), \tag{6}$$

where

$$T_{\Delta\nu}(z_2, z_1; \mu_i) = \int_{\nu_1}^{\nu_2} \exp\left[-\frac{1}{\mu_i}\int_{z_1}^{z_2} k_\nu(z) n(z)\, dz\right] \phi(\nu) \frac{d\nu}{\Delta\nu}. \tag{7}$$

Note that $\phi(\nu)$ denotes the instrumental slit function. We first examine the transmission function (or transmittance). For a given height z, the transmittances are normally available in the form (for VTPR channels, see MCMILLIN *et al.* 1973) $T_{\Delta\nu}(z) = T_{\Delta\nu}(z, \infty; 1)$. There are only two sets of transmittances being generated for VTPR instruments corresponding to 0° (nadir) and 23°47′ zenith angles. However, spectral radiative transfer for cirrus clouds require the clear column angular upward and downward radiances reaching the cloud base and top, respectively. To evaluate these radiances, the angular dependent transmittances from 0° to 90° zenith angles are needed. The maximum VTPR scan angle is about 40° zenith angle. Thus, angular transmittances other than 0° and 23°47′ are also needed for the interpretation of the upward radiances. According to MCMILLIN (1976, personal communication), the angular transmittances for the 15 μm CO_2 channels are approximately given by

$$T_{\Delta\nu}(z, \mu) = [T_{\Delta\nu}(z)]^{f_1}, \tag{8}$$

where

$$f_1 = [\sec(\cos^{-1}\mu)]^{0.1 f_2}. \tag{9}$$

The empirical coefficient f_2 was derived from a four-term polynomial fitting. It is related to the pressure and temperature for a given level and certain empirically determined constants. For water vapor channels 7 and 8, however, angular dependent transmittances are approximated by

$$T_{\Delta\nu}(z, \mu) = [T_{\Delta\nu}(z)]^{\sec(0.77 \cos^{-1}\mu)}. \tag{10}$$

Verification of the empirical angular transmittances was done by comparing the calculated and observed radiances for various scan angle. It appears that up to about 40°, they are reasonably reliable. Since no additional information is available, we have used these empirical equations to evaluate the angular transmittances from 0° to 90° zenith angles for the eight VTPR channels.

Recalling equations (2) and (3), we notice that the gaseous absorption coefficient k_ν is needed to carry out the transfer of infrared radiation in cloud layers composed of absorbing gases. However, k_ν is known only through the absorption line parameters of gases and its values vary greatly with wavenumber in a small spectral interval. It is very difficult, if not impossible, to carry out line-by-line calculations including scattering contributions of cloud particles. Thus, a simpler approach for the gaseous absorption in a scattering layer would be to make use of the known transmittances which have been obtained to a good accuracy by means of line-by-line calculations for inhomogeneous atmospheres in conjunction with satellite sensing. Recognizing the definition of the vertical transmittance we may approximate it by

$$T_{\Delta\nu}(u) \approx \int_{\nu_1}^{\nu_2} \exp(-\bar{k}_\nu u)\frac{d\nu}{\Delta\nu} \approx \sum_{j=1}^{M} w_j \exp(-k_j u), \tag{11}$$

where u is the vertical path length, k_j may be thought of as an equivalent absorption coefficient, w_j is the weight and M denotes the total number of finite terms in the fitting of the transmittances. Once k_j and w_j have been determined, we may consider the transfer of spectral infrared radiation as monochromatic in the sub-spectral interval j and carry out transfer calculations in a cloud layer M times with a new single scattering albedo $\tilde{\omega}_\nu^j$. Here, we assume that the phase function and the absorption and scattering cross sections of ice crystals are independent of the wavenumber within a spectral interval. These assumptions are justified in view of the relatively slow varying refractive indices of ice in the infrared regions as evident in the next section.

Without postulating any assumptions and approximations, the upward radiance at the satellite point of view in completely cloudy conditions may be expressed by

$$I_{\Delta\nu}^C(\infty, \mu_i) = \int_{\nu_1}^{\nu_2} I_\nu^\uparrow(z_t, \mu_i) T_\nu(\infty, z_t; \mu_i)\phi(\nu)\frac{d\nu}{\Delta\nu} + \int_{z_t}^{\infty} B_{\Delta\nu}[T(z)]\, dT_{\Delta\nu}(\infty, z; \mu_i), \tag{12}$$

where T_ν denotes the monochromatic transmittances associated with the upward radiance $I_\nu^\uparrow$ at the cloud top. The second term on the right-hand side is for the clear atmosphere above the cloud layer and it is an exact expression. However, the first term requires approximations in order to take the scattering of cloud particles into account.

On the basis of the exponential fit to the transmission function described in equation (11), let $I_j^\uparrow (j = 1, 2, \ldots, M)$ be the resulting radiances calculated from the transfer program involving cirrus cloud layers, equation (12) can be approximated by

$$I_{\Delta\nu}^C(\infty, \mu_i) \cong \sum_{j=1}^{M} I_j^\uparrow(z_t, \mu_i) w_j T_j(\infty, z_t; \mu_i) + \int_{z_t}^{\infty} B_{\Delta\nu}[T(z)]\, dT_{\Delta\nu}(\infty, z; \mu_i), \quad (13)$$

where $I_j^\uparrow$ at the cloud top is to be evaluated from the upward and downward radiances arising from the molecular absorption and emission reaching the cloud bottom and top using the equivalent absorption coefficients and weights derived from the exponential fit noted in equation (11), and T_j is associated with the exponential function given by equation (11). The success of carrying out such a semi-monochromatic infrared transfer calculation depends solely on the reliability of fitting the satellite transmission functions for the entire atmosphere. The characteristics of transmission functions for the VTPR on board NOAA 4 have been described previously. Owing to the great difficulties experienced in performing the exponential fits of VTPR transmittances, another approach described below was subsequently developed.

Since the variation of the radiative property of clouds over a small spectral interval is much smaller than that of the transmittance and since the spectral transmittances for satellite channels $T_{\Delta\nu}$ are available from the top of the cloud to the top of the atmosphere, we postulate from equation (12) that

$$I_{\Delta\nu}^C(\infty, \mu_i) \cong I_{\Delta\nu}^\uparrow(z_t, \mu_i) T_{\Delta\nu}(\infty, z_t; \mu_i) + \int_{z_t}^{\infty} B_{\Delta\nu}[T(z)]\, dT_{\Delta\nu}(\infty, z; \mu_i), \quad (14)$$

where

$$I_{\Delta\nu}^\uparrow(z_t, \mu_i) = \sum_{j=1}^{M} I_j^\uparrow(z_t, \mu_i), \quad (15)$$

and semi-monochromatic upward radiances $I_j^\uparrow$ are to be evaluated from the procedures described below. The approach described here makes use of the known spectral transmission functions above and below the cloud and utilizes the exponential fitting program for the cloud layer described in Section 3. If the cloud layer is a black body, then equation (14) is an exact expression where $I_{\Delta\nu}^\uparrow$ will be given by the Planck function of the cloud top temperature. For moderately thick and thick clouds, we would think that the approximation is a good one since cloud particles, not the gases within the cloud, dominate the transfer processes.

The solution of the infrared radiative transfer equation given by equation (4) is applicable only to isothermal and homogeneous cloud layers. The question of inhomogeneity of cirrus cloud compositions is a difficult one. In the first place, there have not been many observations available. For the purpose of radiative transfer calculations, it seems that averaged property of cirrus cloud compositions may be appropriate. However, the effect of the non-isothermal structure of clouds on their radiative properties is more critical. Physically, a cold cloud top would reduce the transmitted radiances. It is conceivable, therefore, that an overestimation of upward radiances is likely to take place. Although our main concern is the non-isothermal structure, the transfer program described below may be utilized to investigate, if desirable, the inhomogeneous properties of clouds as well.

In reference to Fig. 2, the cirrus cloud layer is divided into a number of sub-layers, each of which is considered to be isothermal and homogeneous. The optical depth is evaluated from the cloud top to the bottom of the sub-layer. The index l is used to denote the number of the sub-layer. We now apply the solution of the radiative transfer equation given by equation (4) to each sub-layer and sub-spectral interval j to obtain ($\sum_m$ denotes summation over discrete-streams, $(-n, n)$)

$$I_j^l(\tau, \mu_i) = \sum_m L_m^l \phi_m(\mu_i) \exp(-k_m^l \tau) + B_j(T_c^l). \tag{16}$$

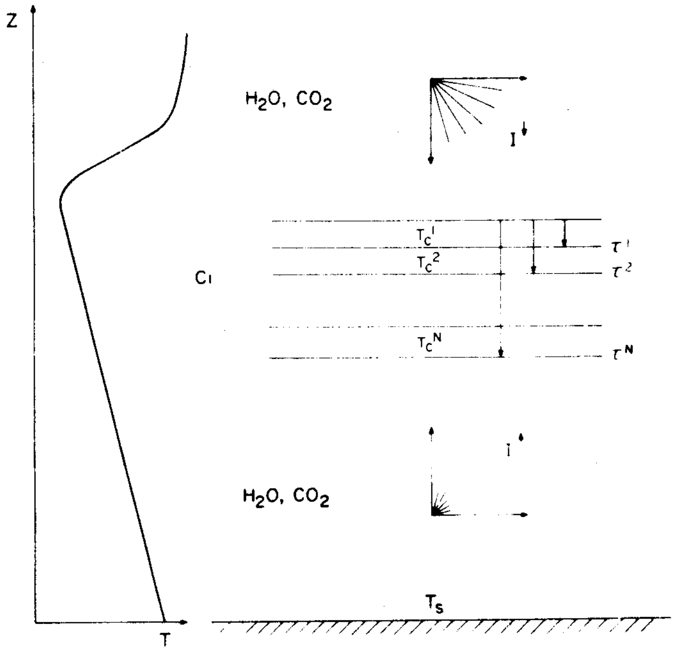

Figure 2
Radiative transfer model for an inhomogeneous cirrus cloudy atmosphere. The cirrus cloud layer is divided into several sub-layers, each of which is considered to be isothermal and homogeneous.

In order to determine the unknown coefficients L_m^l, the radiation continuity relationships are needed in addition to two radiation boundary conditions specified in equations (5) and (6). At the cloud top, the downward radiance has to be equal to that from the molecular atmosphere above, so that

$$I_j^1(0, -\mu_i) = I_{\Delta\nu}^{\downarrow}(z_t, -\mu_i)w_j. \tag{17}$$

Between the layers, the radiances from all directions must be continuous. Thus,

$$I_j^l(\tau^l, \mu_i) = I_j^{l+1}(\tau^l, \mu_i), \quad l = 1, 2, \ldots, N-1, \tag{18}$$

where N denotes the total number of sub-layers. Lastly, the upward radiance has to be equal to that from the molecular atmosphere below to give

$$I_j^N(\tau^N, +\mu_i) = I_{\Delta\nu}^{\uparrow}(z_b, +\mu_i)w_j. \tag{19}$$

Upon inserting the radiance solution expressed by equation (16) into equations (17)–(19), a set of linear equations with unknown coefficients may be obtained. The unknown coefficients can be solved by standard matrix inversion methods. A similar procedure has been employed by LIOU (1975) to evaluate the transfer of solar radiation in inhomogeneous atmospheres.

3. *Some discussions on numerical procedures and calculations*

Calculations for single scattering parameters were first carried out for ice crystals randomly oriented in a horizontal plane according to a previous theoretical model for ice crystal clouds by one of the authors (LIOU, 1972). Employing the size (l = 200 μm, r = 30 μm) and the concentration (0.05 cm^{-3}) of ice cylinders along with the corresponding refractive indices for ice, Mie type scattering computations were made for the central wavenumber of each channel.

Table 2 lists the optical properties for randomly oriented ice cylinders. The real

Table 2

Optical properties of ice cylinders (length 200 μm, width 60 μm) for the central wavenumbers of VTPR channels

Channel	ν(cm^{-1})	n_r	n_i	$\tilde{\omega}_0$	σ_{ext}(10^{-4} cm^{-2})
1	668.5	1.573	0.178	0.527	2.94
2	677.5	1.581	0.194	0.528	2.94
3	695.0	1.583	0.232	0.530	2.94
4	708.0	1.577	0.262	0.532	2.92
5	725.0	1.556	0.305	0.533	2.92
6	747.0	1.522	0.349	0.534	2.92
7	535.0	1.503	0.074	0.554	3.12
8	833.0	1.259	0.409	0.531	2.86

and imaginary parts of the refractive indices are taken from the recent measurements by SHAAF and WILLIAMS (1973). We see that imaginary parts decrease from the shorter wavelength to the longer wavelength in this part of the infrared spectrum with the window wavelength having the largest value. However, the window wavelength is also associated with the smallest value of the real part of the refractive index. The ice cylinders appear to have the largest extinction cross-section and largest single scattering albedo in the rotational band wavelength owing to the smallest imaginary part. The fact that the extinction and single scattering albedo vary insignificantly in these wavenumbers is probably because of the large particle size considered in this study.

Since angular dependent phase functions involve enormous computer time to evaluate, it was decided to perform only two calculations for the 15 μm CO_2 band at 677.5 and 725 cm^{-1}. Thus, phase functions for channels 1–3 and 4–6 use those for these two wavenumbers respectively. In view of the slow varying values of the refractive indices, the simplification appears reasonable. The phase functions are then expanded in a series of Legendre polynomials in the form

$$P_\nu(\mu, \mu') \approx \sum_{l=0}^{N} \tilde{\omega}_l P_l(\mu) P_l(\mu'), \tag{20}$$

where $\tilde{\omega}_0 = 1$ and $\tilde{\omega}_l(l > 1)$ are obtained from the orthogonal properties of the Legendre polynomials. The expanded form is to be used in the transfer calculations.

The transmission curves take into account absorption by three gases simultaneously, with each gas distributed differently and also include the instrumental slit function. These curves cover the entire atmosphere from 0.01 mb to the surface, which introduces a significant pressure variation. All of these factors lead to the non-exponential shape of the transmission curves $T_{\Delta\nu}(u)$ as functions of the major absorber.

Since the spectral transmittances are available above and below the cloud, it was decided to fit the transmittance over the range of pathlength covered by the thickest cirrus cloud to be considered so as to incorporate the gaseous absorption in scattering cloud layers. This was done by selecting a base height of 8 km and a maximum height of 14 km (approximately); the variation in pressure in this range is not enough to produce significant deviation from exponential behavior and the water vapor concentration is small enough to be neglected in calculating the pathlength. The curve was generated by using a constant CO_2 concentration of 5.11×10^{-4} gm cm^{-2}/mb. The transmission was read off the true curve at an altitude of 14 km and considered as 1.0 and the transmission at each subsequent level before this was considered as a fraction of what the value was at 14 km. For CO_2 channels, carbon dioxide is the major absorber, so the neglect of water vapor absorption above a height of about 5 km produces insignificant errors. As for H_2O channels, actual water vapor concentration profiles were used in the fitting program.

A simplified fitting routine was developed which guaranteed the required restriction of w_j and k_j (both have to be positive). The numerical scheme is based on the following iterative process. An initial $T_{\Delta\nu}(u_1)$ and w_1 are chosen (initially $w_1 = 1$) and k_1 is generated from

$$k_1 = \frac{1}{u_1} ln \frac{w_1}{T_{\Delta\nu}(u_1)}. \tag{21}$$

The curve $T'_{\Delta\nu}(u) = w_1 \exp(-k_1 u)$ is then generated point by point; as each new point is generated it is compared with the true value $T_{\Delta\nu}(u)$ to make sure it meets the following conditions

$$\begin{aligned} T'_{\Delta\nu}(u) &< T_{\Delta\nu}(u), \\ \frac{dT'_{\Delta\nu}(u)}{du} &< \frac{dT_{\Delta\nu}(u)}{du}. \end{aligned} \tag{22}$$

The second condition requires only a rough approximation for the derivative. If either of these are not met, a new w_1 is generated (i.e., $w_1 = w_1 \times$ constant), so that the new w_1 is slightly less than the old one, and the above is repeated. If no w_1 can be found, a new u_1 is selected (larger than the old point). When a good w_1 and k_1 are found, the above process is then repeated to produce a w_2 and k_2 using values of $T_{\Delta\nu}(u) - T'_{\Delta\nu}(u)$ instead of $T_{\Delta\nu}(u)$. The number of points needed to fit the curve then depends only on the size of the error tolerance used and on the range of u over which $T_{\Delta\nu}(u)$ is to be fitted, It was possible to fit the curve between 8 km and 14 km to within 2 percent and in most cases much less than 1 percent. The exponential fitting approximation is now restricted in the scattering cloud layer, so that the gaseous absorption within the cloud layer may be included. Note here that spectral transmittances are available both above and below the cloud.

Once the scattering and absorption properties of ice crystals and the spectral absorption characteristics of gases have been determined, we may evaluate the volume single-scattering properties for a mixture of ice crystals and gases within the cirrus clouds. According to equation (2) and the exponential fitting program described above, the single-scattering albedo including gaseous contribution for a spectral interval whose central wavenumber is ν may be written

$$\tilde{\omega}_\nu^j = \beta_s/(\beta_e + nk_j), \quad j = 1,\ldots, M, \tag{23}$$

where β_e denotes the volume extinction cross-section of ice cylinders. The total optical depth for a cloud with a thickness of Δz is given by

$$\Delta\tau_\nu^j = \beta_e \Delta z + uk_j, \quad j = 1,\ldots, M, \tag{24}$$

here we omit the subscript ν in the right-hand side of these two equations. The amount of H_2O (for channels 7 and 8) within cirrus is estimated by using a mean temperature and assuming saturated conditions. CO_2 concentration (for channels 1–6) is assumed uniformly distributed in the atmosphere. Thus, the pathlength of gases within the cloud may be obtained. Finally, it is also assumed that the spectral phase function is

$$P_\nu^j(\mu, \mu') = P_\nu(\mu, \mu'), \tag{25}$$

as described in equation (20). The single-scattering albedo, the total optical depth for a given thickness Δz and the phase function in the form of Legendre polynomials are the basic parameters for carrying out the spectral radiative transfer calculations.

The monochromatic transfer program first generates the required eigenvalues for a set of homogeneous differential equations from the single-scattering albedo and the expanded phase function. LIOU (1973 a, b) pointed out the mathematical and numerical ambiguities involving CHANDRASEKHAR's (1950) method for searching the eigenvalues corresponding to the $2n$ associated homogeneous differential equations, and developed a matrix method directly from the associated homogeneous differential equations for the eingenvalue problem. Recognizing the symmetric relationships between the matrix elements, ASANO (1975) further developed the analytic procedures for the reduction of the rank of the matrix. These developments thus allow the accuracies of eigenvalues to be improved and the computer time involved greatly reduced. Computations presented in this paper follow the numerical procedures outlined by LIOU (1973 a, b) and ASANO (1975) for eigenvalue problems in the discrete-ordinate method for radiative transfer.

Secondly, the program calculates the clear column angular upward and downward radiances arising from the absorption and emission of gases (in forms of transmittances) above and below the cloud as well as the surface contribution. Each model atmospheric profile is divided into 101 levels from the surface to 100 km by means of linear interpolation between the available data on pressure, temperature and humidity. The base of a cirrus is placed at about 8 km, and nonisothermal cloud temperatures are assumed the same as those of the surroundings.

The program then solves the unknown coefficients of proportionality of radiance solutions for an inhomogeneous system of equations. After proper summation over the subspectral weight derived from the exponential fit, the transmitted and reflected spectral radiances at the cloud top and bottom, respectively, can be obtained.

Since the infrared computer program employs 16 discrete streams in the calculations, interpolations of the 16 radiance values are finally needed to obtain 0°, 180° and limb (90°) directions. Note that 0° and 180° directions are used to define the transmissivity and reflectivity of cirrus clouds.

4. Comparison between satellite observations and theoretical simulations for selected cirrus cloud cases

A number of cirrus cloud cases under the NOAA satellite pass were selected for comparisons between the present theoretical calculations and satellite observations. Figure 3 represents the case studies to which the cloud model was applied. Each case

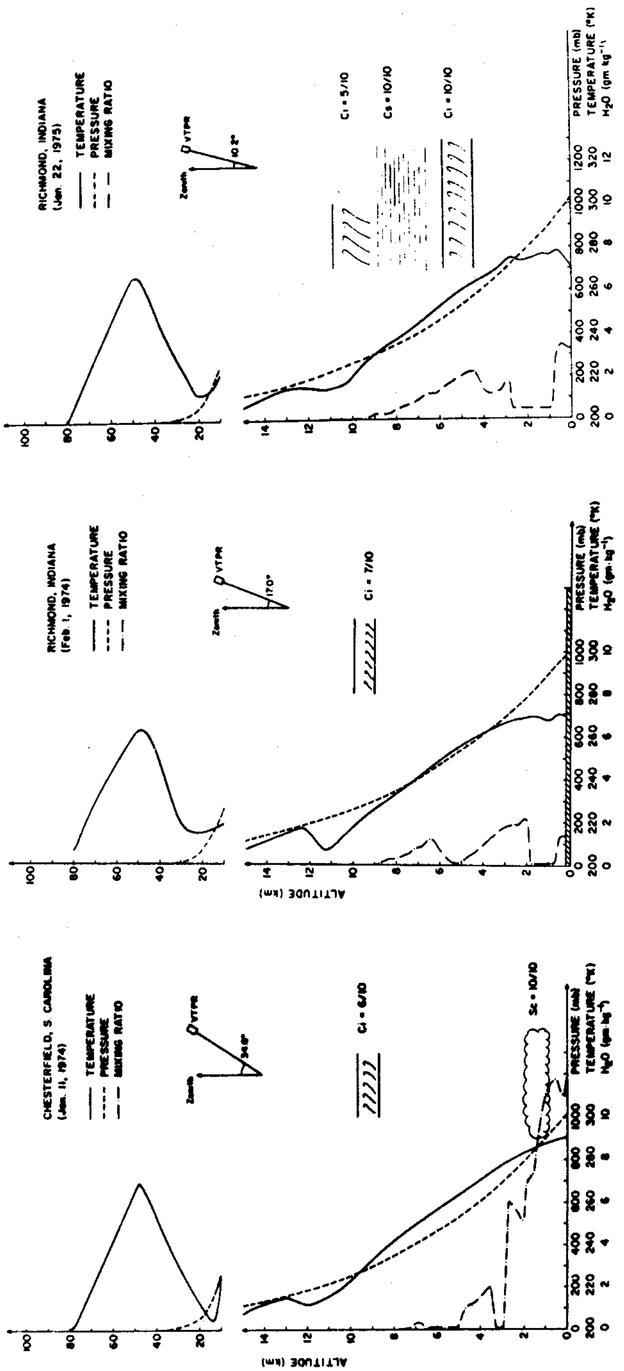

Figure 3

The observed cirrus cloud cases to which the spectral infrared radiative transfer model was applied. The figure depicts the profile of temperature, pressure and water vapor mixing ratio as functions of height and the scan angle of the VTPR instrument.

study is described to include the atmospheric profile, the synoptic situation, and the cloud composition determination. These cases were selected from a group of cases developed by the AFGL during 1974 and 1975. These three case studies were used to test the cloud model program under three conditions and include cirrus with low clouds (11 January 1974), thin cirrus with no low clouds (1 February 1974), and thick cirrus with no low clouds (22 January 1975).

The figure depicts the profile of temperature, pressure, and water vapor versus height for the three cases. Each parameter was interpolated to 101 levels used in the clear column radiance calculations. Input into each profile was the actual temperature and water vapor profiles of the nearest radiosonde station to the point of interest in time and space linearly interpolated to the required pressure heights. Above the level of the radiosonde the temperature, water vapor, height and pressure profiles compiled by McClatchey *et al.* (1971) for a midlatitude winter atmosphere were used for each case study.

The position and ice content of the cloud for each case were estimated from airborne instrumentation equipped with a Knollenberg probe (Knollenberg, 1970) used by AFGL during the cloud physics experiment. These estimates were obtained by an aircraft spiralling downward from 10 km to the approximate time of the satellite pass. Objective analyses of ice contents were also made. The ice content analyses involve best estimates derived from detailed aircraft observations consisting of visual observations, snow intensity, particle size replica, visibility, temperature and photography (Conover, 1976, personal communication).

To use the observed cloud information in the theoretical transfer calculations, the vertical ice content for the ice cloud model employed in this study has to be defined. For randomly oriented ice cylinders in a horizontal plane, it is given by

$$IC = \pi r^2 l \rho_l N \Delta z, \tag{26}$$

where r and l are the radius and the length of the cylinder, respectively, ρ_l the density of ice, N the number density and Δz the thickness of the cloud. Since the radius and the length of the ice cylinder have been specified to obtain the single-scattering parameters, it follows that the 'equivalent' particle number density can be derived. Once the number density has been obtained, the optical depth of ice within the cirrus cloud under consideration is simply equal to $\sigma_{ext} N \Delta z$. Together with the optical depth of the gases within the cloud, the total optical depth represents one of the fundamental parameters in transfer calculations. Having the atmospheric temperature and water vapor profiles from radiosonde, and the cloud structure and composition specified, transfer calculations may now be carried out for these cases.

Below we describe briefly the three cases used in this study. On 11 January 1974, the area of the United States east of the Rocky Mountains was under the influence of a weak low pressure, which at 500 mb had its center in north central Canada. Chesterfield, South Carolina, 34.65 N–80.20 W, was located under the cirrus shield of the frontal system. The location had an observation of overcast stratocumulus with the top at 2 km and a 6/10 cirrus layer from 8.8 to 9.8 km. The temperature, pressure and water vapor profiles were obtained from the 1200 Z Athens, Georgia, sounding. The time of the observation and the satellite pass time was 1521 Z. The cirrus cloud ice content was estimated to be 11 gm m^{-2} in the VTPR field of view.

On 1 February 1974, the area of the United States east of the Rocky Mountains was under the influence of zonal flow at 500 mb with a surface high pressure centered near the Minnesota–Canadian border. Richmond, Indiana, 39.75 N–84.83 W, was located under cirrus clouds south of the main precipitation area. The location had an observation of a thin 7/10 cirrus cover from 9–9.8 km. The temperature, pressure and water vapor profiles were obtained from the 1200 Z Dayton, Ohio, sounding. The time of the observation and satellite pass time was 1526 Z. The cirrus cloud ice content was estimated to be 2 gm m^{-2} in the VTPR field of view. Note that during the above two days the cloud physics observations were under the NOAA 2 satellite. Inspection of the VTPR transmittances of NOAA 2 and NOAA 4 indicates insignificant differences. Thus, we have consistently used the NOAA 4 VTPR transmittances in the present study.

On 22 January 1975, the area of the United States east of the Rocky Mountains was dominated by zonal flow with a cutoff low over New Mexico at 500 mb. Richmond, Virginia, 37.53 N–77.27 W, was located in a general area of scattered fog and overcast cirrus. The location had an observation of 10/10 cirrus from 4.4–5.7 km and 10/10 cirrostratus from 6.7–8.7 km and 5/10 cirrus from 8.7–10.7 km. The temperature, pressure and water vapor profiles were obtained from the Richmond 1200 Z sounding. The cirrus cloud ice content was 6.4 gm m^{-2} for the top layer, 67.8 gm m^{-2} for the middle layer and 104.2 gm m^{-2} for the lower layer in the VTPR field of view. The time of the observation and satellite pass time was 1417 Z.

Figure 4 illustrates comparisons between the observed and computed upward radiances for the three cirrus cloud cases described previously. Note that the VTPR scan angles are denoted in Fig. 3. For the case when thick, layered cirrus clouds are present, the top layer consists of only 50 percent cloud cover. Thus calculations were made by averaging the resulting values for a three- and a two-layer cloud system. The calculated radiances for channels 4-8 are consistently lower than those observed from the NOAA 4 satellite. This, perhaps, indicates that the observed value of the ice content was overestimated. Comparisons between the observed and computed radiances for the single cirrus cloud case reveal systematically higher values based on theoretical simulations. Again, it is perhaps caused by the underestimation of the vertical ice content within the field of view of the VTPR instrument. In our judgment, the systematic behavior of the calculated and observed radiance values for those two cases indicates to certain extents the reliability of the theoretical model for the transfer of spectral infrared radiation. Such systematic differences might be due, in part, to the assumption of the cloud temperature. The final case contains an overcast lower stratocumulus and a light cirrus. It is assumed that the low cloud is a black body so that the transfer calculations start from the cloud top. Comparisons with observed

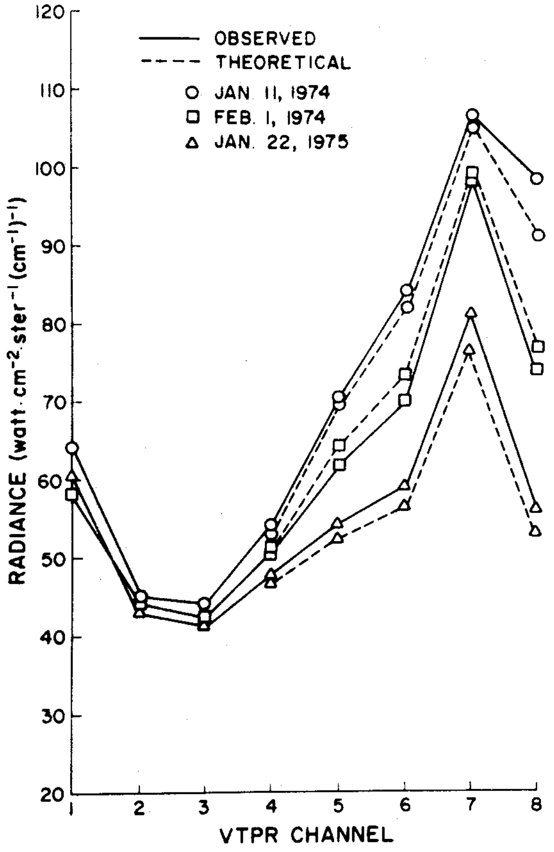

Figure 4
Comparisons between satellite observed and theoretically simulated upward radiances for the three cirrus cloud cases described in Fig. 3.

radiances show good agreement except for channel 8. In order to find the physical reason for the deviation in this particular channel, careful examinations were carried out for transfer calculations. The observed radiance in channel 8 is about 98.3 watts cm^{-2} $ster^{-1}$ $(cm^{-1})^{-1}$. However, utilizing the observed temperature and mixing ratio profiles denoted in Fig. 3, it was found that the clear column radiance at 34.6° zenith angle was only about 99.4 watt cm^{-2} $ster^{-1}$ $(cm^{-1})^{-1}$. In view of the considerable reduction of radiances in all other channels, it seems that the observed value in channel 8 should have been on the order of about 90 watts cm^{-2} $ster^{-1}$ $(cm^{-1})^{-1}$. We note here that this is the only unsatisfactory channel comparison.

Within the uncertainties involved in the measured radiances and atmospheric and cloud parameters, and the assumptions employed in the theoretical model, agreement between satellite observed and theoretically simulated radiances appears satisfactory for these three selected cirrus cloud cases. These comparisons establish our confidence in the theoretical model for the transfer of spectral infrared radiation. It should be noted that since the peaks of the weighing functions in channels 1–3 are all above the possible cirrus cloud location, cloud effects on the upward radiances in these channels can be ignored.

5. *Theoretical results and cloud scene discussions*

In the preceding section, systematic agreement between observed and computed upward radiances appears to establish the reliability of the theoretical program. Thus, we shall present some results of the radiative properties of cirrus clouds and cirrus cloudy atmospheres in this section. We first discuss transmission and reflection properties of cirrus in VTPR channels. Graphs of upward radiances in cirrus cloudy atmospheres are then presented. Finally, possible determination of cloud ice content and amount is described.

It is convenient to define the spectral transmission $T^C_{\Delta\nu}$ and reflection $R^C_{\Delta\nu}$ of a cloud layer as follows (see equations (5) and (6)):

$$\left.\begin{aligned} T^C_{\Delta\nu} &= I^{\uparrow}_{\Delta\nu}(z_t, 1)/I^{\uparrow}_{\Delta\nu}(z_b, 1) \\ R^C_{\Delta\nu} &= I^{\downarrow}_{\Delta\nu}(z_b, -1)/I^{\uparrow}_{\Delta\nu}(z_b, 1) \end{aligned}\right\}. \tag{27}$$

Physically, the transmission defined above represents the actual percentage attenuation of upwelling radiances reaching the cloud base, and it may be employed for the parameterization of cloud radiative properties from a satellite point of view. Note that both transmission and reflection include contributions of cloud emission, which is automatically generated in transfer calculations.

Figure 5 illustrates the spectral reflection of cirrus as a function of the thickness and the vertical ice content for VTPR channels. The left and right-hand graphs are for a tropical (wet) and a midlatitude winter (dry) atmosphere, respectively. The vertical scale applies to the lowermost curves. The scales for other curves are to be obtained by subtracting consecutively a factor of 0.2 such that the horizontal bar on each curve is zero. Cloud infrared reflection appears independent of the atmospheric profile. Its value increases asymptotically with increasing cloud thickness. Cloud reflection arises primarily from scattering of cloud particles. The single-scattering albedos are about the same for channels 4, 5, 6, and 8 resulting in similar reflection patterns. For channel 7, however, a higher single-scattering albedo is derived (see Table 2). This results in larger reflection values as evident in Fig. 5. Note that reflection defined earlier includes emission contribution from clouds, primarily the warmer cloud base. For a 1 km thick cirrus we see that the reflection value already reaches about 0.4.

Figure 6 shows the spectral transmission of cirrus. As in Fig. 5, the vertical scale applies to the lower-most curves. The scales for other curves are to be obtained by subtracting successfully a factor of 0.2 such that the horizontal bar on each curve is 1.

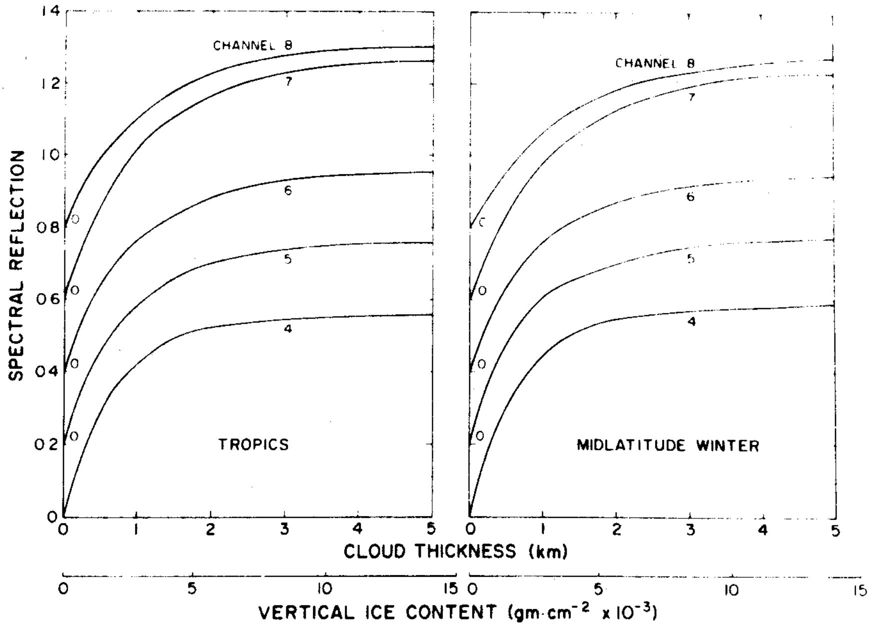

Figure 5
Spectral reflection of cirrus (see text for definition) as a function of the cloud thickness for a tropical and a midlatitude atmosphere. The lower abscissa denotes the ice content scale corresponding to the cloud thickness using a particle number density of 0.05 cm^{-3}.

Cloud infrared transmission decreases with increasing thickness and reaches a value of about 0.3 for a cloud thickness of 5 km for channels 4–6 and 8. As for channel 7, a cloud whose thickness is 5 km transmits about 43 percent of the incident upwelling radiance owing to a larger single-scattering albedo at a wavenumber of 535 cm^{-1}. Inspection of the transmission curves indicates a similar behavior for the CO_2 and window channels. In particular, variations of the spectral transmission of CO_2 channels for all the thicknesses presented here are less than about 3 percent. Even the transmission value of the window channel does not vary by more than 10 percent with respect to those of the CO_2 channels. Larger deviations are seen to take place for larger thicknesses. Since cirrus clouds are not normally thick, we may, to a good approximation, assume that the cloud transmission values are constants for the CO_2 and window channels.

The most important quantity associated with satellite sensing of the atmosphere is the upward radiance at the top of the atmosphere. Upward radiances are the only information for the inference of the structure and composition of the atmosphere.

In Fig. 7, we present the upwelling (nadir) radiance of VTPR channels for two model cirrus cloudy atmospheres representing wet (tropics) and dry (midlatitude winter) conditions, respectively. The solid curves in this figure denote upwelling radiances for clear columns. For the tropical atmosphere, systematic reductions of the upwelling radiance due to the increase of the ice content are illustrated. In reference to Fig. 1, it is seen that the peak of the weighting function for channel 4 is located at about the position of cirrus clouds. Consequently, even a thin cirrus produces a significant signature of radiance reduction. In the water vapor rotational band (smaller wavenumber), larger upwelling radiances arise from the behavior of the Planck function and the emission from water vapor. The systematic reduction caused by the increase of ice content is so distinct that the observed radiances obtained from VTPR channels in a completely cloudy atmosphere may be utilized to evaluate the amount of ice within the field-of-view. This, of course, would rely upon a first estimation of clear column radiances from available atmospheric profiles. Upwelling radiances in a midlatitude winter atmosphere exhibit similar reduction patterns, but their values are lower owing to the lower temperatures in the troposphere. Reduction values in cases when ice contents are 84 and 112 gm m^{-2} are very close, revealing that the cloud is reaching a black-body thermodynamic state.

Having the upward radiance calculated for a series of cirrus ice content values in various model atmospheric profiles, the following procedures seem feasible for the ice content determination. Assume that within the field-of-view of VTPR channels

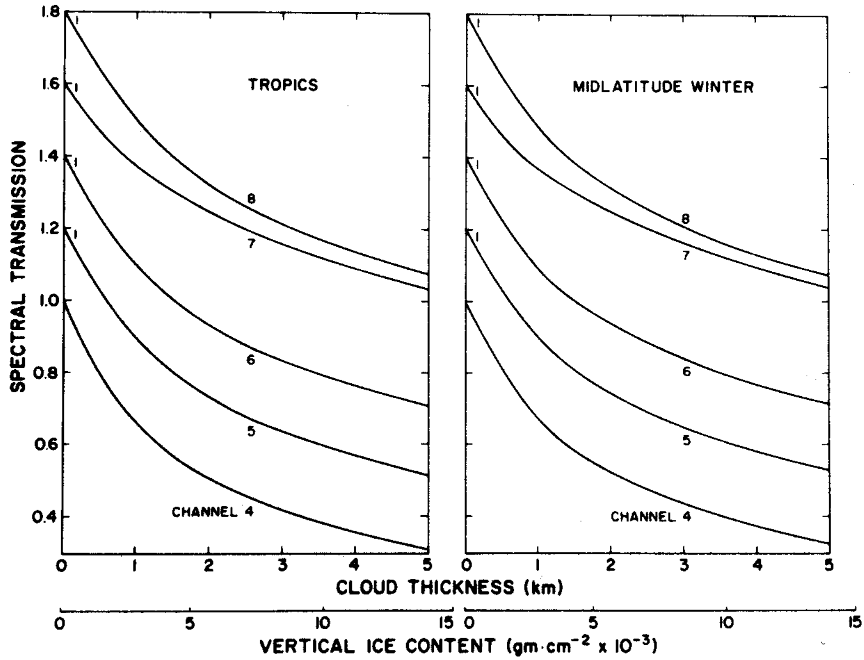

Figure 6
Spectral transmission of cirrus (see text for definition) as a function of the cloud thickness for a tropical and a midlatitude atmosphere. The lower abscissa denotes the ice content scale corresponding to the cloud thickness employing a particle number density of 0.05 cm^{-3}.

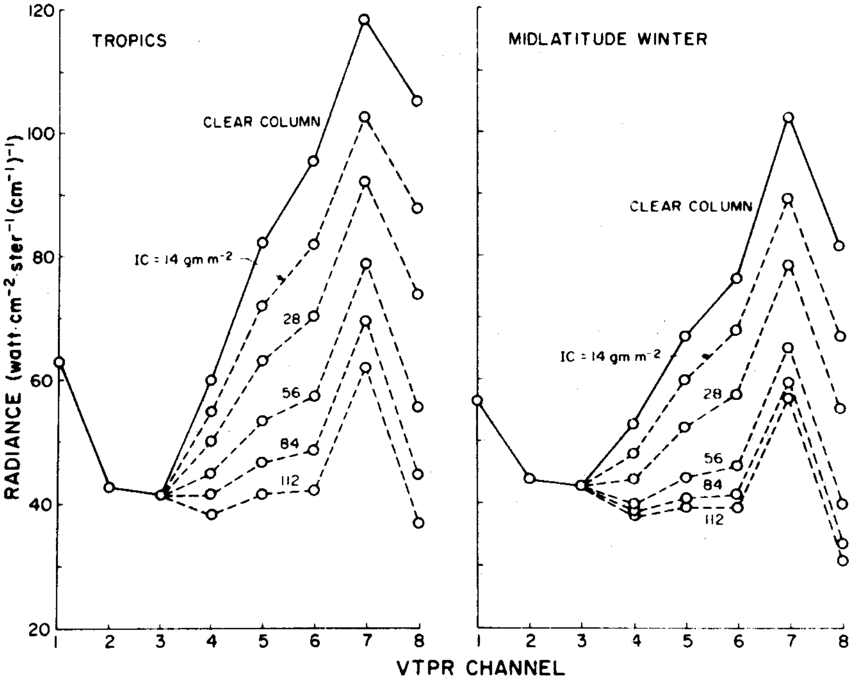

Figure 7
Upwelling (nadir) radiances of VTPR channels for a tropical (left-hand side) and a midlatitude (right-hand side) atmosphere. The solid curves denote upwelling radiances for clear columns. Systematic reduction caused by the increase of the cirrus ice content is shown by dashed curves.

there is η portion of cirrus cloudiness, the upward radiance in a partly cloudy atmosphere is given in the form ($\Delta\nu \to i$).

$$I_i^{PC} = \eta I_i^C + (1 - \eta) I_i^{NC}. \tag{28}$$

It can be shown that for any two cloud-contaminated radiances at distinct wavenumbers that

$$\frac{I_i^{PC} - I_i^{NC}}{I_{i+1}^{PC} - I_{i+1}^{NC}} \Bigg/ \frac{I_i^C - I_i^{NC}}{I_{i+1}^C - I_{i+1}^{NC}} = 1, \tag{29}$$

where $I_i^C < I_i^{PC} < I_i^{NC}$, $i = 4, 5, 6, 7, 8$.

As mentioned previously, clear column radiances are to be estimated. Estimation may be accomplished by employing available atmospheric profile analyses such as those provided by NMC or from climatology. Although equation (24) now contains two unknown parameters I_i^C and I_{i+1}^C, these two variables are both functions of the ice content. Based on transfer calculations, for a given ice content in a specified atmosphere, both values may be derived. Upon inserting a series of ice content values, we may obtain a best estimate such that equation (29) holds. This procedure can be carried out for pairs of adjacent channels. Consequently, each pair gives an ice content estimation. By further inspection of these estimations, a final value for ice content which satisfies all the channel observations could be determined. This could be accomplished by examining the mean and standard deviation with respect to l according to equation (29) for all available channel observations. Once an ice content value has been chosen, I_i^c is known and, from equation (28), the cloud cover information can be calculated. An averaged value of η may be subsequently obtained such that it satisfies all the channel observations.

The above procedures were followed for the three cirrus cloud cases described in Section 4. The best ice content estimates that we obtained were 10, 8, and 112 gm m^{-2} for 11 January 1974, 1 February 1974 and 22 January 1975, respectively, whereas the *in situ* aircraft observations gave 11, 2 and 176 gm m^{-2}, respectively. Since these are the only three available cirrus cloud cases, it is not possible to perform significant statistical error analyses.

It should be emphasized that the semi-empirical method for ice content estimation makes use of the results from transfer calculations and assume a prior knowledge of the clear column radiance. The later assumption relies upon reliable atmospheric profiles which are unknown in cloudy conditions. Hence, the best estimation for the profile, which is needed, may lead to the uncertainty in the ice content determination. Nevertheless, the procedure described here, at least, represents an objective and workable means for recovering cloud information from satellite sensing.

6. *Conclusions*

A theoretical model for the transfer of spectral infrared radiation in atmospheres containing ice crystals and absorbing gases has been developed. The model allows the inhomogeneity of the cirrus cloudy atmosphere, which consists of optically active gases and cirrus clouds, to be treated approximately. It also takes into account the gaseous absorption in scattering cloud layers and the wavenumber dependence of radiative transfer. In addition, it considers the non-isothermal and inhomogeneous properties of cloud layers. It was applied to VTPR channels of the NOAA 4 satellite to simulate upward radiances at the top of the atmosphere in cirrus cloud conditions. Available transmittances for each channel were employed and modified so as to incorporate into the spectral infrared transfer program.

Comparisons between satellite observed and theoretically simulated radiances reveal systematic agreement for selected cirrus cloud cases involving thin and thick cirrus, and thin cirrus with stratocumulus below. Theoretical calculations employ observed atmospheric profiles from radiosondes, and the best estimated cloud parameters from aircraft observations under the satellite pass. These comparisons give confidence in the theoretical model for the spectral infrared transfer in cloudy atmospheres.

Utilizing the spectral infrared transfer model, radiative properties of cirrus cloudy atmospheres in VTPR channels were investigated for a number of model atmospheric profiles. The spectral transmission and reflection, defined as the ratios of the upwelling radiance at the cloud bottom to that at the cloud top and to the downwelling radiance at the cloud bottom, respectively, were calculated. We show that the effect of the atmospheric profile on cloud transmission and reflection is insignificant and that close resemblance of these two parameters is found for VTPR channels 4–6 and 8. Systematic reduction patterns of the upwelling radiance caused by the increase of the ice content are clearly illustrated for VTPR channels employing tropical and midlatitude atmospheric profiles. Finally, procedures are outlined for estimating the cirrus ice content in partly covered conditions. Employing three cirrus cloud cases in which satellite observed radiances are available, we demonstrate that the ice content and the amount of cloud cover may be estimated objectively.

Although we have illustrated in this paper an objective way of deriving the cloud ice content and amount, the method is to be verified in view of a number of assumptions made. However, verification of satellite cloud sensing techniques requires carefully designed field experiments in which reliable cloud parameters could be obtained from aircraft measurements under the satellite pass. With sufficient cases from which cloud parameters may be derived locally, inter-comparisons with satellite estimated values may be carried out to establish the statistical significance of the satellite sensing technique.

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