Parameterization of the Radiative Properties of Clouds

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ABSTRACT

Reflection, transmission and absorption of solar radiation by four cloud types (low cloud, middle cloud, high cloud and stratus) are computed as functions of the solar zenith angle and cloud liquid water/ice content. The reflection, transmission and emission of infrared radiation by cirrus clouds are calculated as functions of the cloud ice content. The plane-parallel radiative transfer program employed is based on the discrete-ordinate method with applications to inhomogeneous atmospheres covering the entire solar and infrared spectra and taking into account the gaseous absorption in scattering atmospheres. The resulting values of the solar radiative properties of clouds are fitted with known mathematical functions involving the solar zenith angle and cloud liquid water/ice content as variables. Effects of the atmospheric profile are discussed and the effects of surface reflectivity on the solar radiative properties of clouds are parameterized in terms of the water vapor absorbtivity below the cloud, ground reflection and average cloud reflection. Parameterized equations for the infrared flux reflectivity, transmissivity and emissivity of cirrus clouds are also presented as functions of the cloud ice content.

1. Introduction

The most important regulators of the radiation balance are clouds, which regularly occupy about 50% of the sky on a global scale. Clouds absorb and scatter the incoming solar radiation and emit thermal infrared radiation. The amount of energy absorbed and/or emitted represents one of the prime sources determining the stability of cloud layers and is further associated with the general circulation of the atmosphere. Moreover, the albedo of the earth-atmosphere system depends crucially on such parameters as cloud cover, cloud composition and cloud structure.

Reflection, transmission and absorption of solar radiation by various cloud types imbedded in atmospheres containing water vapor were extensively investigated by Liou (1976), and Liou *et al.* (1978) have recently extended the radiative transfer program to include the absorption contribution of oxygen, ozone and carbon dioxide and the absorption and scattering contributions by aerosols typical of those in clear atmospheres. The radiation program, as utilized in this work, has taken into account the inhomogeneity of the plane-parallel atmosphere, the wavelength dependence of solar radiation and the gaseous absorption in scattering layers. Calculations have been carried out to cover the entire solar spectrum on a band-by-band basis.

Although the radiation characteristics of clouds, whose horizontal extents are much greater than their vertical depths, can be evaluated to yield good accuracies by means of currently existing transfer methods, the computational time requirement and the effort involved make the comprehensive transfer program impracticable for studies of the weather, climate and radiation balance of the earth-atmosphere system. Owing to the intricacy of the cloud interaction with the solar radiation field of the atmosphere, a set of prescribed values for reflection and transmission parameters has normally been used in the study of the thermodynamic properties of the atmosphere. Manabe (1975), for example, assumed that the reflection of high, middle and low clouds were 20, 48 and 89%, respectively, regardless of the cloud composition and solar zenith angle.

It is the purpose of this paper to provide theoretically derived empirical equations for the reflection, transmission and absorption of solar radiation by clouds of different type as functions of the solar zenith angle and cloud liquid water/ice content. The known mathematical functions are derived by means of numerical fits of the precalculated reflection, transmission and absorption values for a surface with zero reflectivity. Parameterization of the effects of surface reflection and absorption by water vapor between the cloud and the earth's surface is given in terms of the water vapor absorptivity, ground reflection and average cloud reflection. Parameterized equations for the infrared flux reflectivity, transmissivity and emissivity of a

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cirrus cloud are also presented as functions of the cloud vertical ice content.

2. Radiative transfer in a cloudy atmosphere

a. Basic radiation scheme

The basic radiation scheme used is based on the plane-parallel model recently developed by Liou *et al.* (1978), Freeman and Liou (1978) and Roewe and Liou (1978). The model utilizes the discrete-ordinate method for radiative transfer, originally introduced by Chandrasekhar (1950). The method has been found quite satisfactory because it allows for the solutions of the integral-differential transfer equation to be explicitly derived and can easily take into account the distribution, thickness and types of clouds and aerosols, making the scheme well suited to this study.

The transfer equations describing the radiation field for the azimuthally independent diffuse intensity I may be written

$$\mu \frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \tfrac{1}{2}\tilde{\omega}_0 \int_{-1}^{+1} P(\mu,\mu')I(\mu,\mu')d\mu' - \begin{cases} \tfrac{1}{4}\tilde{\omega}_0 F_0 P(\mu,\mu_0)\exp(-\tau/\mu_0) & \text{(solar)} \\ (1-\tilde{\omega}_0)B_\nu(T) & \text{(infrared)}, \end{cases} \quad (1)$$

where τ represents the optical depth, μ and μ_0 the cosine of the emergent and solar zenith angles, respectively, $\tilde{\omega}_0$ the single-scattering albedo, F_0 the incident solar flux, T the temperature, P the normalized phase function and B_ν the Planck function. All parameters except T, μ and μ_0 are wavelength or wavenumber dependent.

Employing the discrete-ordinate method for radiative transfer, the solutions of the transfer equations as shown by Liou (1973) are given by

$$I(\tau,\mu_i) = \sum_j L_j\phi_j(\mu_i)\exp(-k_j\tau) + \begin{cases} Z(\mu_i)\exp(-\tau/\mu_0) & \text{(solar)} \\ B_\nu(T) & \text{(infrared)}, \end{cases} \quad (2)$$

where i and $j(-n,n)$ denote the discrete streams (positive upward and negative downward) used in approximating the basic transfer equation, and ϕ_j and k_j represent the eigenfunction and eigenvalue, respectively. The Z function is associated with Chandrasekhar's H function, the single-scattering albedo and the expanded phase function in terms of Legendre polynomials, and the L_j's are coefficients determined from the boundary conditions.

The solutions of the radiative transfer equations given by Eq. (2) are applicable to a homogeneous and isothermal layer. In order to apply such solutions to inhomogeneous atmospheres, the atmosphere is divided into a number of sublayers each of which may be considered homogeneous and isothermal. At the top of the atmosphere there is no downward diffuse intensity so that

$$I_1(0, -\mu_i) = 0. \quad (3)$$

Between the sublayers, the intensities from all directions must be continuous. Thus,

$$I_l(\tau_l,\mu_i) = I_{l+1}(\tau_1,\mu_i), \quad l = 1, 2, \cdots, N-1, \quad (4)$$

where N is the total number of sublayers and τ_l represents the optical depth from the top of the atmosphere to the layer l. At the bottom of the atmosphere, the upward intensity is given by

$$I_N(\tau_N, +\mu_i) = \begin{cases} (A_s/\pi)[2\pi \sum_{i=1}^{n} a_i\mu_i I(\tau_N, -\mu_i) + \pi\mu_0 F_0 \exp(-\tau_N/\mu_0)] & \text{(solar)} \\ B_\nu(T_s) & \text{(infrared)}, \end{cases} \quad (5)$$

where A_s represents the surface albedo, a_i and μ_i are Gauss weighting factors and zeros of the Legendre polynomials, respectively, n is the number of discrete streams and T_s the surface temperature.

Upon inserting the intensity solution into the boundary conditions in Eqs. (3)–(5), a set of linear equations is obtained from which the unknown L_j for each sublayer may be determined by means of a matrix inversion technique. After the constants of proportionality have been derived, the intensity distribution at any level in the atmosphere may be evaluated.

The upward and downward fluxes are then given by

$$F^{\uparrow}(\tau) = 2\pi \sum_{i=1}^{n} a_i\mu_i I(\tau,\mu_i),$$

$$F^{\downarrow}(\tau) = -2\pi \sum_{i=1}^{n} a_i\mu_i I(\tau, -\mu_i) - S(\tau, -\mu_0), \quad (6)$$

where

$$S(\tau, -\mu_0) = \begin{cases} \pi\mu_0 F_0 \exp(-\tau/\mu_0) & \text{(solar)} \\ 0 & \text{(infrared)}. \end{cases} \quad (7)$$

b. Solar and infrared spectra and model atmosphere

The primary gaseous absorbers considered in the solar spectrum are water vapor, ozone, carbon dioxide and molecular oxygen. The range of the solar spectrum considered in this study was that from 0.2 to 3.4 μm. The particular bands chosen, the primary absorber in each band and the fractional solar flux in each band (from Thekaekara, 1976) are given in Table 1. Incorporation of water vapor,

TABLE 1. Solar bands, absorbers and fractional solar flux.

Band (μm)	0.3	0.5	0.7	0.94	1.10	1.38	1.87	2.70	3.20
Absorber	O_3	O_3	O_2	H_2O	H_2O	H_2O	H_2O	CO_2/H_2O	H_2O
Fractional solar flux	0.0872	0.2696	0.2034	0.1347	0.0892	0.1021	0.0622	0.0300	0.0216

oxygen and carbon dioxide absorption in scattering atmospheres is accomplished by the exponential fitting of band transmissivities in which the pressure dependence on absorption in an inhomogeneous atmosphere is taken into account. The exponential fitting uses the solar band transmissivities derived by Liou and Sasamori (1975) in which the equivalent absorption coefficients and the corresponding weights for optically active gases are derived. These values are utilized to compute the single-scattering albedo and total optical depth for each atmospheric layer. Moreover, Mie scattering calculations for the central wavelengths of the bands are made for the aerosol and cloud particles. The clear atmosphere aerosol loading has a ground visibility of 23 km (Liou *et al.* 1978). The vertical resolution of the model atmosphere is varied according to spectral band and cloud structure. In the solar bands, 15 layers are used, with their thickness varied to better resolve the clouds and lower layers of the troposphere.

The primary gaseous absorbers considered in the infrared spectrum are water vapor, carbon dioxide and ozone. The infrared absorption bands used in this study are listed in Table 2. The Goody random model was used to compute flux transmission in all infrared spectral bands except the water vapor continuum (Roewe and Liou, 1978). The pressure variation over a vertical transmission path through the atmosphere is accounted for by the Curtis-Godson approximation. The scattering parameters for cirrus particles were obtained from the cylinder model proposed by Liou (1972).

The atmospheric profiles used were taken from the report by McClatchey *et al.* (1972) who compiled the water vapor, ozone, pressure, molecular density and temperature profiles for tropical (0–30°), winter and summer midlatitude (30–60°) and winter and summer arctic (60–90°) atmospheres. The concentration of the uniformly mixed gases of interest, carbon dioxide, molecular oxygen and air, were taken to be 5.11×10^{-4}, 0.236 and 1.02 g cm^{-2} mb^{-1}, respectively, constant with season and latitude.

The effects of clouds on the transfer of solar and infrared radiation are determined by the particle phase (liquid water or ice), concentration, size and size distribution. These factors combine to determine the single-scattering albedo, phase function and extinction cross section.

Four different cloud types (cumulus, altostratus, stratus and cirrus) were utilized in this research. The cloud-base heights utilized for these four cloud types were 1.7, 4.2, 1.4 and 4.6 km, respectively, obtained from climatology. The cloud temperatures were assumed to be the same as the environment from which the saturation water vapor path lengths within the cloud were estimated. Thicknesses of the cloud were allowed to vary above the base height for each cloud type. The cirrus clouds were considered to be composed exclusively of ice cylinders, randomly oriented horizontally in space based on several laboratory and field observations. For the purpose of light scattering calculations, ice cylinders were assumed to be monodispersed with a mean length of 200 μm, a mean radius of 30 μm and a mean concentration of 0.05 cm^{-3}. This ice cloud model (Liou, 1972) is the only kind available at the present time. The radiative properties of cirrus would of course depend on the assumption of the cloud compositions used in scattering calculations. Further research regarding the model of single-scattering properties of ice crystals is definitely required in order to investigate the uncertainty of the radiative properties of cirrus that may be encountered in the present calculations. The drop size distributions for the cumulus, altostratus and stratus clouds were based on the observations of Battan and Reitan (1957), Diem (1948) and Singleton and Smith (1960), respectively. Mean water/ice contents for Cu, As, St and Ci are 0.33, 0.24, 0.78 and 0.05 gm^{-3}, respectively. These observations were also summarized by Mason (1971). Within the infrared spectrum all clouds except cirrus clouds were considered to be blackbodies.

c. *Radiative properties of clouds*

As described previously, the radiation model calculates the transfer of solar radiation in a series of absorption bands. To obtain the reflection, transmission and absorption of the cloud for the entire solar spectrum, proper summation over the

TABLE 2. Infrared absorption bands and fractional infrared flux.

Absorption band	Wavenumber range (cm^{-1})	Fractional infrared flux
1. H_2O rotational	40–900	0.615
2. 15 μm CO_2	582–752	0.150
3. H_2O continuum	800–1200	0.160
4. 9.6 μm O_3	1000–1065	0.024
5. 6.3 μm H_2O	1200–2200	0.051

flux densities in each subspectral region weighted by the appropriate percentage of solar flux is required.

The reflection r may then be defined as the ratio of the reflected flux density to the incident solar flux density normal to the cloud top. Hence, the reflection of a cloud layer for the entire solar spectrum is given by

$$r = \sum_i [F_i^{\uparrow}(\tau_t)/F_i^{\downarrow}(\tau_t)](f_i/S_0), \tag{9}$$

where f_i denotes the amount of solar flux density in the ith spectral band, S_0 the solar constant and τ_t the optical depth at the cloud top.

Similar to the above definition, the transmission of a cloud layer may be defined as the ratio of the downward flux density at the cloud base to the incident solar flux density normal to the cloud top. Thus, the transmission of a cloud layer for the entire solar spectrum is given by

$$\tilde{t} = \sum_i [F_i^{\downarrow}(\tau_b)/F_i^{\downarrow}(\tau_t)](f_i/S_0), \tag{10}$$

where τ_b denotes the optical depth at the cloud base, and the notation $\tilde{t}$ is used here to distinguish from the notation t used by Chandrasekhar (1950) (see the Appendix).

Moreover, the absorption α of solar flux density within a cloud layer can be evaluated from the net flux density divergence between the top and bottom of the cloud. Therefore, the total absorption within a cloud layer for the entire solar spectrum is given by

$$\alpha = \sum_i \{[F_i(\tau_t) - F_i(\tau_b)]/F_i^{\downarrow}(\tau_t)\}(f_i/S_0). \tag{11}$$

The radiation model also calculates the transfer of infrared (IR) radiation in a series of absorption bands. Reflectivity, transmissivity and emissivity of the cloud for the entire IR spectrum may also be defined in terms of summations over the flux densities in each subspectral region. The flux reflectivity r_f of the cirrus cloud is defined as the ratio of the downward flux density at the cloud base to the incident IR flux density at the cloud base when cloud emission is not considered. The omission of cloud emission is to ensure that the cloud reflectivity arises from the scattering and absorption of cloud particles only. The IR cloud reflectivity in this case is downward. Thus, if cloud emission were not removed in the calculations, it would give an additional but undesirable contribution to the downward reflected component. Therefore, the flux reflectivity of the cirrus cloud for the entire IR spectrum is given by

$$r_f = \sum_i [F_i^{\downarrow}(\tau_b)/F_i^{\uparrow}(\tau_b)](f_i/R_0), \tag{12}$$

where f_i is the amount of IR flux in the ith spectral band and R_0 the total IR flux.

Likewise, the flux transmissivity t_f is defined as the ratio of the upward flux density at the cloud top to the incident IR flux density at the cloud base when there is no cloud emission contribution. Thus, the flux transmissivity of the cirrus cloud for the entire IR spectrum is given by

$$t_f = \sum_i [F_i^{\uparrow}(\tau_t)/F_i^{\uparrow}(\tau_b)](f_i/R_0). \tag{13}$$

Furthermore, the average flux emissivity of the cloud layer ϵ_f may be defined in terms of the emission from the top and the base of the cloud. The flux emissivity at the cloud top ϵ_{ft} is defined as the ratio of the emitted flux density from the cloud top to the blackbody flux density for the cloud-top temperature. Similarly, the flux emissivity at the cloud base ϵ_{fb} is defined as the ratio of the emitted flux density from the cloud base to the blackbody flux density for the temperature of the cloud base. Hence, the average flux emissivity of a cirrus cloud for the entire infrared spectrum is given by

$$\epsilon_f = (\epsilon_{ft} + \epsilon_{fb})/2, \tag{14}$$

where

$$\epsilon_{ft} = \sum_i [F_i^{\uparrow}(\tau_t)/\pi B_{\bar{\nu}_i}(T_t)](f_i/R_0),$$

$$\epsilon_{fb} = \sum_i [F_i^{\downarrow}(\tau_b)/\pi B_{\bar{\nu}_i}(T_b)](f_i/R_0),$$

and T_t and T_b are the temperatures at the cloud top and base, respectively. Both ϵ_{ft} and ϵ_{fb} are calculated by letting $F^{\uparrow}(\tau_b)$ and $F^{\downarrow}(\tau_t)$ equal zero, i.e., emission from the cirrus cloud layer only without the contributions from the atmospheres above and below.

3. Parameterization of reflection, transmission and absorption of solar radiation by cloud layers

Computations were made for various cloud thicknesses, holding the cloud base at a constant height for each case. The vertical liquid water content W of which cloud thickness is given by $W = w\Delta z$, where w is the mean water/ice content denoted earlier and Δz the geometrical cloud thickness. The ranges of cloud thicknesses used in the radiative transfer calculations of this study for Cu, As, St and Ci are, respectively, 0.15–2.25, 0.1–1.5, 0.05–0.75 and 0.1–2.9 km. The corresponding ranges of vertical liquid water/ice contents are 49.5–742.5, 24.0–360.0, 39.0–585.0 and 5.2–150.4 g m^{-2}, respectively.

The resulting reflection, transmission and absorption by the cloud layer were initially obtained for cumulus and stratus clouds in tropical, midlatitude winter and subarctic winter atmospheres with a zero surface albedo. Figs. 1 and 2 show reflection, transmission and absorption of solar radiation by the cloud layer as functions of the cosine of the solar zenith angle for a cumulus and stratus cloud

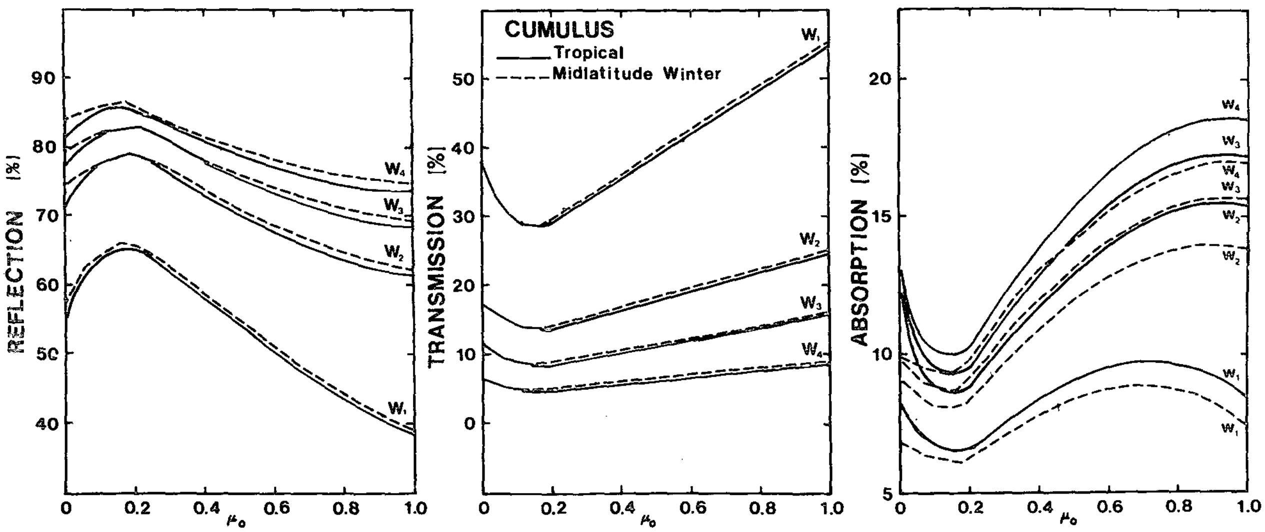

FIG. 1. Reflection, transmission and absorption of solar radiation by a cumulus cloud in tropical and midlatitude winter atmospheres.

having vertical liquid water contents of 50 (W_1), 150 (W_2), 250 (W_3) and 450 (W_4) g m^{-2}, respectively. The reflection, transmission and absorption values for both cloud types show no significant dependence on the atmospheric profile. In each case the variance is less than 2%. The relative insensitivity of the two atmospheric profiles on cloud absorption is a result of the small difference of water vapor amount within the clouds. In addition, the overlapping effect of liquid water absorption also reduces the influence of water vapor in the clouds. The cloud thickness or the vertical liquid water content, however, is a significant parameter whose change greatly affects the reflection, transmission and absorption of solar radiation. For change in vertical liquid water content of 400 g m^{-2} (0.5 km change in cloud thickness), reflection varies by as much as 43%, transmission 56% and absorption 12% when the sun is overhead the stratus cloud.

Fig. 3 depicts the solar radiative properties for several cases of cumulus and stratus clouds having the same vertical liquid water content. The variations in reflection, transmission and absorption for the two cloud types are due to cloud composition, primarily the particle size distribution. The particle size distributions utilized in this investigation are such that the cumulus has a significantly greater number density of particles with diameters <12 μm, while the stratus has a slightly greater number density of particles with diameters >12 μm.

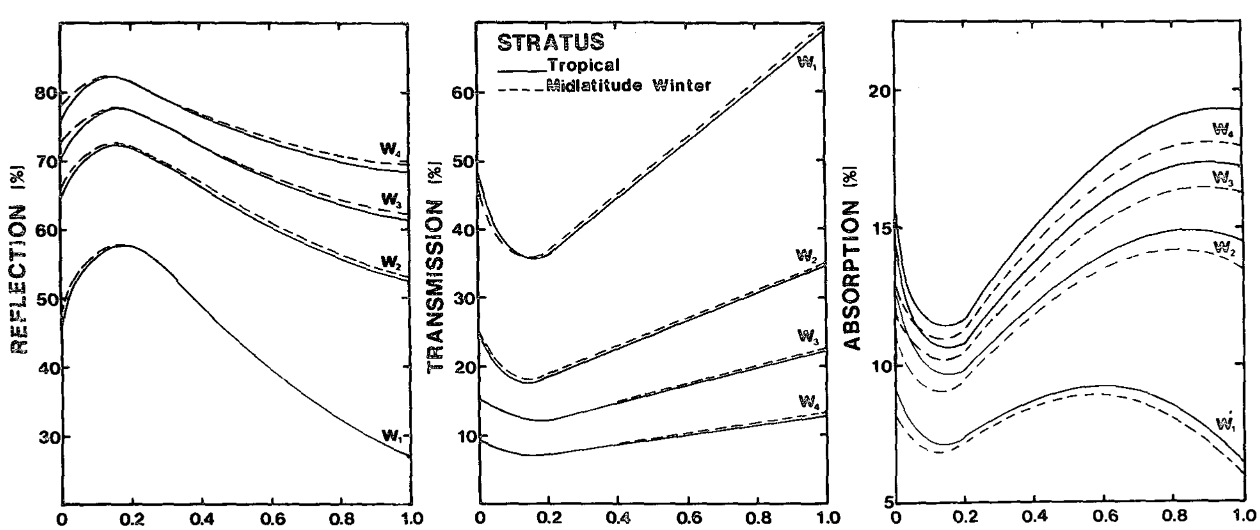

FIG. 2. As in Fig. 1 except by a stratus cloud.

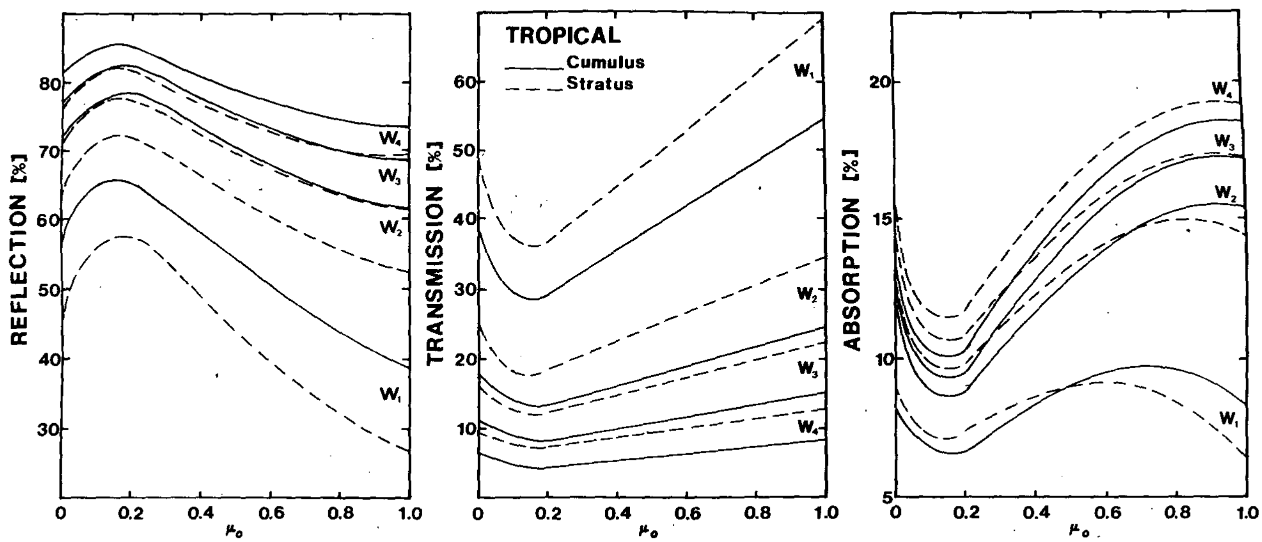

FIG. 3. As in Fig. 1 except by cumulus and stratus clouds having the same vertical liquid water content.

From Fig. 3 it is evident that the cloud reflection, transmission and absorption depend significantly on the cloud type (cumulus or stratus), with variations exceeding 15% when the sun is overhead.

On the basis of the above findings, calculations were made for each of the four cloud types in a tropical atmosphere with a zero surface albedo. The cloud reflection, transmission and absorption values were computed as functions of the cosine of solar zenith angle, μ_0, and vertical liquid water/ice content W. The solar radiative properties were computed for six values of μ_0 (0.01, 0.2, 0.4, 0.6, 0.8, 1.0) and 15 thicknesses of each cloud type over the ranges denoted previously. Fig. 4 illustrates a three-dimensional plot of the resulting reflection, transmission and absorption of solar radiation by a stratus cloud.

The reflection, transmission and absorption surfaces for each of the cloud types were then fit to an approximating polynomial equation using a bivariate regression analysis. The coefficients of each of the approximating predictors within the polynomial equation were calculated in such a way that the sum of the square of the differences between the actual values of the surface and values computed from the polynomial equation was a minimum. The following polynomial equation was found to yield the best fit of the data:

$$S(\mu_0,W) = \sum_{i=0}^{3} \sum_{j=0}^{3} b_{ij}\mu_0^i W^j, \tag{15}$$

where $S(\mu_0,W)$ denotes the cloud reflection, transmission or absorption, b_{ij} are the predictor coefficients and W is in units of 10^2 g m^{-2}. Tables 3–6 list the coefficients which were computed for a cumulus, altostratus, stratus and cirrus cloud, respectively. The rms error of the approximating functions is less than 2% for reflection and transmission and less than 1% for absorption.

4. Parameterization of the surface reflection

The parameterization of reflection, transmission and absorption of solar radiation by clouds described in the previous section does not include the effect of multiple reflections between the earth's albedo surface and the cloud base. The importance of these multiple reflections in determining the solar flux reaching the surface, especially in climate models, which predict snow and ice cover, is noted by Schneider and Dickinson (1976).

To include the surface reflection and the atmospheric effect between the cloud and the earth's surface, the surface is assumed to reflect according to Lambert's law with a surface albedo of A_g. Rayleigh scattering between the cloud and surface is not considered and the absorption between the cloud and surface is assumed to be mainly due to water vapor. We define the average reflection, transmission and absorption of solar radiation by cloud layers in the forms

$$\bar{s} = 2\int_0^1 r(\mu_0)\mu_0 d\mu_0, \tag{16}$$

$$\bar{t} = 2\int_0^1 \tilde{t}(\mu_0)\mu_0 d\mu_0, \tag{17}$$

$$\bar{\alpha} = 2\int_0^1 \alpha(\mu_0)\mu_0 d\mu_0. \tag{18}$$

Note that $\bar{s}$ is equivalent to the spherical albedo which represents the reflecting power of the entire planet.

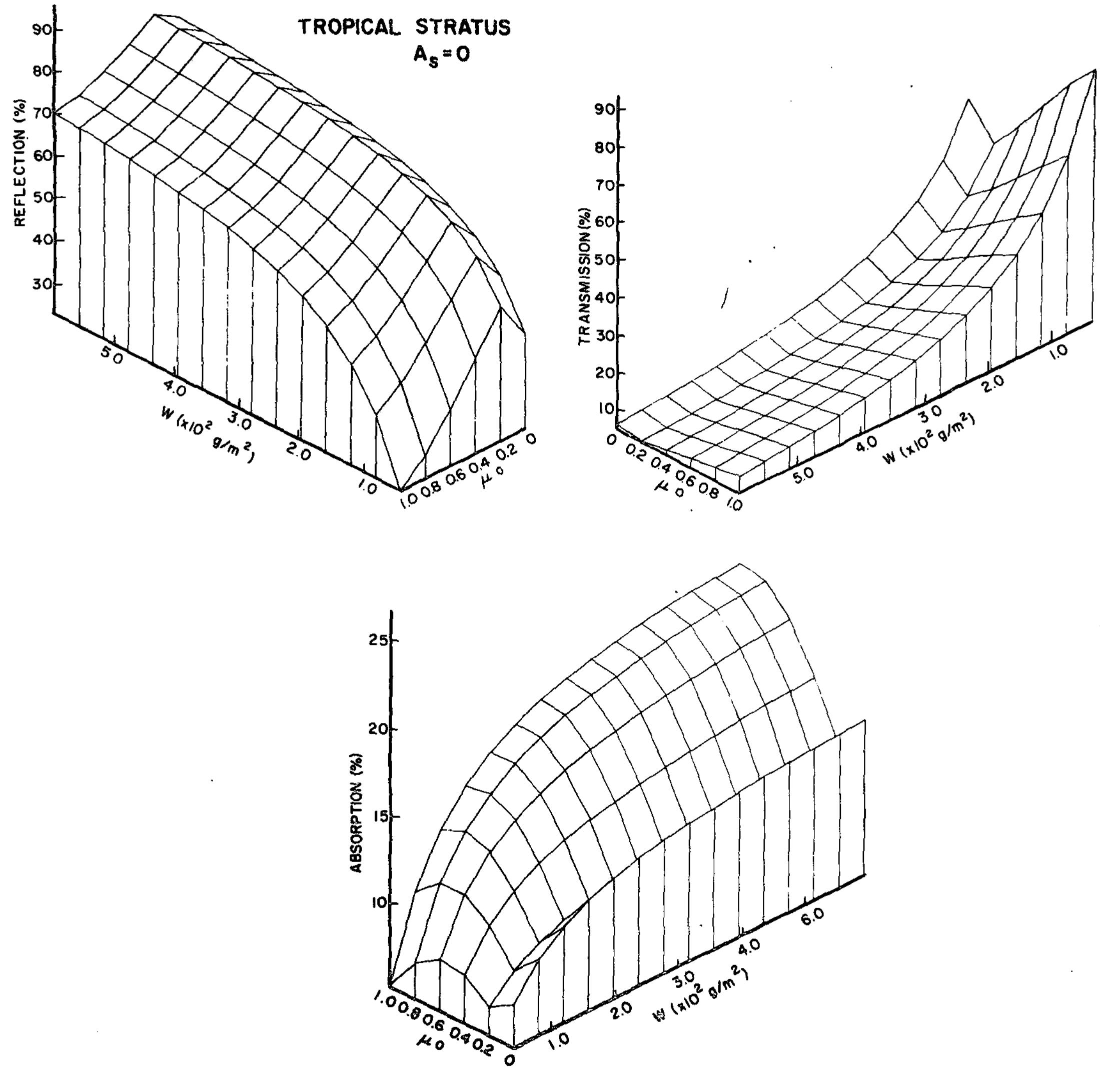

FIG. 4. Three-dimensional reflection, transmission and absorption surfaces of a stratus cloud as functions of the vertical liquid water content W and the cosine of solar zenith angle μ_0.

Fig. 5 depicts the surface reflection contribution to the upward flux density reaching the cloud base. This additional fraction of the upward flux density reaching the cloud bottom may be expressed in terms of the infinite series

$$\begin{aligned} f(\mu_0) &= \tilde{t}(\mu_0)[(1-a)^2A_s + (1-a)^4A_s{}^2\bar{s} \\ &\quad + (1-a)^6A_s{}^3\bar{s}^2 + \cdots] \\ &= \tilde{t}(\mu_0)(1-a)^2A_s[1 + (1-a)^2A_s\bar{s} \\ &\quad + (1-a)^4A_s{}^2\bar{s}^2 + \cdots] \\ &= \tilde{t}(\mu_0)\frac{(1-a)^2A_s}{1-(1-a)^2A_s\bar{s}}, \end{aligned} \tag{19}$$

where a represents the total water vapor absorption between the cloud base and the earth's surface. As defined in Eq. (19), $f(\mu_0)$ represents the additional upward flux density at the cloud base due to the surface reflection contribution. Thus, $f(\mu_0)$ multiplied by the average cloud transmission $\bar{t}$ gives the additional contribution to the cloud reflection component. Similarly, the additional cloud transmission component may be obtained by multiplying $f(\mu_0)$ (upward) by the average cloud reflection $\bar{s}$. If we let the superscript asterisk denote parameters which include the surface effect, the actual cloud reflection, transmission and absorption values are then given, respectively, by

$$r^*(\mu_0) = r(\mu_0) + f(\mu_0)\bar{t}, \tag{20}$$

$$\tilde{t}^*(\mu_0) = \tilde{t}(\mu_0) + f(\mu_0)\bar{s}, \tag{21}$$

$$\alpha^*(\mu_0) = \alpha(\mu_0) + f(\mu_0)\bar{\alpha}. \tag{22}$$

In the Appendix we show that Eqs. (20) and (21) are essentially equivalent to those derived by

TABLE 3. Approximating predictor coefficients b_{ij}. These coefficients were derived for a cumulus cloud whose thickness was varied upward from 0.15 to 2.25 km with a constant base height of 1.7 km. A tropical inhomogeneous atmospheric profile with a zero surface albedo was employed in the calculations (see Section 2b for further information regarding the model atmosphere used).

	j			
i	0	1	2	3
		Reflection		
0	0.51771E + 00	0.18270E + 00	−0.35851E − 01	0.23478E − 02
1	0.72387E + 00	−0.27109E + 00	0.51303E − 01	−0.34428E − 02
2	−0.21499E + 01	0.75225E + 00	−0.14588E + 00	0.97306E − 02
3	0.12157E + 01	−0.40120E + 00	0.78145E − 01	−0.52224E − 02
		Transmission		
0	0.43642E + 00	−0.22727E + 00	0.44702E − 01	−0.29117E − 02
1	−0.66319E + 00	0.35415E + 00	−0.70708E − 01	0.46403E − 02
2	0.19014E + 00	−0.10256E + 01	0.20574E + 00	−0.13534E − 01
3	−0.10157E + 01	0.54533E + 00	−0.10912E + 00	0.71667E − 02
		Absorption		
0	0.66103E − 01	0.34702E − 01	−0.69662E − 02	0.44253E − 03
1	−0.79484E − 01	−0.73614E − 01	0.17572E − 01	−0.10799E − 02
2	0.29293E + 00	0.25047E + 00	−0.55398E − 01	0.35139E − 02
3	−0.22077E + 00	−0.13272E + 00	0.28740E − 01	−0.17990E − 02

Chandrasekhar (1950) for the reflected and transmitted intensities in the planetary problem based on the principles of invariance.

The water vapor absorption a depends on the water vapor path length and atmospheric pressure. Liou and Sasamori (1975) modified the spectral absorptivities of water vapor and carbon dioxide

TABLE 4. Approximating predictor coefficients b_{ij}. As in Table 3, except that these coefficients were derived for an altostratus cloud whose thickness was varied upward from 0.1 to 1.5 km with a constant base height of 4.2 km.

	j			
i	0	1	2	3
		Reflection		
0	0.64457E + 00	0.28785E + 00	−0.12069E + 00	0.16673E − 01
1	0.44388E + 00	−0.45496E + 00	0.18695E + 00	−0.26672E − 01
2	−0.14875E + 01	0.13141E + 01	−0.54825E + 00	0.77121E − 01
3	0.85585E + 00	−0.71346E + 00	0.29823E + 00	−0.41922E − 01
		Transmission		
0	0.30735E + 00	−0.35373E + 00	0.14797E + 00	−0.20208E − 01
1	−0.44042E + 00	0.50188E + 00	−0.20975E + 00	0.28630E − 01
2	0.13537E + 01	−0.15738E + 01	0.66248E + 00	−0.90739E − 01
3	−0.73396E + 00	0.85256E + 00	−0.35858E + 00	0.49077E − 01
		Absorption		
0	0.70594E − 01	0.40901E − 01	−0.16982E − 01	0.21345E − 02
1	−0.28640E − 01	−0.18917E − 01	0.11167E − 01	−0.37025E − 03
2	0.19500E + 00	0.19298E + 00	−0.86520E − 01	0.98269E − 02
3	−0.15133E + 00	−0.10728E + 00	0.47143E − 01	−0.53524E − 02

TABLE 5. Approximating predictor coefficients b_{ij}. As in Table 3, except that these coefficients were derived for a stratus cloud whose thickness was varied upward from 0.05 to 0.75 km with a constant base height of 1.4 km.

	j			
i	0	1	2	3
		Reflection		
0	0.38094E + 00	0.26622E + 00	−0.62460E − 01	0.49964E − 02
1	0.88882E + 00	−0.39997E + 00	0.95133E − 01	−0.76756E − 02
2	−0.26293E + 01	0.11593E + 01	−0.28004E + 00	0.22779E − 01
3	0.14876E + 01	−0.64913E + 00	0.15951E + 00	−0.13106E − 01
		Transmission		
0	0.57723E + 00	−0.32665E + 00	0.76160E − 01	−0.60763E − 02
1	−0.88652E + 00	0.52284E + 00	−0.12434E + 00	0.10020E − 01
2	0.25788E + 01	−0.15559E + 01	0.37421E + 00	−0.30349E − 01
3	−0.13906E + 01	0.85278E + 00	−0.20753E + 00	0.16969E − 01
		Absorption		
0	0.63909E − 01	0.48691E − 01	−0.11011E − 01	0.86647E − 03
1	−0.25430E − 01	−0.11109E + 00	0.26393E − 01	−0.21154E − 02
2	0.11770E + 00	0.36211E + 00	−0.85429E − 01	0.68359E − 02
3	−0.13761E + 00	−0.18136E + 00	0.42026E − 01	−0.33420E − 02

for strong and weak bands based on measurements of Howard *et al.* (1956) and gave a general expression in the form

$$a_i = \frac{1}{\Delta\nu}\,[C_i + D_i \log_{10}(X_i + X_{0i})], \tag{23}$$

where

$$X_i = uP^{K_i/D_i}, \quad X_{0i} = 10^{-C_i/D_i}.$$

TABLE 6. Approximating predictor coefficients b_{ij}. As in Table 3, except that these coefficients were derived for a cirrus cloud whose thickness was varied upward from 0.1 to 2.9 km with a constant base height of 4.6 km.

	j			
i	0	1	2	3
		Reflection		
0	0.25264E + 00	0.45462E + 00	−0.29492E + 00	0.82536E − 01
1	−0.29273E + 00	0.22029E + 01	−0.24834E + 01	0.82674E + 00
2	−0.24531E + 00	−0.63610E + 01	0.75864E + 01	−0.25896E + 01
3	0.35097E + 00	0.38933E + 01	−0.49188E + 01	0.17187E + 01
		Transmission		
0	0.78722E + 00	−0.66348E + 00	0.44495E + 00	−0.12500E + 00
1	0.73255E + 00	−0.43772E + 01	0.47094E + 01	−0.15549E + 01
2	−0.68508E + 00	0.11764E + 02	−0.13716E + 02	0.46796E + 01
3	0.15453E + 00	−0.70865E + 01	0.87472E + 01	−0.30566E + 01
		Absorption		
0	0.49388E − 01	0.13715E + 00	−0.10122E + 00	0.28402E − 01
1	−0.19622E + 00	0.11882E + 01	−0.12267E + 01	0.40299E + 00
2	0.24118E + 00	−0.23149E + 01	0.28776E + 01	−0.10168E + 01
3	−0.94420E − 01	0.11305E + 01	−0.15772E + 01	0.58577E + 00

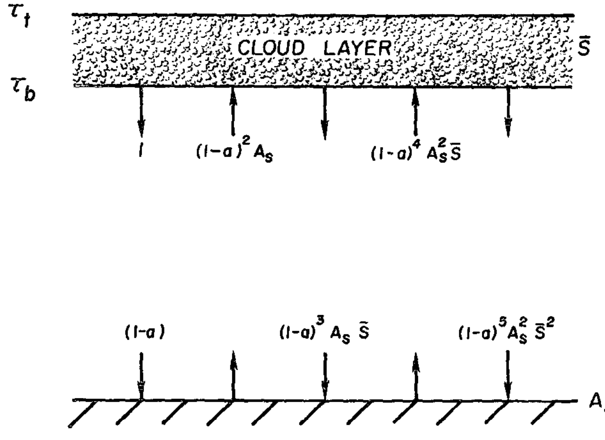

FIG. 5. Surface reflection contribution to the upward flux density at the cloud base.

In the above equations, $\Delta\nu$ is the spectral width of the absorption band, u the water vapor path length, P the atmospheric pressure, and C, D and K are empirical constants. Thus, the total water vapor absorption between the cloud and the surface may be obtained by

$$a(u,\bar{P}) = \sum_i a_i(u,\bar{P})[f_i/S_0], \tag{24}$$

where the summation is for all the water vapor absorption bands in the solar spectrum and $\bar{P}$ denotes the mean pressure between the cloud and surface.

Eqs. (20)–(22) were used to compute the reflection, transmission and absorption values for nonzero surface albedos. These values were compared with those calculated from the inhomogeneous transfer program including the surface reflection. Fig. 6 shows this comparison for a cumulus cloud in a tropical atmosphere with a surface albedo of 0.4. Comparisons were made for different cloud types, atmospheres and surface albedos and revealed that differences between the two methods were generally less than about 2%. It should be noted that since the calculations involve an additional reflection effect from a surface having an albedo of 0.4, the sum of absorption, reflection and transmission should be greater than unity as evident from this figure. The surface multiple-reflection effect is especially pronounced when clouds are thin. We also note that for a nonreflecting surface the absorption, reflection and transmission of a single-cloud layer are conserved. This is evident from the calculations depicted in Figs. 1–4.

Calculations were also made using the surface reflection parameterization as described previously except that the water vapor absorption between the cloud base and the earth's surface was neglected, i.e., a is zero in Eq. (19). The values for reflection and transmission showed no significant difference from those which considered the absorption by water vapor. However, the error in absorption values was ~2–3% greater.

It was shown in the previous section that the error of the parameterization of the solar radiative properties of clouds was less than 2% for each of the four cloud types. In addition, Fig. 6 shows that the error of the surface reflection parameterization is also less than 2%. Comparisons between reflection, transmission and absorption values calculated from the inhomogeneous transfer program and from the combination of the two parameterizations described previously were also examined. We found that the difference of values was normally less than 3% for the four cloud types utilized in this study.

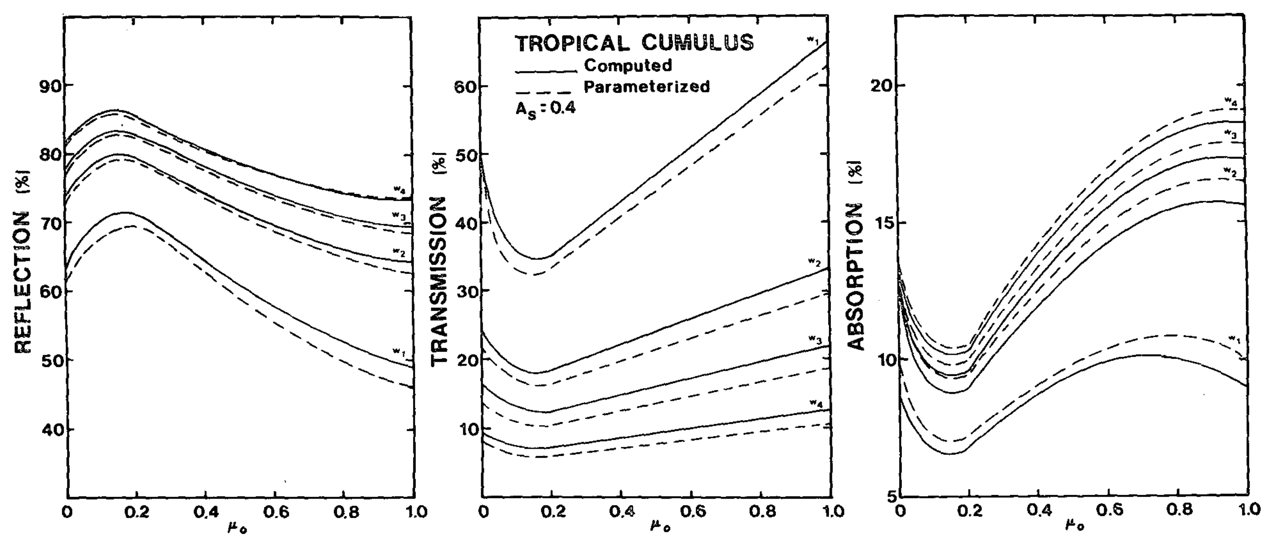

FIG. 6. Comparison of the reflection, transmission and absorption of solar radiation by a cumulus cloud as calculated by the transfer program and the albedo parameterization.

5. Parameterization of infrared reflectivity, transmissivity and emissivity of cirrus clouds

Since cirrus clouds may not be considered blackbodies as are other cloud types, their infrared radiative properties are quite important in studies of the weather, climate and radiation balance of the earth-atmosphere system. The infrared portion of the radiative transfer model was used to obtain the flux densities necessary to compute the IR reflectivity, transmissivity and emissivity by a cirrus cloud. These IR radiative properties of a cirrus cloud were computed over the range of vertical ice contents denoted previously. The flux density of radiation emitted by the atmosphere and entering the cloud base, $F^{\uparrow}(\tau_b)$, and the flux density of radiation due to atmospheric emission entering the cloud top, $F^{\downarrow}(\tau_t)$, were set equal to zero when calculating the emissivity of the cloud so that the effect of atmospheric emission was not included. On the other hand, the Planck function within the cloud layer was set equal to zero when calculating the reflectivity and transmissivity in order to exclude the effect of the cloud emission.

The plots of the computed radiative properties are displayed in Fig. 7. From these plots it can be seen that each of the radiative properties become nearly constant beyond a vertical ice content of 100 g m^{-2} ($\sim$2 km geometrical thickness). Consequently, computations beyond 150 g m^{-2} were not required. There is also a significant difference between the emissivity of the cloud at its base and top. This is due to the temperature gradient within the cloud layer ($T_b > T_t$). Since the emission of the cloud is dependent upon its temperature, the emission of the cloud sublayers below the cloud top will be greater than the emission of the sublayer at the cloud top. Upon taking into account the warmer base and colder top temperatures, the cloud emissivity is less at the cloud base than at the cloud top. The mean cirrus emissivity is obtained by averaging the cloud-top and cloud-base emissivity values.

In local thermodynamic equilibrium conditions in which a system has uniform temperature and isotropic radiation, Kirchhoff's laws may be applicable and we would anticipate an equilibrium between emission and absorption such that emissivity is equal to absorptivity. In the present calculations, however, the temperature of the cloud system is nonisothermal and radiation is neither isotropic. As a result of the significant temperature gradient and strong anisotropic radiation within the cloud, we find that the sum of the mean emissivity, reflectivity and transmissivity is greater than unity for $W > 50$ g m^{-2}. It is noted that the nonlocal thermodynamic equilibrium state (in reference to the cloud as a whole system) is particularly apparent for thick clouds because of the greater temperature lapse rate.

The reflectivity, transmissivity and emissivity curves were fit to an approximating polynomial equation using a least-squares technique similar to that described in Section 3. The following polynomial equation provided the best fit of the data:

$$R(W) = \sum_{i=0}^{5} c_i W^i, \quad (25)$$

where $R(W)$ denotes the flux reflectivity, transmissivity or emissivity of the cirrus cloud, c_i the predictor coefficients and W the vertical ice content in units of 10^2 g m^{-2}.

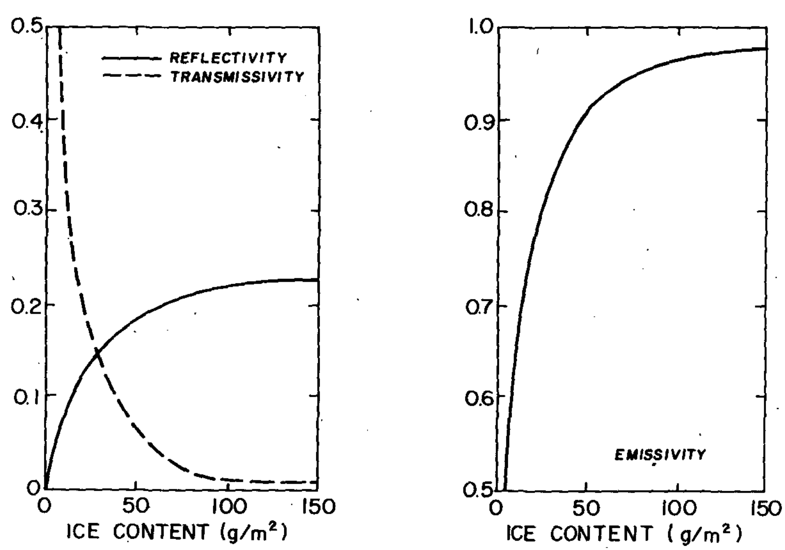

FIG. 7. Infrared transmissivity, reflectivity and emissivity of a semitransparent cirrus cloud.

Table 7 lists the coefficients, which were computed for the various infrared radiative properties of the cirrus cloud. In each case the rms error of the approximating functions is less than 1%.

6. Conclusion

Clouds are the most important atmospheric elements involved in the complex interactions of the radiation field. In this study, the cloud effects were carefully reproduced by the comprehensive radiative transfer program and then parameterized into simplifying empirical-theoretical equations for both the solar and infrared radiative properties of clouds.

By employing various different atmospheric profiles, the effect of the atmospheric profile on these radiative properties of clouds is shown to be insignificant. However, the cloud type, which is represented by a distinct particle size distribution, does have a significant influence on the values. The reflection, transmission and absorption of solar radiation by four cloud types are parameterized by polynomial equations in terms of the solar zenith angle and the cloud vertical liquid water/ice content. The additional effect of surface reflection on the reflection, transmission and absorption of solar radiation by clouds is further parameterized in which water vapor absorption between the cloud and the earth's surface is taken into account. Finally, parameterized equations for the IR reflectivity, transmissivity and emissivity of cirrus clouds are expressed in terms of the cloud vertical ice content. The parameterized equations for the radiative properties of clouds would be useful and practicable in conjunction with the study of the climate and climatic changes of the earth-atmosphere.

After this paper was written and submitted an interesting study of the parameterization scheme for the radiative properties of clouds appeared (Stephens, 1978a,b). Stephens used a two-stream approximation to evaluate the reflection and transmission and divided the solar spectrum into two parts. The optical depth in the two-stream equation is expressed in terms of liquid water content for various cloud types. Since the two-stream equation is quite inaccurate for optically thin clouds, he then tuned the single-scattering albedo and backscattering fraction parameter to match with more exact computations in terms of a series of empirical coefficients. The fitting was done for all the cloud types considered. Thus, this parameterization scheme provided no dependence on the cloud type, which is quite different from our present approach. Moreover, three steps were required in his method to evaluate reflection and transmission values. These involved two sets of two-stream equations for two solar parts, correlation between the optical depth and liquid water content, and the empirical fitting of the coefficients appeared in the two-stream equations. However, our approach is straightforward in that parameterization is done once and for all by fitting the resulting computations utilizing the two-dimensional polynomial equation in terms of the cosine of the solar zenith angle and liquid water content. In the present study, reflection, transmission and absorption for each cloud type for a given μ_0 and W can be immediately evaluated by inserting the predictor coefficients into the polynomial equations. In addition, our parameterization of surface reflection is mathematically and physically rigorous with verification from more reliable transfer calculations. Stephens' method, on the contrary, appears to have a number of glaring omissions: the neglect of water vapor absorption between the cloud and surface, the neglect of the surface reflection contribution on cloud absorption, and the incompleteness of the cloud transmission and reflection equations. Finally, Stephens provided empirical equations for the emissivity of water clouds. In the present study, however, we only consider cirrus as non-black clouds. Stephens noted the difference between the upward (cloud top) and downward (cloud base) emissivities of clouds. We have also discussed this difference in Section 2c.

TABLE 7. Approximating predictor coefficients c_i for a cirrus cloud in the infrared region. Cloud parameters used are the same as in Table 6.

i	Reflectivity	Transmissivity	Emissivity
0	0.30619E − 01	0.73597E + 00	0.28568E + 00
1	0.81134E + 00	−0.38162E + 01	0.35222E + 01
2	−0.18995E + 01	0.83288E + 01	−0.76275E + 01
3	0.24900E + 01	−0.91066E + 01	0.83096E + 01
4	−0.15805E + 01	0.48765E + 01	−0.44405E + 01
5	0.37581E + 00	−0.10141E + 01	0.92209E + 00

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APPENDIX

Deviation of Eqs. (20) and (21) from Chandrasekhar's Formulations

Chandrasekhar (1950) has formulated the reflected and transmitted intensities including the effect of a surface which reflects according to Lambert's law. They are given, respectively, by

$$I^*(0;\mu,\phi) = I(0;\mu,\phi) + \frac{A_s}{1-A_s\bar{s}}\mu_0 F_0 \gamma_1(\mu_0)\gamma_1(\mu), \quad (A1)$$

$$I^*(\tau_1;-\mu,\phi) = I(\tau_1;-\mu,\phi) + \frac{A_s}{1-A_s\bar{s}}\mu_0 F_0 \gamma_1(\mu_0)s(\mu)/\mu, \quad (A2)$$

where A_s denotes the surface albedo, and

$$\gamma_1(\mu) = e^{-\tau_1/\mu} + t(\mu)/\mu, \quad (A3)$$

$$\bar{s} = 2\int_0^1 s(\mu)d\mu, \quad (A4)$$

$$s(\mu) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^1 S(\mu,\phi;\mu_0,\phi_0)d\mu_0 d\phi_0, \quad (A5)$$

$$t(\mu) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^1 T(\mu,\phi;\mu_0,\phi_0)d\mu_0 d\phi_0, \quad (A6)$$

$$S(\mu,\phi;\mu_0,\phi_0) = \frac{4\mu}{F_0}I(0;\mu,\phi), \quad (A7)$$

$$T(\mu,\phi;\mu_0,\phi_0) = \frac{4\mu}{F_0}I(\tau_1;-\mu,\phi). \quad (A8)$$

The reflected and transmitted flux densities may be expressed by

$$\begin{aligned} F^{\uparrow *}(0) &= \int_0^{2\pi}\int_0^1 I^*(0;\mu,\phi)\mu d\mu d\phi \\ &= \int_0^{2\pi}\int_0^1 I(0;\mu,\phi)\mu d\mu d\phi + \frac{A_s}{1-A_s\bar{s}} \\ &\quad \times \mu_0 F_0\gamma_1(\mu_0)\int_0^{2\pi}\int_0^1 \gamma_1(\mu)\mu d\mu d\phi, \end{aligned} \quad (A9)$$

$$\begin{aligned} F^{\downarrow *}(\tau_1) &= \int_0^{2\pi}\int_0^1 I^*(\tau_1;-\mu,\phi)\mu d\mu d\phi \\ &= \int_0^{2\pi}\int_0^1 I(\tau_1;-\mu,\phi)\mu d\mu d\phi + \frac{A_s}{1-A_s\bar{s}} \\ &\quad \times \mu_0 F_0\gamma_1(\mu_0)\int_0^{2\pi}\int_0^1 s(\mu)/\mu\mu d\mu d\phi. \end{aligned} \quad (A10)$$

We first define the reflection and total transmission parameters, respectively, as follows:

$$\begin{aligned} \bar{t}(\mu_0) &= \bar{t}^{\mathrm{dir}}(\mu_0) + \bar{t}^{\mathrm{dif}}(\mu_0) \\ &= e^{-\tau_1/\mu_0} + F^{\downarrow}(\tau_1)/(\pi\mu_0 F_0), \end{aligned} \quad (A11)$$

$$r(\mu_0) = F^{\uparrow}(0)/(\pi\mu_0 F_0), \quad (A12)$$

where

$$F^{\downarrow}(\tau_1) = \int_0^{2\pi}\int_0^1 I(\tau_1;-\mu,\phi)\mu d\mu d\phi, \quad (A13)$$

$$F^{\uparrow}(0) = \int_0^{2\pi}\int_0^1 I(0;\mu,\phi)\mu d\mu d\phi. \quad (A14)$$

Next, we examine the terms which appear in (A3) and (A5) and we find

$$\frac{t(\mu_0)}{\mu_0} = \frac{1}{4\pi\mu_0}\int_0^{2\pi}\int_0^1 T(\mu_0,\phi_0;\mu,\phi)d\mu d\phi, \quad (A15)$$

$$\frac{s(\mu_0)}{\mu_0} = \frac{1}{4\pi\mu_0}\int_0^{2\pi}\int_0^1 S(\mu_0,\phi_0;\mu,\phi)d\mu d\phi. \quad (A16)$$

On the basis of the Helmholtz's principle of reciprocity (Chandrasekhar, 1950, Chap. 17, Section 52), we have

$$T(\mu_0,\phi_0;\mu,\phi) = T(\mu,\phi;\mu_0,\phi_0), \quad (A17)$$

$$S(\mu_0,\phi_0;\mu,\phi) = S(\mu,\phi;\mu_0,\phi_0). \quad (A18)$$

Thus, substituting (A8) into (A15) and (A7) into (A16) we obtain

$$\begin{aligned} \frac{t(\mu_0)}{\mu_0} &= \frac{1}{\pi\mu_0 F_0}\int_0^{2\pi}\int_0^1 I(\tau_1,-\mu,\phi)\mu d\mu d\phi \\ &= \frac{F^{\downarrow}(\tau_1)}{\pi\mu_0 F_0}, \end{aligned} \quad (A19)$$

$$\begin{aligned} \frac{s(\mu_0)}{\mu_0} &= \frac{1}{\pi\mu_0 F_0}\int_0^{2\pi}\int_0^1 I(0,\mu,\phi)\mu d\mu d\phi \\ &= \frac{F^{\uparrow}(0)}{\pi\mu_0 F_0}. \end{aligned} \quad (A20)$$

By virtue of the definitions of the total transmission and reflection given in (A11) and (A12) we find

$$\gamma_1(\mu_0) = \bar{t}^{\mathrm{dir}}(\mu_0) + \bar{t}^{\mathrm{dif}}(\mu_0) = \bar{t}(\mu_0), \quad (A21)$$

$$s(\mu_0)/\mu_0 = r(\mu_0). \quad (A22)$$

The integrals appearing in the last term of (A9) and (A10) may now be written as

$$\int_0^{2\pi}\int_0^1 \gamma_1(\mu)\mu d\mu d\phi = 2\pi\int_0^1 \bar{t}(\mu)\mu d\mu = \pi\bar{\bar{t}}, \quad (A23)$$

$$\int_0^{2\pi}\int_0^1 s(\mu)/\mu\mu d\mu d\phi = 2\pi\int_0^1 r(\mu)\mu d\mu = \pi\bar{s}, \quad (A24)$$

where $\bar{\bar{t}}$ is the total global transmission and $\bar{s}$ represents the global reflection, i.e., the spherical albedo.

By means of (A13), (A14), (A21), (A23) and (A24), Eqs. (A9) and (A10) may be rewritten in the forms

$$F^{\uparrow *}(0) = F^{\uparrow}(0) + \frac{A_s}{1-A_s\bar{s}}\pi\mu_0 F_0\bar{\bar{t}}\bar{t}(\mu_0), \quad (A25)$$

$$F^{\downarrow *}(\tau_1) = F^{\downarrow}(\tau_1) + \frac{A_s}{1 - A_s\bar{s}}\pi\mu_0 F_0 \bar{s}\tilde{t}(\mu_0). \quad \text{(A26)}$$

Dividing (A25) and (A26) by $\pi\mu_0 F_0$, we obtain

$$r^*(\mu_0) = r(\mu_0) + f(\mu_0)\bar{t}, \quad \text{(A27)}$$

$$\tilde{t}^*(\mu_0) = \tilde{t}(\mu_0) + f(\mu_0)\bar{s}, \quad \text{(A28)}$$

where

$$f(\mu_0) = \frac{A_s\tilde{t}(\mu_0)}{1 - A_s\bar{s}}. \quad \text{(A29)}$$

These are equations that we have derived based on the ray tracing technique for the purpose of parameterizing the surface reflection effect when the absorption between the cloud layer and the surface is neglected (i.e., $a = 0$).

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