Remote Sensing of the Thickness and Composition of Cirrus Clouds from Satellites

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ABSTRACT

A retrieval technique is presented for the determination of the surface temperature, the thickness and transmissivity of cirrus clouds, and the fraction of the cirrus cloudiness by means of four observed upwelling radiances in the 10 μm window region. On the basis of radiative transfer calculations for mean wavenumbers of 900, 950, 1100 and 1150 cm^{-1}, assumptions are made in the theoretical retrieval analyses that water vapor effects above cirrus clouds are negligible and that ratios of the transmissivities are linear functions of the cloud thickness. Error analyses employing climatological data reveal that independent random errors in temperature and humidity profiles introduce insignificant errors in the four resulting parameters. The resulting errors caused by random errors in the expected upwelling radiances, however, depend upon their standard deviations. Once the thickness and the transmissivity at a given wavenumber of a cirrus cloud have been determined, we illustrate that the vertical ice content may be estimated assuming that ice particles are randomly oriented in a horizontal plane.

1. Introduction

Cirrus clouds have been noted to introduce serious difficulties in remote sensing of atmospheric temperature and humidity profiles and surface conditions, owing to their semi-transparent appearance in the visible as well as in the infrared region. It is extremely important, therefore, to derive the reliable and accurate radiation properties of cirrus clouds from an independent set of sounding frequencies in conjunction with the atmospheric parameter evaluation. Moreover, determination of the vertical ice content over the global atmosphere is equally important from the point of view of climatology studies and, perhaps, numerical weather prediction.

The paper by Houghton and Hunt (1971) apparently was the first one to explore the passive remote sensing possibility of ice clouds by means of two wavelengths in the far infrared. Liou (1974) discussed emission and transmission properties of cirrus clouds in the 10 μm window region in conjunction with the remote sensing potential from satellites. Bunting and Conover (1974) proposed a simple means for the estimation of the vertical ice content of cirrus clouds assuming the exponential attenuation of IR radiation.

With respect to the atmospheric parameter determination, Chahine (1974) presented a numerical method to derive vertical temperature profiles in cloudy atmospheres by means of radiances obtained from two partially overlapping fields of view. Taylor (1974) described an approach employing soundings at two different zenith angles for the determination of temperature profiles in the presence of clouds. Both of these two important studies concerning the remote sensing of cloudy atmospheres involved attempts to remove the cloud effect rather than to incorporate it into the radiative transfer equation.

Cirrus clouds consist of non-spherical ice crystals of various sizes, possibly randomly oriented in a horizontal plane. Other unknown variables include the ice-crystal concentration, the cloud thickness and the location of the cloud in the atmosphere. Moreover, for a given upwelling radiance at the satellite point of view, there will be additional unknown variables associated with the atmospheric temperature and gaseous profiles. In view of a large number of unknown variables, parameterization equations describing the radiation field of an atmosphere containing cirrus clouds have to be formulated. Retrieval of the unknown parameters, each of which may represent a combination of several variables, may be carried out from satellite radiance observations.

In this paper, I would like to present some hypothetical analyses and calculations concerning the determination of the structure and composition of cirrus clouds from a set of synthetic radiance observations in the window region.

2. Theoretical analyses

We assume that within the field of view of the satellite radiometer the atmosphere contains η portion of cloudiness. The monochromatic upwelling radiance measured by the satellite radiometer at the top of a

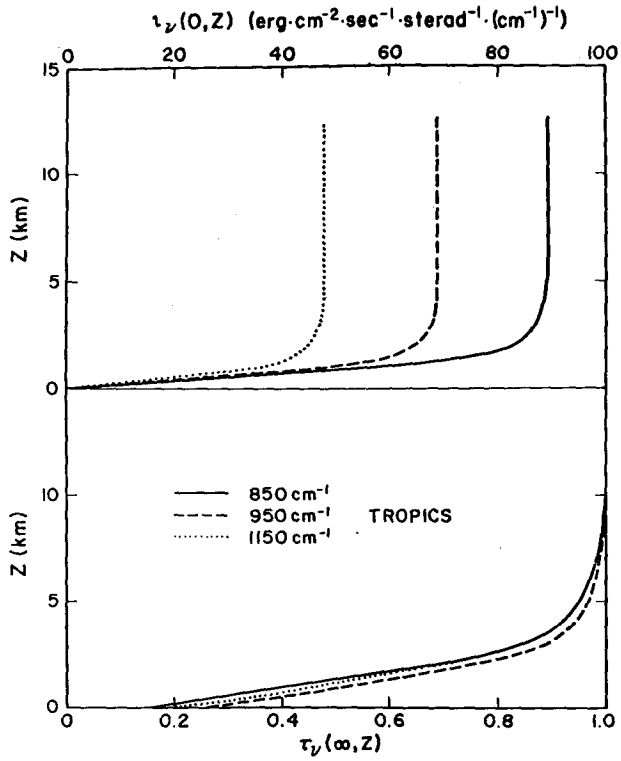

FIG. 1. Values of the integral term and transmission function shown in Eq. (2). The integral term is evaluated from the surface to the level z, while the transmission function of water vapor is calculated from the top of the atmosphere to the level z. The calculations are carried out for wavenumbers of 850, 950, 1150 cm^{-1} employing a tropical atmosphere.

partially cloudy atmosphere is given by

$$\tilde{I}_\nu^{pc}(\infty)=\eta I_\nu^c(\infty)+(1-\eta)I_\nu^{nc}(\infty), \tag{1}$$

where the upwelling radiance arising from the clear portion of the atmosphere is simply

$$I_\nu^{nc}(\infty)=B_\nu(T_s)\tau_\nu(\infty,0)+\int_{z=0}^{z=\infty} B_\nu[T(z)]d\tau_\nu(\infty,z). \tag{2}$$

And the upwelling radiance due to the cloudy portion of the atmosphere may be expressed as

$$I_\nu^c(\infty)=\tau_\nu^c I_\nu(z_b)\tau_\nu(\infty,z_t)+\int_{z=z_t}^{z=\infty} B_\nu[T(z)]d\tau_\nu(\infty,z). \tag{3}$$

In this equation the cloud transmissivity τ_ν^c is defined as the ratio of the upwelling radiance at the cloud top, i.e., $I_\nu(z_t)$, to that at the cloud base, and it can be written as

$$\tau_\nu^c=I_\nu(z_t)/I_\nu(z_b), \tag{4}$$

where z_b and z_t denote the cloud-base and cloud-top heights, respectively, and the upwelling radiance reaching the cloud base is given by

$$I_\nu(z_b)=B_\nu(T_s)\tau_\nu(z_b,0)+\int_{z=0}^{z=z_b} B_\nu[T(z)]d\tau_\nu(z_b,z). \tag{5}$$

In Eqs. (3)–(5), T_s is the surface temperature, τ_ν the transmission function of gases which will be discussed later, and the Planck function in the wavenumber domain is

$$B_\nu(T)=a\nu^3/(e^{b\nu/T}-1), \tag{6}$$

with $a=1.1272\times10^{-5}$ erg cm^2 s^{-1} and $b=1.4389$ cm K. It should be noted that $I_\nu(z_t)$ in Eqs. (3) and (4) must be evaluated from the transfer equation including absorption, emission and scattering of gases and cloud particles subject to the radiation boundary conditions of the top and base of the cloud concerned (Liou, 1974).

Cirrus clouds are normally fairly high in the atmosphere with their top heights on the order of 10 km or higher. Thus, we may select spectral channels in the window region where the effect of water vapor absorption above the cirrus cloud layer can be neglected such that

$$\left.\begin{aligned}\int_{z=z_t}^{z=\infty} B_\nu[T(z)]d\tau_\nu(\infty,z)\approx 0\\ \tau_\nu(\infty,z_t)\approx 1\end{aligned}\right\}. \tag{7}$$

Justification of Eq. (7) is illustrated in Fig. 1 (see also Section 3a for water vapor absorption). We have employed a tropical atmospheric profile (McClatchey *et al.*, 1971) in the calculations. The upper diagram represents the results of the first equation in (7) for the integration limits from the surface to a level z. Three wavenumbers of 850, 950 and 1150 cm^{-1} were used. It is evident that the integral term in Eq. (2) has negligible contribution above a height of about 5 km, which is normally lower than the base heights of cirrus clouds. The lower diagram represents the transmission term in Eq. (2). Clearly, we see that above about 8 km, the transmission function τ_ν of water vapor for a moist atmosphere is very close to 1. Consequently, we would expect that our first assumptions can be applied to other atmospheric profiles without difficulties.

Thus the observed upwelling radiance at the top of a partially cloudy atmosphere denoted in Eq. (1) becomes

$$\tilde{I}_\nu^{pc}(\infty)\approx(1-\eta+\eta\tau_\nu^c)\{B_\nu(T_s)\tau_\nu(z_b,0)+\int_{z=0}^{z=z_b} B_\nu[T(z)]d\tau_\nu(z_b,z)\}, \tag{8}$$

where the unknown parameters are the fraction of cloudiness, the surface temperature, the cloud transmissivity which is wavenumber dependent, and the temperature profile and the transmission function of water vapor. Eq. (8) expresses the upwelling radiance for a single wavenumber. However, a satellite instrument can distinguish only finite bandwidths $\Delta\nu(\nu_1,\nu_2)$

with the instrumental slit function $\phi(\nu_1,\nu_2)$, which we shall ignore in the following theoretical development. The finite bandwidth is normally so small that the variation of Planck function with respect to the wavenumber can be neglected. This is particularly evident in the 10 μm window region. Thus, the upwelling spectral radiance may be written as

$$\tilde{I}^{pc}_{\Delta\nu}(\infty)=\int_{\nu_1}^{\nu_2}\tilde{I}^{pc}_{\nu}(\infty)\frac{d\nu}{\Delta\nu}=(1-\eta+\eta\tau^c_{\Delta\nu})\{B_{\Delta\nu}(T_s)\tau_{\Delta\nu}(z_b,0)+\int_{z=0}^{z=z_b}B_{\Delta\nu}[T(z)]d\tau_{\Delta\nu}(z_b,z)\}. \quad (9)$$

Here we also note that the variation of cloud transmissivity within a small spectral interval is relatively small in view of the slow varying refractive indices of ice in the window region. Consequently, it is physically reasonable to use a mean wavenumber to calculate the spectral cloud transmissivity.

If prior knowledge of the temperature and humidity profiles in the atmosphere were available, say from climatological data, then $\tau_{\Delta\nu}(z_b,0)$ and the integral term in Eq. (9) could be evaluated. With a measurement of the upwelling radiance from the satellite radiometer in the window channel, there are only three unknown parameters, η, $\tau^c_{\Delta\nu}$ and T_s. How many radiance measurements are required to determine all these variables?

We shall now examine the cloud transmissivity $\tau^c_{\Delta\nu}$ defined in Eq. (4). Calculations of the transmissivity as a function of the cirrus cloud thickness and ice content were carried out by Liou and Stoffel (1976). It was assumed that the cirrus cloud was composed of long circular cylinders randomly oriented in space with a base height of 8 km and an isothermal temperature of −36°C. Mean wavenumbers of 900, 950, 1100 and 1150 cm^{-1} were chosen in this investigation. Fig. 2 shows the ratios of the transmissivities of 950, 1100 and 1150 cm^{-1} to the transmissivity of 900 cm^{-1}. It is evident that to a good approximation, these curves are fairly close to a linear function of the thickness.

On the basis of the theoretical radiative transfer calculations, we postulate that the transmissivities of cirrus clouds for wavenumbers in the window region can be scaled as

$$\tau^c_i=(a_i\Delta z+b_i)\tau^c, \quad (10)$$

TABLE 1. Linear coefficients for cloud transmissivities.

$\bar{\nu}$(cm^{-1})	i	a_i	b_i
900	1	0	1
950	2	−0.02	0.99
1100	3	−0.25	1.23
1150	4	−0.04	1.01

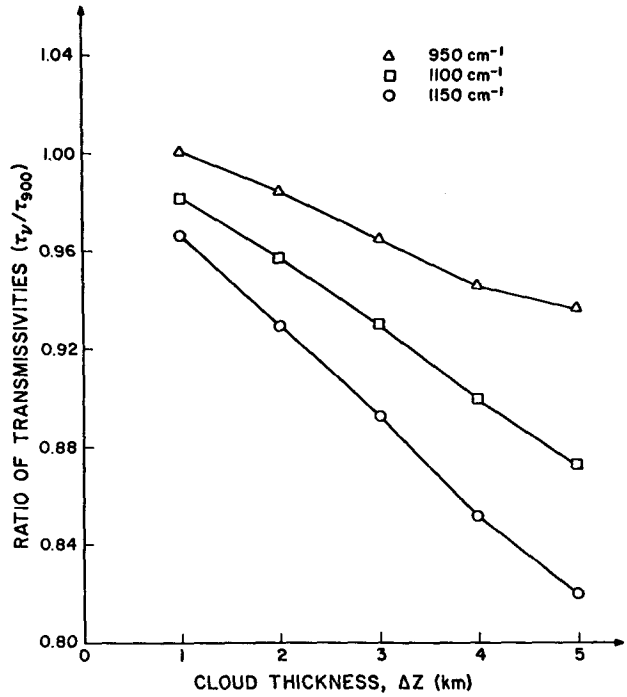

FIG. 2. Ratios of the cloud transmissivity of 950, 1100 and 1150 cm^{-1} to that of 900 cm^{-1}. The cloud transmissivities are computed from the radiative transfer program including scattering, absorption and emission of water vapor and cloud particles.

where we change the wavenumber index $\Delta\nu$ to i. In Eq. (10), τ^c represents the transmissivity at a reference wavenumber (900 cm^{-1} in Fig. 2), and a_i and b_i are certain constants given in Table 1.

In view of the linear approximation for the ratios of the transmissivities, the cloud fraction η, the surface temperature T_s, the cloud thickness Δz and the cloud transmissivity τ^c at the reference wavenumber may be determined from four radiance measurements in the window region provided that the atmospheric temperature and humidity profiles are given, for example, from climatological data. For the convenience of the following discussions, let

$$\left.\begin{aligned} I^*_i&=\int_{z=0}^{z=z_b}B_i[T(z)]d\tau_i(z_b,z)\\ \tau^*_i&=\tau_i(z_b,0)\end{aligned}\right\}, \quad (11)$$

with * denoting values to be evaluated from the climatological data. Thus from Eq. (9) we have the following expressions for the four upwelling radiances:

$$\tilde{I}^{pc}_i=[1-\eta+\eta(a_i\Delta z+b_i)\tau^c][B_i(T_s)\tau^*_i+I^*_i],\quad i=1,2,3,4. \quad (12)$$

Defining

$$Q_i(T_s)=\tilde{I}^{pc}_i/[B_i(T_s)\tau^*_i+I^*_i], \quad (13)$$

Eq. (12) can be written in the form

$$Q_i=1-\eta+\eta(a_i\Delta z+b_i)\tau^c, \quad i=1,2,3,4, \tag{14}$$

a nonlinear equation consisting of four unknown parameters, η, τ^c, Δz and T_s. First, we find that η can be eliminated to yield

$$(Q_1-1)[(a_i\Delta z+b_i)\tau^c-1]-(Q_i-1)(\tau^c-1)=0, \quad i=2,3,4. \tag{15}$$

Second, τ^c can be omitted in Eq. (15) to obtain

$$\frac{(Q_1-Q_2)[(Q_1-1)(a_i\Delta z+b_i)-(Q_i-1)]}{(Q_1-Q_i)[(Q_1-1)(a_2\Delta z+b_2)-(Q_2-1)]}-1=0, \quad i=3,4. \tag{16}$$

Finally, from the two equations in (16) we eliminate Δz to give

$$\frac{[Q_1(b_3-b_2)+Q_2(1-b_3)+Q_3(b_2-1)][Q_1(a_2-a_4)+Q_2a_4-Q_4a_2]}{[Q_1(b_4-b_2)+Q_2(1-b_4)+Q_4(b_2-1)][Q_1(a_2-a_3)+Q_2a_3-Q_3a_2]}-1=0. \tag{17}$$

Eq. (17) represents a nonlinear equation consisting of an unknown parameter T_s, which can be determined from a set of four upwelling radiances $\tilde{I}_i^{pc}$ measured in a partially cloudy atmosphere. On deriving the surface temperature, the cloud thickness, the cloud transmissivity at the reference wavenumber and the fraction of cloudiness can be subsequently evaluated. In the next section, we shall investigate the possible errors of the resulting parameters caused by the use of the climatological temperature and humidity profiles and the possible random errors in the expected upwelling radiances.

3. Computational analyses

In order to test the idea outlined in the previous section, computational analyses have been carried out. Water vapor absorption properties are first discussed in Section 3a. In 3b, a synthetic partially cloudy atmosphere is constructed. Error analyses are then presented in 3c. Some notes are finally given in 3d for the estimation of the vertical ice content of cirrus clouds.

a. Water vapor absorption in the 10 μm window

The transmission function for water vapor in the 10 μm window region is due to the selective absorption by weak lines and the continuous absorption, and can be written

$$\tau_\nu=\tau_\nu(\text{selective})\times\tau_\nu(\text{continuum}). \tag{18}$$

The selective absorption by weak lines is well understood (see, e.g., Goody, 1964). However, continuous absorption in the 10 μm window has been a subject of considerable speculation. In recent years, it seems to have been established that the water vapor continuum is caused by the broadening of foreign gases and the *e*-type (water dimmer) absorption (Bignell, 1970; Burch, 1970). Thus the absorption coefficient of the continuum may be expressed as

$$k(T,p,e)=k_1(T)p+k_2(T)e, \tag{19}$$

where p is the total pressure and e the partial vapor pressure. Based on Bignell's measurements, k_2 is much greater than k_1, although the temperature dependence of k_2 has not been investigated completely.

With these sources of absorption taken into consideration the transmission function for water vapor may be written as

$$\tau_\nu(u)=\exp[-\eta_\nu u^{\frac{1}{2}}-(k_1p+k_2e)u], \tag{20}$$

where u is the vertical path length of water vapor and the weak line parameter

$$\eta_\nu=(\pi\alpha\sigma)^{\frac{1}{2}}/\delta, \tag{21}$$

with σ the mean line intensity, δ the mean line spacing and α the mean line width. These weak line parameters have been given by Roach and Goody (1958) based on fitting the statistical band model from observed data. According to Bignell (1970) and Burch (1970), the *e*-type absorption continuum is dominant in the 10 μm window region. Furthermore, comparisons of synthetic IRIS (Infrared Interferometer Spectrometer) radiances computed from Bignell's water vapor continuum data with observed radiances gave good agreement (Kunde *et al.*, 1974). In view of this evidence and in view of the fact that the absorption coefficient is continuous, we have computed water vapor transmissivities employing the mean wavenumbers of 900, 950, 1100 and 1150 cm^{-1} in conjunction with radiative transfer calculations reported in this paper.

b. Synthetic partially cloudy atmosphere

To test the accuracy of the procedures described in Section 2, a synthetic partially cloudy atmosphere has been constructed (Fig. 3). It is assumed that within the field of view of the satellite radiometer, there is 70% of cirrus cloudiness whose thickness is 2 km, and the base of the cirrus cloud is located at 8 km in the atmosphere. Based on radiative transfer calculations, we obtain a transmissivity of 0.675 for a 2 km cirrus cloud at a wavenumber of 900 cm^{-1}.

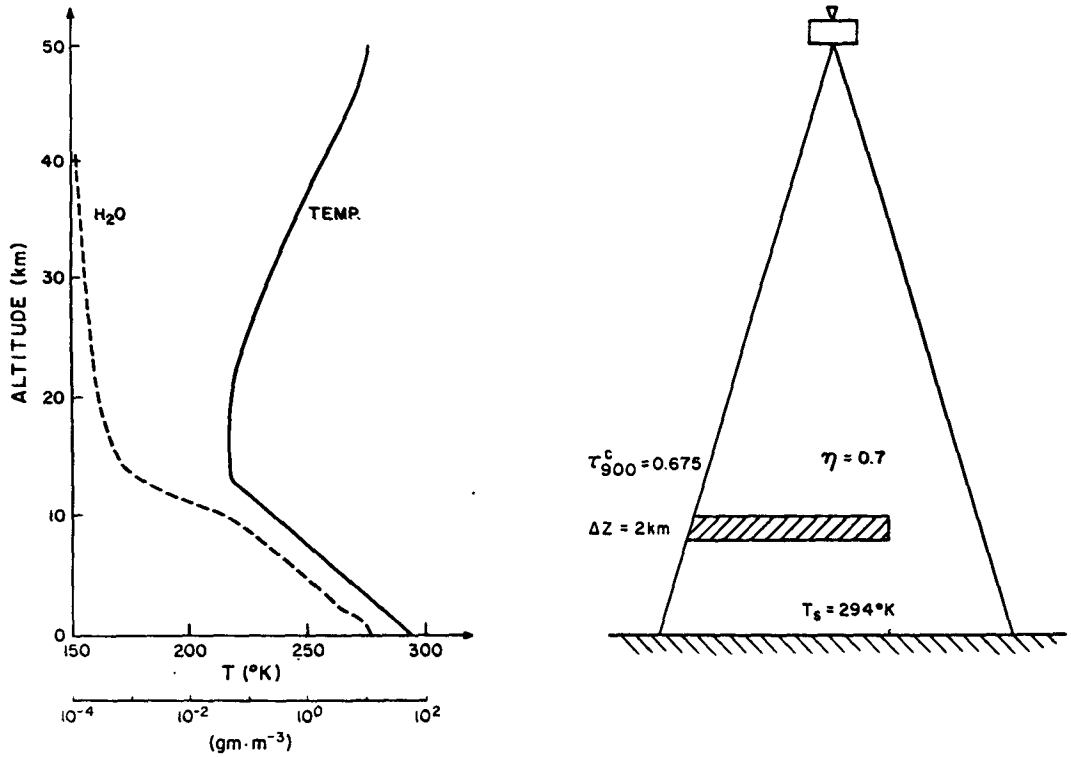

FIG. 3. A diagram for the synthetic partially cloudy atmosphere. The climatological temperature and humidity profiles for a mid-latitude summer atmosphere are used in the analyses.

Moreover, a surface temperature of 294 K is employed. With respect to the atmospheric temperature and humidity profiles, we use the tabulated climatological data by McClatchey *et al.* (1971) for the mid-latitude summer atmosphere.

With the transmissivity at 900 cm^{-1} given, the rest of the transmissivities can be evaluated according to Eq. (10). Hence, we can theoretically calculate the expected upwelling radiances in the partially cloudy atmosphere for four wavenumbers by means of Eqs. (1)–(3). Once the upwelling radiances have been obtained, we proceed to Eqs. (14)–(17) to retrieve the surface temperature, the cloud thickness, the transmissivity at 900 cm^{-1} and the fraction of cirrus cloudiness.

c. *Error analyses*

Eq. (17) represents a complicated nonlinear equation which contains the unknown surface temperature. We may write

$$F(T_s)=0. \tag{22}$$

Since the function is a high-order nonlinear equation, we would expect that it consists of a number of roots for T_s in the $(0,\infty)$ domain. However, there will be only one root that is our desired solution. Fig. 4 illustrates the behavior of Eq. (22) for the conditions given in Section 3b with the realistic range of T_s from 240 to 340 K. In addition to the desired root of 294 K, we find that 263 K also satisfies Eq. (22). Hence, care should be taken in the error analysis to disregard the undesirable roots. Fortunately, from a number of error analyses exercises, we found that the correct value always lies at the zero intersection of the right-hand curve in Fig. 4.

The first error analyses were done for the upwelling radiance. Random numbers were arbitrarily selected and added to the upwelling radiances calculated exactly from the synthetic atmosphere. The retrieval procedures using the upwelling radiances with random errors were carried out for the surface temperature, and subsequently for the cloud thickness, the reference cloud transmissivity and the cloud fraction. We define the following statistical terms:

$$\Delta I_m=\frac{1}{N}\sum_{i=1}^{N}\Delta I_i/I_i, \tag{23}$$

$$\Delta I_{rms}=\left[\sum_{i=1}^{N}(\Delta I_i/I_i-\Delta I_m)^2\right]^{\frac{1}{2}}. \tag{24}$$

Table 2 lists the resulting parameters in terms of these two variables. The second line of this table shows that 1.2% random error produces almost negligible errors in the four resulting parameters. A random error of 11.2% introduces deviations of about 3.6 K,

TABLE 2. Random errors in the measured radiances.

ΔI_m	ΔI_{rms}	T_S(°K)	ΔZ(km)	τ^c_{900}	η
0	0	294	2	0.675	0.7
0.012	0.002	294.346	1.924	0.684	0.702
0.056	0.010	295.745	1.655	0.719	0.710
0.112	0.020	297.549	1.385	0.760	0.719
0.002	0.003	297.486	2.187	0.644	0.690

0.61 km, 0.08 and 0.02 for the surface temperature, the cloud thickness, the reference cloud transmissivity and the fraction of cloudiness, respectively. Other experiments employing different sets of random numbers were also carried out. It was found that a large value of ΔI_{rms} would introduce large errors in the four resulting parameters. The fifth line of this table illustrates an example as to how the root mean square affects the present analysis. We see that a small random error of 0.2% having a larger rms (0.3%) produces about the same errors in the four resulting parameters as the case when a random error of 11.2% with a small rms of 2% is introduced.

It is known that the temperature and humidity profiles derived from radiosonde are normally not available in ocean areas, particularly in the Southern Hemisphere. The best guesses of the temperature and humidity profiles in cloudy conditions, perhaps, may be obtained from climatology. Deviations of the climatological values from the true profiles are likely to occur. Thus, it is of importance to examine how the errors in the temperature and humidity profiles affect the resulting recovered parameters developed previously. (Note that if true temperature and humidity profiles are available, such error analyses are obviously not needed.)

In order to investigate the effect of utilizing climatological temperature and humidity profiles, three experiments were carried out: (i) random errors from 0 to 5 K and from 0 to 10 K were added to the temperature profile, (ii) random errors from 0 to 5% and from 0 to 10% were added to the relative humidity profile, and (iii) independent random errors similar to (i) and (ii) for both temperature and humidity profiles were used. Results are shown in Fig. 5. Errors in the temperature profile apparently introduce large errors in the resulting four parameters. For random errors from 0 to 5 K in the temperature profile, we note that deviations of about 9 K, 1 km, 0.07 and 0.006 are produced for the surface temperature, the cloud thickness, the reference transmissivity and the cloud fraction, respectively. On the other hand, random errors from 0 to 5% in the humidity profiles introduce 5 K, 0.5 km, 0.04 and 0.003 for the resulting four parameters, respectively. Note that the + and − signs in these diagrams indicate the positive and negative errors, respectively. When independent errors are added to both the temperature and humidity profiles, we find that errors in the four resulting parameters are greatly reduced. The reason for this may be caused by the compensating effect of opposite errors for the temperature and humidity profiles as evident from Fig. 5. It should be noted that simultaneous deviations of the true temperature and humidity profiles from the climatological means are to be expected in the atmosphere. Hence we would anticipate that the use of climatological data in the

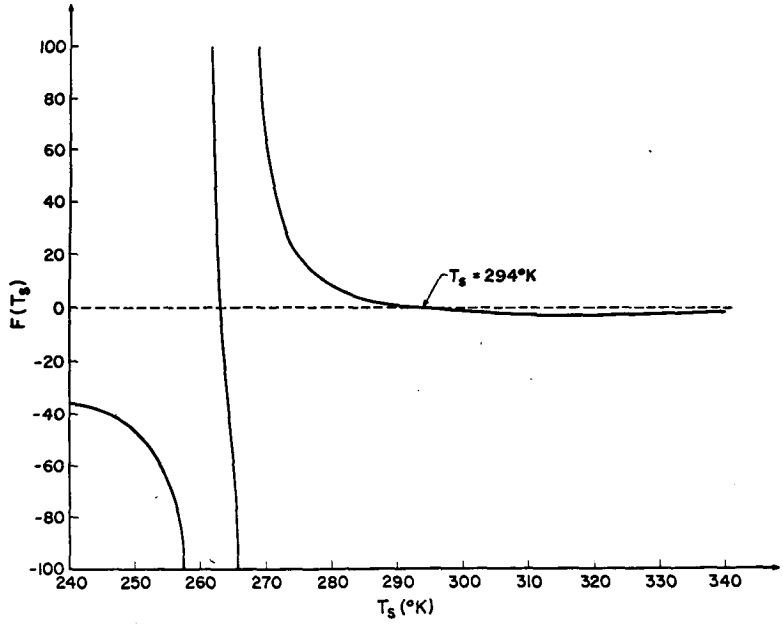

FIG. 4. Behavior of the nonlinear equation (17) as a function of the surface temperature.

present retrieval analysis introduces insignificant errors in cloud parameters.

In the development of the retrieval technique, the assumption has been made that the cloud is high in the atmosphere so that radiative effects of water vapor above the cloud can be neglected. Hence the method should be applied as such and cases when low cumulus and/or stratus are present are to be disregarded. Since visible and IR cloud pictures are routinely available from operational satellites, discrimination between low and high clouds appears possible. There are cases when high and low clouds occur simultaneously in the atmosphere. In these situations, low clouds, which are generally optically thick, may be considered as surfaces and to a good approximation the theoretical approach reported here may also be applicable. Note that we have employed 900, 950, 1100 and 1150 cm^{-1} in the theoretical analysis to demonstrate that information associated with cirrus may be derived from satellite radiance observations. Of course, other spectral bands within the window region ought to be explored to ensure the optimum bands, which could be proposed for future satellite experiments concerning the detection of cirrus cloud compositions and structure in the atmosphere.

d. Note on the determination of cirrus ice content

In the preceding analyses, we have shown that the surface temperature, the cloud thickness, the reference cloud transmissivity and the fraction of cloudiness may be inferred from four radiance measurements in the window region. Once a reliable reference cloud transmissivity has been derived, it seems possible to *estimate* the vertical ice content following the procedures described below.

Based on the theoretical radiative transfer calculations, we may construct a diagram (Fig. 6) consisting of the transmissivity at a given wavenumber and the optical depth at that given wavenumber. We note that the transmissivity is nothing but an attenuation parameter. (For a cirrus cloud whose optical depth is less than about 0.5 it appears that an exponential attenuation may be applied.) By means of this curve constructed prior to the retrieval experiment, a transmissivity derived from the previous analysis should correspond to an optical depth.

We now express the optical depth as

$$\tau(\bar{\nu})=\Delta z\beta_{\text{ext}}(\bar{\nu}), \tag{25}$$

where $\beta_{\text{ext}}(\bar{\nu})$ denotes the volume extinction cross section for a given mean wavenumber $\bar{\nu}$. Now with the information of the cloud thickness which has been determined simultaneously with the cloud transmissivity, the important volume extinction cross section may be estimated.

Since cirrus clouds are composed ice particles on the order of 100–1000 μm (see, e.g., Heymsfield and Knollenberg, 1972), an extinction parameter (usually denoted as Q_{ext}) of about 2 may be adopted. Consequently, the extinction cross section is mainly a function of the size and concentration of ice particles and may be written as

$$\beta_{\text{ext}}\approx Q_{\text{ext}}\bar{A}N, \tag{26}$$

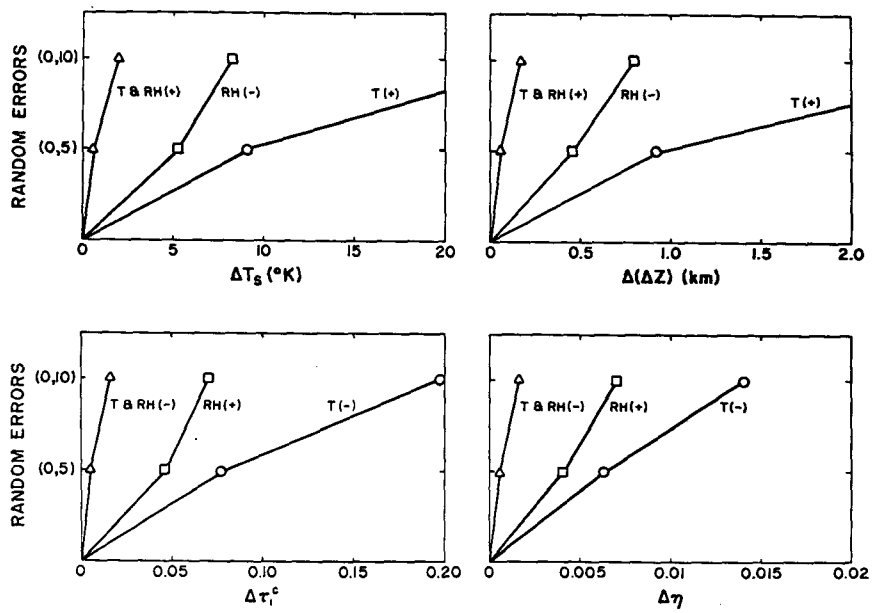

FIG. 5. Error analyses for the use of the climatological temperature and humidity profiles. The abscissa and ordinate represent random errors and deviations of the four parameters from the true values, respectively.

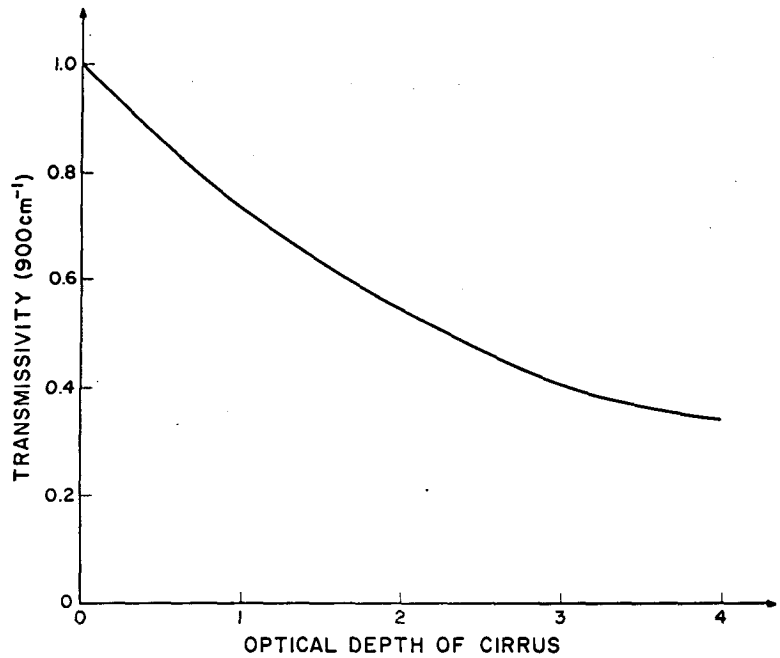

FIG. 6. Transmissivity as a function of optical depth based on radiative transfer calculations.

where N and $\bar{A}$ denote the concentration and the mean cross-section area of ice particles, respectively. For ice crystals randomly oriented in a horizontal plane, we should have $\bar{A}\approx\bar{l}\bar{d}$, with $\bar{l}$ and $\bar{d}$ representing the mean length in the major and minor axes, respectively.

Finally, the vertical ice content can be written in the form

$$\mathrm{IC}=\rho_{\mathrm{I}}\bar{V}N\Delta z, \tag{27}$$

where ρ_{I} represents the density of the ice and $\bar{V}$ the mean volume of ice particles.

If we let $\xi=\bar{V}/\bar{A}$, the above equation for the vertical ice content is given by

$$\mathrm{IC}\approx\rho_{\mathrm{I}}\frac{\beta_{\mathrm{ext}}}{Q_{\mathrm{ext}}}\xi\Delta z. \tag{28}$$

For long circular ice cylinders randomly oriented in a horizontal plane, we have $\xi=\pi\bar{d}/4$. Here, ρ_{I}, Q_{ext} and ξ are known quantities. Thus, a determination of β_{ext} and Δz gives an estimation on the vertical ice content.

4. Conclusion

We have presented a retrieval method for the determination of surface temperature, cirrus cloud thickness, cirrus cloud transmissivity at a reference wavenumber and the fraction of cirrus cloudiness within the field of view from four expected upwelling radiance observations in the 10 μm window region. Theoretical analyses assume that water vapor effects above the cirrus cloud are negligible and that the ratios of transmissivities are linear functions of the cloud thickness. Justifications of these two assumptions are given on the basis of radiative transfer calculations.

Computational analyses employing the climatological means show that random errors in both the temperature and humidity profiles produce insignificant errors in the four resulting parameters. Random errors in the expected upwelling radiances also introduce small errors in the resulting parameters provided that the rms random errors are relatively small as compared with the mean value. Hence, from a theoretical point of view, the retrieval procedures demonstrated appear feasible for the evaluation of cirrus cloud thickness and transmissivity at a reference wavenumber in the 10 μm window. Once the thickness and transmissivity at a given wavenumber of a cirrus cloud have been derived, we note that the vertical ice content of that cirrus cloud may be estimated.

The retrieval technique described in the preceding sections, in principle, should be applicable to cirrus cloudy atmospheres for the quantitative estimation of cirrus thickness and ice content from passive satellite observations. Although a ground-based active remote sensing method employing a laser source has been recently developed for the detection of cirrus, it is of local nature and is to be verified for the successful determination of thickness and ice content. However, cirrus clouds cover a large portion of the planet. It is therefore vitally important to explore passive remote sensing techniques from which cirrus cloud composition and structure may be derived from

satellites on a routine basis over the global scale. There has been hardly any investigation on the recovery of cloud properties from passive satellite sensing owing to the complexity of the cloud interaction with the radiation field of the atmosphere. This is particularly apparent for the high, semi-transparent cirrus clouds. This paper represents an approach based on which the thickness and ice content of cirrus may be evaluated by utilizing four pieces of radiance information in the 10 μm window region.

Any satellite sensing technique for the recovery of atmospheric composition and structure is subject to verification by utilizing real data. The data source that may be of use in connection with the present theoretical analysis would be the IRIS experiment on board Nimbus 4 as reported by Kunde *et al.* (1974). IRIS measured the thermal emission of the earth's atmosphere and surface from 400–1600 cm^{-1} with an apodized spectral resolution of 2.8 cm^{-1}. It seems possible to select a set of four spectral intervals in the 750–1250 cm^{-1} region to carry out the analyses described in this paper provided that the synoptic and cloud information of a cirrus cloudy atmosphere is available. Unfortunately, satellite cloud experiments require *in situ* information of the cloud thickness and composition, which are normally not available under the satellite pass. Perhaps the major thrust of the present paper is to illustrate that certain cirrus cloud parameters could be evaluated from a set of radiance observations in the 10 μm window region. Such a theoretical demonstration seems to warrant a possibility of pre-satellite experiments for the quantitative determination of cirrus cloud composition and thickness.

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