NOTES AND CORRESPONDENCE

Comments on "Solar Heating Rates: The Importance of Spherical Geometry"

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The paper by Lary and Balluch (1993, hereafter LB) purports to assess the importance of spherical geometry for solar heating rates at zenith angles greater than 90° (i.e., for subhorizon sun conditions). The model used [their Eq. (1)] is indeed the correct equation for time-independent radiative transfer under conditions of spherical symmetry. However, it is incorrect for radiation transport in a spherically symmetric medium with an external radiation source (i.e., the sun). This equation was first presented by Chandrasekhar (1960, p. 23), who stated, "and when, further, no radiation from the outside is incident, the intensity and the source function will be functions only of the distance r and the inclination θ to the radius vector." Clearly, when there is an external nonisotropic (i.e., solar) source present, the conditions as stated by Chandrasekhar are not valid.

The spherically symmetric transfer equation has been correctly employed by Hummer and Rybicki (1971), Yorke (1980), and Balluch (1990) in stellar modeling. However, Yorke (1980) further stated that this equation would be applicable for the radiative properties of a dust cloud immersed in an external nonisotropic radiation field. This statement is clearly incorrect as the azimuthal symmetry would not be present, violating the conditions as set forth in Chandrasekhar's statement above.

The correct equation for the specific intensity in a spherically symmetric medium with a solar source can be found in texts such as Sobolev (1975) and Lenoble (1985) and will not be repeated here. Instead, we note that in a later paper not referenced by LB, Dahlback and Stamnes (1991, hereafter DS) present results for the azimuthally averaged intensities in a spherical atmosphere, acknowledging the need to consider the azimuthal *asymmetry* in the presence of a solar source. However, for an isotropic scattering function DS state in appendix A that "the driving term becomes isotropic and the [specific] intensity must therefore also be azimuth independent. This is consistent with what we would expect on physical grounds." In fact, this is not what one would expect on physical grounds. Azimuthal asymmetry in the intensity distribution can be produced by both the scattering phase function and the attenuation of the direct solar beam. While it is true that for the case of isotropic scattering the azimuthally independent intensity term is the only term needed to compute the multiple scattering source function, the resulting intensity distribution is still azimuthally dependent (except in the case of an overhead sun) due to the different attenuation of the direct solar beam at various azimuthal angles for a fixed polar angle. Since DS appear to only consider the influence of the phase function upon the specific intensity's azimuthal dependence, it is unclear whether their azimuthal averaging was performed correctly.

A second serious problem in LB is their representation of the source function shown in Eq. (6), which clearly omits the direct solar irradiance term that gives rise to the azimuthal dependence. In the discussion following Eq. (6), the procedure for single scattering out of the sun is described, but no equations are given. Therefore, it is unclear whether or not allowance has been made for the azimuthal dependence arising from the attenuation of the direct solar beam. Under any conditions the single scatter source term should have been included in Eq. (6).

The azimuthal dependence in the single scatter may readily be seen by considering the expression for the incident solar zenith angle θ_0' at a point along a line of sight (see Fig. 1). The line of sight is defined by the radial distance r_0 between the observer and the center of the sphere, the polar view angle θ with respect to

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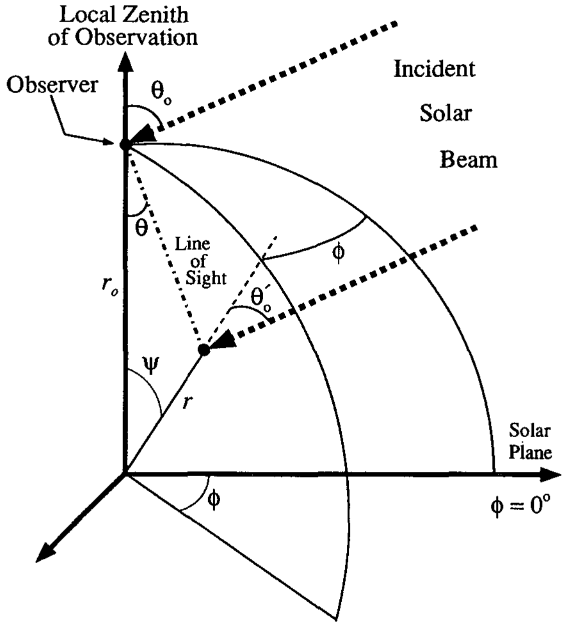

FIG. 1. Geometry for determining incident solar zenith angle θ_0' along a desired line of sight. The solar plane is in the plane of the paper.

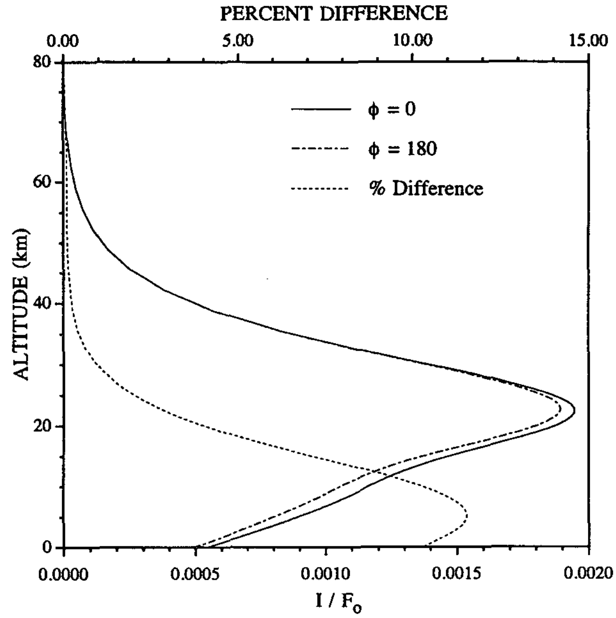

FIG. 2. The downward traveling intensity (normalized to the spectral solar irradiance) and the azimuthal difference, both as a function of altitude within a spherical atmosphere. The observation angle is 115°, and ϕ = 0° represents radiation traveling in an azimuthal direction parallel to the direct solar beam.

the local zenith of observation, and the azimuthal angle ϕ with respect to the solar plane. The point of single scatter is specified by the radial distance r from the center of the sphere, the angle ψ between the radius vector to the point of scatter and the local zenith of observation, and ϕ. The angle θ_0' is given by the law of cosines,

$$\cos\theta_0' = \cos\theta_0 \cos\psi + \sin\theta_0 \sin\psi \cos\phi, \quad (1)$$

where θ_0 is the solar zenith angle with respect to the local zenith of observation, and ψ is given by

$$\psi = \sin^{-1}\left(\frac{r_0}{r}\sin\theta\right) - \theta . \quad (2)$$

We can see that for a fixed value of θ, θ_0, and r, θ_0' is still dependent upon ϕ. If one were to rotate about the zenith of observation by changing ϕ but keeping θ constant, there would be a change in the solar zenith angle θ_0' leading to a change in the attenuation of the direct solar beam for different azimuthal angles. Thus, the single scatter intensity would change with changes in ϕ. The azimuthal dependence introduced into the single scatter intensity would also cause the total intensity to have an azimuthal dependence. When the flat atmosphere approximation is used, r_0/r is unity and θ_0' reduces to θ_0. Since θ_0' in the flat atmosphere case is independent of ϕ, the scattering phase function is the only source of azimuthal asymmetry in a horizontally homogeneous atmosphere.

To illustrate the azimuthal asymmetry caused by the attenuation of the direct solar beam in a spherical atmosphere, we present calculations, for an isotropic scattering function, of the specific intensity as a function of altitude above the earth's surface at two selected view angles, using the method of Herman et al. (1994).

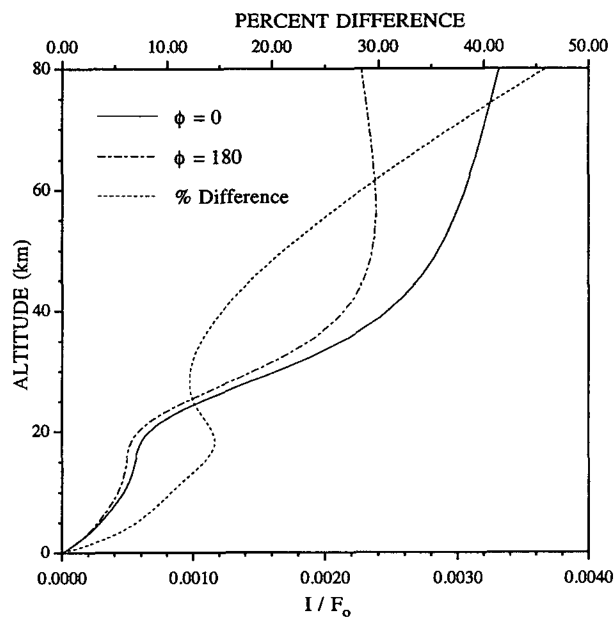

FIG. 3. The same as Fig. 2 except for an observation angle of 65° (radiation traveling in the upward direction).

Figure 2 shows the results at a view angle of 115° (radiation traveling in the downward direction) for the 0° and 180° azimuthal directions. Figure 3 is the same as Fig. 2 except for a view angle of 65° (radiation traveling in the upward direction). For these computations the solar zenith angle θ_0 was 85°, the wavelength was 312.5 nm, the surface reflectance was 0.0, and the total ozone amount was 325 DU. These choices were made to be consistent with the calculations of LB. From Fig. 2 we can see that at altitudes below 30 km the variation in intensity is greater than 1% and is as large as 10% at the surface. Figure 3 shows that at altitudes greater than 3 km the variation in intensity is greater than 5% for the upward traveling radiation and is as large as 45% at 80 km, where the upward beams are near their peak values, and these differences represent an appreciable amount of energy.

Finally, the authors state that "the motivation of this study is to assess the importance of spherical geometry for solar heating rates at zenith angles greater than 90° (the hashed region shown in Fig. 1)." This figure is totally misleading, as it is drawn completely out of scale. It leads to the conclusion that direct solar transmission through the limb contributes greatly to solar heating on the back side of the atmosphere. In reality, with a very thin shell of an atmosphere, direct transmission across the limb has negligible effect on the backside (night side) of the earth.

In view of the above, we question the validity of the basic equations developed for radiative transfer in a spherical atmosphere illuminated by a nonisotropic source and the accuracy of the results for isotropic scattering cases as presented in LB.

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