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A Three-Parameter Approximation for Radiative Transfer in Nonhomogeneous Atmospheres: Application to the O_3 9.6-μm Band

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A three-parameter scaling approximation is developed in conjunction with random models for the computation of transmittance along nonhomogeneous atmospheres. In addition to the scaled path length and pressure, a third parameter is introduced to account for intermediate absorption. With reference to the results derived from line-by-line calculations, we assess the accuracy of this scaling approximation in different atmospheric conditions. For the O_3 9.6-μm band where the Curtis-Godson and other conventional scaling methods usually produce large errors, the present model can achieve relative accuracies within ~1% in transmittance and the downward surface flux. Accuracy of ~0.15 K/d can be obtained in cooling rate calculations. The present three-parameter approximation is well suited for radiative transfer parameterizations in which the demand for high accuracy is required in the transmittance and flux calculations.

1. INTRODUCTION

Cooling due to infrared radiation has an important effect on thermal structure of the stratosphere, trace gas distributions [*Harwood and Pyle*, 1980], and the evolution of atmospheric eddies via radiative damping [*Fels*, 1982]. In addition to the CO_2 15-μm band, the O_3 9.6-μm band is also very important in the generation of cooling in the stratosphere. Also, ozone may influence tropospheric climate through both radiative and dynamical interactions between the troposphere and the stratosphere [*Ramanathan and Dickinson*, 1979]. Furthermore, in the remote sensing experiments that involve the 9.6-μm channels, the errors in transmittance can lead to erroneous estimates of the total ozone amount [*Kuriyan et al.*, 1978].

Accurate and efficient calculations for radiative transfer due to absorption and emission of the O_3 9.6-μm band have been found to be difficult. This is because the ozone profile is characterized by the combination of large concentration at low pressure and small concentration at high pressure. For this reason, the two-parameter Curtis-Godson (CG) and other conventional scaling approximations, which may work well for the H_2O and CO_2 bands, fail to provide accurate results in nonhomogeneous atmospheres for the O_3 9.6-μm band [*Curtis*, 1952; *Walshaw and Rodgers*, 1963; *Kratz and Cess*, 1988].

A number of methodologies have been proposed to improve the two-parameter scaling approximation by introducing a third parameter [*Goody*, 1964; *Armstrong*, 1968; *Yamamoto et al.*, 1972; *Kuriyan et al.*, 1978]. However, none of these attempts have been applied to narrow-band and wide-band models and assessed by line-by-line computations. In the present study, we present a three-parameter approximation in conjunction with the Malkmus random band model for calculating radiative fluxes and heating rates. Results computed from a line-by-line program are utilized as a diagnostic tool for assessing the approximation. We also compare the present three-parameter approximation with the CG scaling in terms of the computed transmittances, fluxes, and cooling rates due to the O_3 9.6-μm band in different atmospheres.

In section 2 the improved CG approximations for a single line are reviewed. The narrow-band model that is used in the present study to compute transmittance is described in section 3. In section 4 we formulate a three-parameter scaling approximation in the context of a narrow-band model. In section 5 we examine the errors produced by the CG and present three-parameter approximations for the O_3 9.6-μm band. Finally, conclusions are given in section 6.

2. REVIEW OF THE IMPROVED CURTIS-GODSON APPROXIMATION FOR A SINGLE LINE

For a single Lorentz line, the absorption coefficient can be written as

$$k_\nu = Sf_\nu = \frac{S}{\pi}\frac{\alpha}{\nu^2 + \alpha^2}, \tag{1}$$

where f_ν is the normalized line shape factor, ν is wave number, α is the Lorentz line half width, and S is the line intensity. Absorption and transmission are determined by the optical depth defined by

$$\tau_\nu = \int k_\nu \, du = \int k_\nu q \, dp/g, \tag{2}$$

where u is path length, q is the mixing ratio of the absorber, p is the pressure, and g is the gravitational acceleration.

In the calculation of atmospheric transmission the vertical nonhomogeneity must be accounted for. The conventional approach that has been used is the so-called Curtis-Godson (CG) method [*van de Hulst*, 1945; *Curtis*, 1952; *Godson*, 1953]. The CG approach reconstructs a homogeneous path for which the transmittance is approximately the same as that for the actual path, namely,

$$\tau_\nu \simeq \frac{\bar{S}_{CG}}{\pi}\frac{\bar{\alpha}_{CG}}{\nu^2 + \bar{\alpha}_{CG}^2}u, \tag{3}$$

where the two parameters $\bar{\alpha}_{CG}$ and $\bar{S}_{CG}$ are derived on the basis of matching the transmittance in the strong- and weak-line limits, which yields

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$$\bar{S}_{CG} = \int S\, du \Big/ \int du, \tag{4}$$

$$\tilde{\alpha}_{CG} = \int S\alpha\, du \Big/ \int S\, du. \tag{5}$$

Since the CG approximation reduces a nonhomogeneous path to a homogeneous one, it can be easily incorporated into a band model, which is usually developed for a homogeneous path. However, as pointed out by *Curtis* [1952] and *Walshaw and Rogers* [1963], the CG approximation is likely to be inadequate for a combination of large amounts of gas at a low pressure with small amounts at a high pressure, that is, a typical ozone distribution. Walshaw and Rogers also concluded that the CG approximation is not very accurate for flux and cooling rate calculations involving the water vapor rotational band.

There have been a number of attempts to improve the CG approximation. *Goody* [1964] proposed a model consisting of two superimposed Lorentz lines using a relationship between the two line intensities in order to reduce the number of variables from four to three. Employing the cosine transform of the optical depth and equating the three lowest order moments, Goody obtained the three unknown parameters in his two-line model. By matching one additional term as compared to the CG approximation, the three-parameter model improves the absorptance significantly, especially near the line center. Note that the two parameters of the single Lorentz line that occurs in the CG approximation can be determined by equating the two lowest order moments of the cosine transform of the optical depth [*van de Hulst*, 1945].

Using a different approach, *Armstrong* [1968] expanded the line shape factor in a Taylor series about a pressure point $\bar{p}$. By substituting this series in (2) for the optical depth, $\bar{p}$ can be chosen such that the first-derivative term vanishes, so that the zeroth-order term will make the dominant contribution to τ_ν. The requirement that the first-derivative term vanishes leads to the condition that $\bar{p}$ is the CG mean pressure. The result is independent of line shape. The use of the zeroth-order term in this approach is essentially equivalent to the CG approximation. Furthermore, a correction term, the next nonvanishing term in the series, may be used for a given case to improve the transmittance calculations. Armstrong also applied Gaussian quadrature to the pressure integration occurring in (2). The first-order Gaussian result is equivalent to the CG approximation in the case when the line intensity is a constant. The second-order Gaussian result has a similar form to *Goody*'s [1964] "three-parameter" model, except that the "parameters" in *Armstrong*'s [1968] two-line model are all determined a priori and require no specific solutions. Armstrong demonstrated that the second-order Gaussian result is better than the corrected CG result derived from the optimized Taylor series. *Kuriyan et al.* [1978] developed an improved CG scheme by modifying (3) such that the constant $\tilde{\alpha}^2_{CG}$ in the denominator is replaced by a function in the form $\tilde{\alpha}^2(\nu) = (a\nu^2 + b)/(\nu^2 + c)$, where the three free parameters are to be determined from supplementary conditions.

The improved CG approximations discussed above are all successful in decoupling the wave number from the pressure integration. However, unlike the CG scaling, a homogeneous path that can be represented by the parameters of a single line has not been constructed. For this reason, these methods are not practical for atmospheric transmittance calculations.

Yamamoto et al. [1972] noted that the CG approximation overestimates absorption in the intermediate range between weak- and strong-line limits. According to their interpretation the equivalent half width used in the CG approximation appears too large to give an exact result for intermediate absorption. It follows that the CG approximation may be improved if a smaller equivalent half width is used. Yamamoto et al. showed that the absorptance of a series of superimposed Lorentz lines with different half widths for a nonhomogeneous path length can be reduced to that for a homogeneous path length that is characterized by the equivalent line intensity and half width in the forms

$$\bar{S} = \int S\, du \Big/ \int du \equiv \bar{S}_{CG}, \tag{6}$$

$$\tilde{\alpha}^{\varepsilon} = \int S\alpha^{\varepsilon}\, du \Big/ \int S\, du, \tag{7}$$

where the third parameter ε is introduced to reduce the equivalent half width defined by the CG approximation in (5). Thus the value ε should be smaller than 1 so that the equivalent half width may be adequately applied to intermediate absorptions that are neither strong nor weak. Using a single Lorentz line and a curve fitting technique, *Yamamoto et al.* [1972] developed an analytic form: $\varepsilon = [\chi_{CG}/(1 + \chi_{CG})]^n$, where $\chi_{CG} \simeq \bar{S}_{CG}u/2\pi\tilde{\alpha}_{CG}$ and n is a constant to be determined from the information of the vertical distribution of absorbing gases. For a single line, Yamamoto et al. derived a value of n of 1.6 for H_2O and CO_2 and 3.2 for O_3.

In section 4 we wish to develop a three-parameter approximation in connection with the Malkmus band model to account for the effects of pressure and temperature variations on the transfer of infrared radiation.

3. Band Model Used in the Development

The two most widely used narrow-band models are those of *Goody* [1952] and *Malkmus* [1967]. Based on the results illustrated by *Kratz and Cess* [1988], the Malkmus band model yields band absorptances that are more comparable to those derived from line-by-line calculations for the O_3 9.6-μm band. For this reason, we will use the Malkmus band model in connection with our development of a three-parameter correction.

The Malkmus random model using the Lorentz line shape may be described by the spectral transmittance in the form

$$T_{\bar{\nu}}(u) = \exp\left\{-\frac{\pi\alpha}{2\delta}\left[\left(1 + \frac{4\bar{S}u}{\pi\alpha}\right)^{1/2} - 1\right]\right\}, \tag{8}$$

where $\bar{S}$ is the mean line intensity within the spectral interval $\Delta\nu$ and δ is the mean line spacing. Equation (8) is equivalent to the original Malkmus equation by noting that $S_E = 4\bar{S}/\pi$ and $\delta_E = 4\delta/\pi$. The model consists of two parameters, $\bar{S}/\delta$ and $\pi\alpha/\delta$, which can be computed from line parameters via the following relations

$$\bar{S}/\delta = \sum S_i/\Delta\nu, \tag{9}$$

$$\pi\alpha/\delta = \left[2 \sum (S_i\alpha_i)^{1/2}\right]^2 \Big/ \left(\Delta\nu \sum S_i\right), \quad (10)$$

where S_i and α_i denote the individual line intensities and Lorentz line half widths, respectively, and the summation is over all lines within $\Delta\nu$. Equations (9) and (10) can be derived by matching the results under the strong- and weak-line limits. The line parameter data required in the narrow-band models are obtained from the 1982 version of the AFGL data tape [*Rothman et al.*, 1983].

For the present purpose the narrow-band calculations were carried out over a spectral interval of 5 cm^{-1} width [*Robertson et al.*, 1981], so that the distributions of intensities, widths, and spacings of absorption lines do not vary significantly across each interval. Moreover, within this interval there are a sufficient number of lines to yield meaningful statistics [*Crisp et al.*, 1986; *Kratz and Cess*, 1988].

For the computation of cooling rates above about 40 km where gases are at low pressure, it becomes important to incorporate the combined effects of the Lorentz and Doppler broadening, that is, the Voigt profile, in infrared transfer calculations. In order to incorporate the Voigt line shape into the narrow-band model we follow the approach proposed by *Fels* [1979]. In this approach, the shape of a Voigt line profile is parameterized as a rectangular Doppler core with Lorentz wings in the form

$$f(\nu) = \begin{cases} C & |\nu| \le \nu_0 \\ \alpha/\pi\nu^2 & |\nu| > \nu_0 \end{cases}, \quad (11)$$

where ν is the distance from the line center, and the normalization requirement for the line shape factor gives

$$C = \frac{1}{2\nu_0} - \frac{\alpha}{\pi\nu_0^2}. \quad (12)$$

The value ν_0 depends on both the Lorentz and the Doppler widths and can be assumed to be

$$\nu_0 = \frac{2}{\pi}(1 + \xi)\alpha + \beta\alpha_D, \quad (13)$$

where the Doppler width $\alpha_D = \bar{\nu}\sqrt{2RT}/c$ with $\bar{\nu}$ is the mean wave number in the spectral interval $\Delta\nu$, R is the gas constant, T is the temperature, and c is the speed of light. The two parameters β and ξ are adjustable parameters.

For the line shape defined in (11), the transmittance for the Malkmus random model can be expressed analytically and is given by

$$T_{\bar{\nu}}(u) = \exp\left\{-\frac{2}{\pi}(uab)^{1/2}\tan^{-1}\left[\frac{2}{\pi}\left(\frac{\delta}{\nu_0}\right)(uab)^{1/2}\right] - \frac{1}{2\left(\frac{\delta}{\nu_0}\right)}\ln\left[\frac{1 + 4ua\left(\frac{\delta}{\nu_0}\right)\nu_0 C}{1 + \frac{4}{\pi^2}uab\left(\frac{\delta}{\nu_0}\right)^2}\right]\right\}, \quad (14)$$

where $a = \bar{S}/\delta$ and $b = \pi\alpha/\delta$, which can be obtained from (9) and (10), $\delta = \Delta\nu/n$, where n is the total number of lines within $\Delta\nu$, and C and ν_0 can be determined from (12) and (13), respectively, with the Lorentz half width given by $\alpha = b\delta/\pi$. Equation (14) differs from the equation developed by *Fels* [1979] in that a and b are defined by (9) and (10). The present model is referred to as the Malkmus-Fels band model. In order to use (8) or (14) for the calculations of fluxes and cooling rates in nonhomogeneous atmospheres due to the O_3 9.6-μm band, an accurate scaling approximation is required.

4. Formulation of a Three-Parameter Approximation For Band Models

We first define the mean line strength for n absorption lines randomly distributed in a spectral interval $\Delta\nu$ in the form

$$\bar{S}(T) = \sum_{i=1}^{n} S_i(T)/n. \quad (15)$$

As shown in (8), the transmittance from the Malkmus random model can be expressed as a function of two parameters, $\bar{S}/\delta$ and $\pi\alpha/\delta$, apart from the path length, u. Following *Yamamoto et al.* [1972], the scaled line intensity and half width in a nonhomogeneous path length may be written

$$\bar{\bar{S}} = \int \bar{S}(T)\, du \Big/ \int du, \quad (16)$$

$$\tilde{\alpha}^{\varepsilon} = \int \alpha^{\varepsilon}(T, p)\bar{S}(T)\, du \Big/ \int \bar{S}(T)\, du, \quad (17)$$

where we have introduced a third parameter ε in a manner similar to a single line. Determination of ε will be discussed later. We may scale the path length u instead of the intensity $\bar{S}(T)$ such that

$$\tilde{u} = \int \bar{S}(T)/\bar{S}(T_r)\, du, \quad (18)$$

where T_r denotes an arbitrary reference temperature. Since $\bar{\bar{S}}u = \bar{S}(T_r)\tilde{u}$ in (8), equation (18) is equivalent to (16).

Using the two parameters defined in (9) and (10), equation (17) may be expressed by

$$\left(\frac{\pi\tilde{\alpha}}{\delta}\right)^{\varepsilon} = \int \left\{\left[2 \sum (S_i\alpha_i)^{1/2}\right]^{2\varepsilon} \Big/ \left(\sum S_i\right)^{\varepsilon - 1}\right\} du \Big/ \left[(\Delta\nu)^{\varepsilon} \int \sum S_i\, du\right]. \quad (19)$$

We use the reference pressure and temperature so that

$$\left(\frac{\pi\tilde{\alpha}}{\delta}\right)^{\varepsilon} \Big/ \left(\frac{\pi\alpha}{\delta}\right)_r^{\varepsilon} = \frac{1}{\tilde{u}} \int \frac{\left[\sum \sqrt{S_i(T)\alpha_i(T, p)}\right]^{2\varepsilon}}{\left[\sum \sqrt{S_i(T_r)\alpha_i(T_r, p_r)}\right]^{2\varepsilon}} \frac{\left[\sum S_i(T_r)\right]^{\varepsilon - 1}}{\left[\sum S_i(T)\right]^{\varepsilon - 1}} du, \quad (20)$$

where the subscript r denotes the reference condition.

For a given scaled half width $\tilde{\alpha}$, we may set the temperature of the equivalent homogeneous atmosphere at T_r in order to define the scaled pressure $\tilde{p}$ such that

$$\left(\frac{\pi\tilde{\alpha}}{\delta}\right)^{\varepsilon} = \left(\frac{\pi\alpha}{\delta}\right)_r^{\varepsilon} \tilde{p}^{\varepsilon}/p_r^{\varepsilon}. \tag{21}$$

It follows from (20) that $\tilde{p}$ may be expressed as

$$\tilde{p}^{\varepsilon} = \frac{p_r^{\varepsilon}}{\tilde{u}} \int \frac{\left[\sum \sqrt{S_i(T)\alpha_i(T, p)}\right]^{2\varepsilon}}{\left[\sum \sqrt{S_i(T_r)\alpha_i(T_r, p_r)}\right]^{2\varepsilon}} \frac{\left[\sum S_i(T_r)\right]^{\varepsilon-1}}{\left[\sum S_i(T)\right]^{\varepsilon-1}} du. \tag{22}$$

The scaled pressure contains both pressure and temperature adjustments along the path.

To simplify the computational procedure, we may separate the pressure and temperature dependence in the half width and write $\alpha_i(T, p) = \alpha_i'(T)p/p_r$, where $\alpha_i'(T) = \alpha_i(T_r, p_r)(T_r/T)^{1/2}$. Then we obtain

$$\tilde{p}^{\varepsilon} = \frac{1}{\tilde{u}} \int p^{\varepsilon} \frac{\left[\sum \sqrt{S_i(T)\alpha_i'(T)}\right]^{2\varepsilon}}{\left[\sum \sqrt{S_i(T_r)\alpha_i'(T_r)}\right]^{2\varepsilon}} \frac{\left[\sum S_i(T_r)\right]^{\varepsilon-1}}{\left[\sum S_i(T)\right]^{\varepsilon-1}} du. \tag{23}$$

At this point we obtain an equivalent homogeneous path with the scaled path length $\tilde{u}$, the scaled pressure $\tilde{p}$, and temperature T_r for the nonhomogeneous atmosphere in a spectral interval $\Delta\nu$.

The temperature-dependent terms in (18) and (23) may be parameterized in terms of the following polynomial form:

$$\ln \sum S_i(T) = \sum_{n=0}^{N} C_n(T - 250)^n, \tag{24}$$

$$\ln \left[\sum \sqrt{S_i(T)\alpha_i'(T)}\right]^2 = \sum_{n=0}^{N} C_n'(T - 250)^n, \tag{25}$$

where C_n and C_n' are empirical constants. It suffices to use $N = 2$ in the polynomial fitting to obtain accuracy within 1%.

The Malkmus random model given in (8) can now be employed to calculate the transmittance for a nonhomogeneous path length as follows:

$$T_{\bar{\nu}} = \exp\{-\tfrac{1}{2}\bar{b}[(1 + 4\bar{a}\tilde{u}/\bar{b})^{1/2} - 1]\}, \tag{26}$$

where

$$\bar{a} = \left(\frac{\bar{S}}{\delta}\right)_r, \quad \bar{b} = \left(\frac{\pi\alpha}{\delta}\right)_r \frac{\tilde{p}}{p_r}. \tag{27}$$

In (27), $\tilde{p}$ is to be determined by taking the $1/\varepsilon$ root of (23).

The parameter ε in (23) cannot be derived analytically and must be obtained by numerical means. From the analysis presented by *Yamamoto et al.* [1972] for a single line, ε can be related to a parameter defined by $\chi_{CG} = \bar{S}_{CG}u/2\pi\tilde{\alpha}_{CG}$. We may use the parameter, χ_{CG}, to develop an expression for ε for the spectral band. Using curve fitting for spectral absorptance computed from a line-by-line method and taking into consideration the weak- and strong-line limits, we find

$$\varepsilon = \left(1 + \frac{1}{\chi_{CG}}\right)^{-n(\chi_{CG})}, \tag{28}$$

where the index, n, is a function of χ_{CG} and can be determined by the following expression:

$$n(\chi_{CG}) = c\left[1 + \left(1 + \frac{1}{\chi_{CG}}\right)^{-1}\right], \tag{29}$$

where the coefficient, c, has a value of 1.3 for O_3 and 1 for H_2O and CO_2. It follows that in the case of the weak-line limit, that is, $\chi_{CG} \to 0$, we have $\varepsilon \to 0$. Thus as expected, absorption is independent of the line width. In the strong-line limit, χ_{CG} is large, so that $\varepsilon \cong 1$. In this case, the equivalent half width is equal to that of the CG approximation. For intermediate absorption, ε ranges from 0 to 1. Further, using the CG scaling for the random band model, we have

$$\chi_{CG} = \frac{p_r}{8} \frac{\left[\int \sum S_i(T)\, du\right]^2}{\int p\left[\sum \sqrt{S_i(T)\alpha_i'(T)}\right]^2 du}. \tag{30}$$

With the parameters $\tilde{u}$ and $\tilde{p}$ defined in (18) and (23), respectively, the vertical nonhomogeneity of the atmosphere is approximately accounted for in the random band model. The application of the Malkmus-Fels random model to a nonhomogeneous atmosphere may be made by employing (14) at temperature T_r and pressure $\tilde{p}$ with the scaled path length $\tilde{u}$, where we set $T_r = \int T\, du/\int du$ to account for the temperature dependence of the Doppler half width. Letting $\varepsilon = 1$ in (23), we have the CG approximation for the random band model.

5. Comparison With Line-by-Line Calculations

In the O_3 9.6-μm band (980–1100 cm^{-1}), since the width $\Delta\nu$ (120 cm^{-1}) is relatively narrow, the upward and downward fluxes in the height coordinate may be expressed in the form [*Liou*, 1980]

$$F_{\bar{\nu}}^{\uparrow}(z) = \pi B_{\bar{\nu}}(T_s)T_{\bar{\nu}}^f(0, z) + \int_0^z \pi B_{\bar{\nu}}[T(z')] \frac{d}{dz'} T_{\bar{\nu}}^f(z', z)\, dz', \tag{31a}$$

$$F_{\bar{\nu}}^{\downarrow}(z) = \int_{z_\infty}^{z} \pi B_{\bar{\nu}}[T(z')] \frac{d}{dz'} T_{\bar{\nu}}^f(z, z')\, dz', \tag{31b}$$

where $B_{\bar{\nu}}$ is the spectrally integrated Planck function, z_∞ is the height at the top of the atmosphere (TOA), T_s is the surface temperature, and the diffuse transmittance between levels z and z' is defined by

$$T_{\bar{\nu}}^f(z, z') = \frac{1}{\Delta\nu} \int_{\Delta\nu} d\nu \int_0^1 2 \exp[-\tau_\nu(z, z')/\mu]\mu\, d\mu, \tag{31c}$$

TABLE 1. Comparison of Diffuse Transmittances (0–60 km) for the O_3 9.6-μm Band (980–1100 cm^{-1})

	MLS	SAW	TRO	USS
Line by line	0.6103	0.5632	0.6866	0.6219
Present model	0.6093	0.5592	0.6836	0.6198
	(−0.2)	(−0.7)	(−0.4)	(−0.3)
Malkmus with	0.5624	0.5333	0.6256	0.5782
CG scaling	(−7.8)	(−5.3)	(−8.9)	(−7.0)

The percentage error is in parentheses. MLS, SAW, TRO, and USS are abbreviations for mid-latitude summer, subarctic winter, tropical, and U.S. Standard Atmosphere, respectively.

TABLE 2. Comparison of the Downward Surface Fluxes (W m^{-2}) due to the O_3 9.6-μm Band (980–1100 cm^{-1})

	MLS	SAW	TRO	USS
Line by line	4.54	2.86	3.67	3.44
Present model	4.48	2.86	3.64	3.41
	(−1.3)	(0.0)	(−0.8)	(−0.9)
Malkmus with	4.92	3.04	4.09	3.75
CG scaling	(8.4)	(6.3)	(11.4)	(9.0)

The percentage error is in parentheses.

where $\tau_\nu(z, z')$ is the optical depth between z and z'. The net flux $F_{\bar\nu} = F_{\bar\nu}^{\uparrow} - F_{\bar\nu}^{\downarrow}$. The heating rate is then

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho_a C_p}\frac{dF_{\bar\nu}(z)}{dz}, \tag{31d}$$

where ρ_a is the air density and C_p is the specific heat at constant pressure.

In the line-by-line calculations the line data employed are identical to those used in the narrow-band models. The absorption line was assumed to follow the Voigt function and was cut off at a wave number 260 times the Lorentz half width from the center. Following the work of *Chou and Kouvaris* [1986], the absorption coefficients were precomputed at a wave number interval of 0.005 cm^{-1} in the O_3 9.6-μm band for 19 pressures with $\Delta\log_{10} p = 0.2$ and for three temperatures (200, 250, and 300 K). The absorption coefficient at the conditions other than those 19 pressures and 3 temperatures can be interpolated linearly with pressure and exponentially with temperature. The diffuse transmittance was computed from beam transmittance through the use of a fourth-order Gaussian quadrature. We use four atmospheric profiles, including mid-latitude summer, subarctic winter, tropical, and U.S. Standard atmospheres [*McClatchey et al.*, 1971], to cover a variety of temperatures and ozone concentrations in the computation of radiative transfer. In the vertical the atmosphere was equally divided into 60 homogeneous layers between the surface and the TOA, set at 60 km. The temperature, the ozone mixing ratio, and the logarithm of pressure are assumed to be linear in the height coordinate.

To calculate the diffuse transmittance by using the random model with the scaling approximation developed in the preceding section, we first divide the absorption band $\Delta\nu$ into 24 intervals, each of which consists of 5 cm^{-1}. In finite difference form we have

$$T_{\bar\nu}^{f}(z, z') = \frac{1}{24}\sum_{i=1}^{24} 2\sum_{j=1}^{4} \mu_j a_j T_{\bar\nu,i}[\bar u/\mu_j, \bar p(\mu_j)], \tag{32}$$

where μ_j and a_j are Gaussian points and weights for angular integration, $\bar u$ and $\bar p$ are scaled path length and pressure, and $T_{\bar\nu,i}$ is the transmittance from the random model. In the two-parameter CG approximation the pressure correction is independent of the cosine of the zenith angle μ. However, in the present three-parameter approach, the pressure correction is a function of μ through the parameter $\chi_{CG}(u/\mu)$.

For the O_3 9.6-μm band, the virtual equivalence of the Malkmus narrow-band model with line-by-line calculations for homogeneous paths makes it convenient to evaluate the scaling approximation for nonhomogeneous paths using band models. Table 1 summarizes the diffuse transmittances for the whole column (0–60 km) in different atmospheres, as determined by employing the line-by-line calculation, the narrow-band model with the three-parameter scaling, and the narrow-band model with the CG scaling. The three-parameter scaling can achieve an overall accuracy within 1%, whereas the CG approximation can produce errors greater than 8% in the tropical atmosphere. From Table 1 we conclude that the proper scaling approximation is important for the transmittance calculation and that the present three-parameter scaling appears to be sufficiently accurate for any atmospheric conditions.

Table 2 shows a comparison of the downward flux at the surface, $F^{\downarrow}(0)$, for the O_3 9.6-μm (980–1100 cm^{-1}) band in different atmospheres. The CG technique produces about a 10% overestimate of $F^{\downarrow}(0)$. The present three-parameter scaling reduces the relative error by a factor of 10. In the calculations for surface fluxes we have not incorporated the Voigt line shape into the Malkmus model since the use of a Voigt versus Lorentz line shape has little significance (less than 0.2%) [*Kratz and Cess*, 1988].

In order to obtain a more meaningful comparison of radiation models for the upward flux at the top of the atmosphere we use the difference of surface and top upward fluxes in which the surface contribution is removed. Table 3 lists this upward flux difference due to the O_3 9.6-μm band, as computed by different models for a number of atmospheric conditions. It can be seen that errors due to the present three-parameter approximation are less than 2%, while the errors due to the CG scaling can be as large as 16% (1.3 W/m^2) in the tropical atmosphere.

Figures 1–4 show the comparison of heating rates due to the O_3 9.6-μm band in mid-latitude summer, subarctic winter, tropical, and U.S. Standard atmospheres, respectively. The heating rate profiles calculated using the line-by-line method are shown in parts (*a*) of these figures, while the corresponding error profiles for the results computed from

TABLE 3. Comparison of the Reduction of Upward Top Fluxes (W m^{-2}) due to the 9.6-μm Band (980–1100 cm^{-1})

	MLS	SAW	TRO	USS
Line by line	8.53	4.15	7.81	7.72
Present model	8.62	4.23	7.97	7.82
	(1.0)	(1.9)	(2.0)	(1.3)
Malkmus with	9.59	4.50	9.12	8.63
CG scaling	(12.4)	(8.4)	(16.7)	(11.8)

The percentage error is in parentheses.

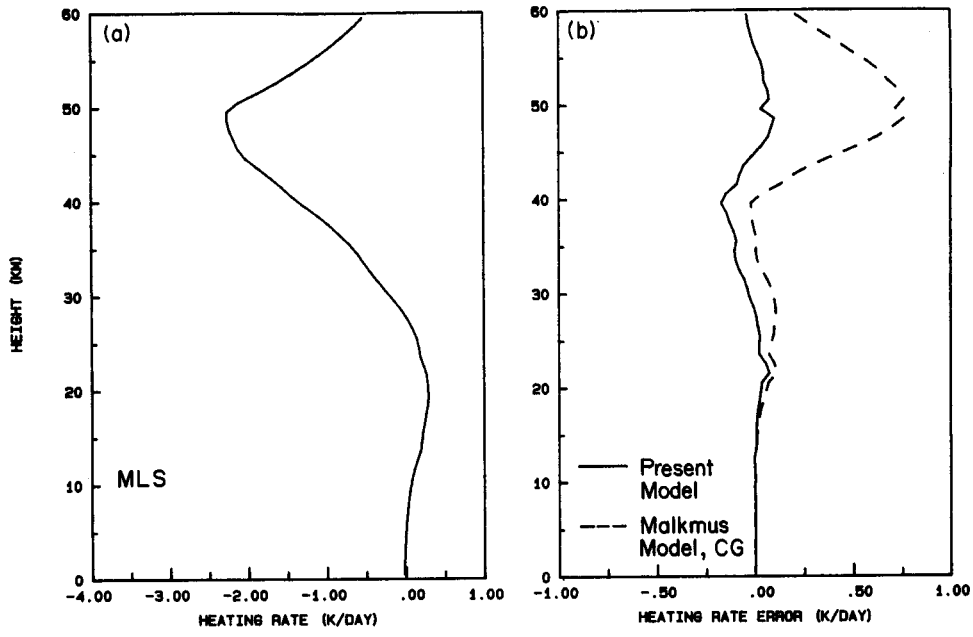

Fig. 1. (*a*) Heating rate profile due to the O_3 9.6-μm band in the mid-latitude summer atmosphere (MLS) computed from a line-by-line method. (*b*) Heating rate error profiles produced by the Malkmus model with Curtis-Godson (CG) scaling (dashed curve) and by the present model (solid curve).

the Malkmus random model with CG scaling and from the Malkmus-Fels random model with the present three-parameter scaling are shown in parts (*b*). Above 40 km the error introduced by the Malkmus model with the CG approximation is very significant (maximum about 0.85 K/d). This is because the Doppler broadening significantly affects the cooling rate in this region. Between about 20 to 30 km where the pressure broadening still dominates the line shape, the error, which is less than 0.2 K/d, is caused by the CG approximation. This result is consistent with the conclusions presented by *Kratz and Cess* [1988]. At about 35 km the Malkmus model in combination with the CG scaling gives the same cooling as do the line-by-line calculations. This can be explained by the fact that the error due to the use of Lorentz line shape at that height region is cancelled by the error caused by the CG scaling [*Ramanathan and Coakley*, 1978]. Compared with the Malkmus random model with the CG approximation, the Malkmus-Fels random model with the present three-parameter scaling gives an overall improvement of the calculated heating rates throughout the entire atmosphere, except in the vicinity of the 35-km region where the approximate Voigt line shape given in (11) fails to produce the Voigt dip. This dip is defined as the region where the cooling computed from the Voigt line becomes

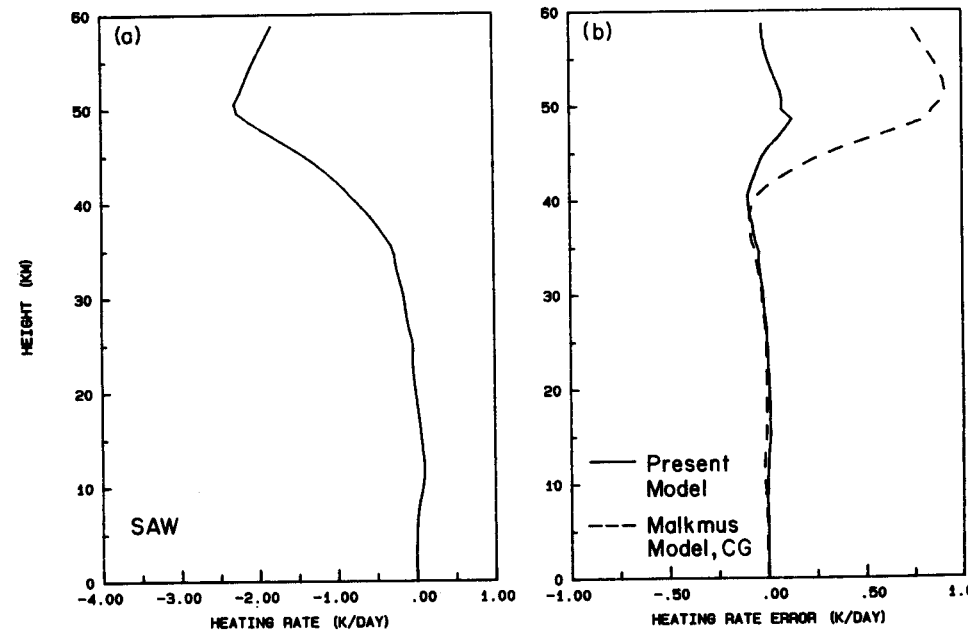

Fig. 2. Same as Figure 1, except for the subarctic winter atmosphere (SAW).

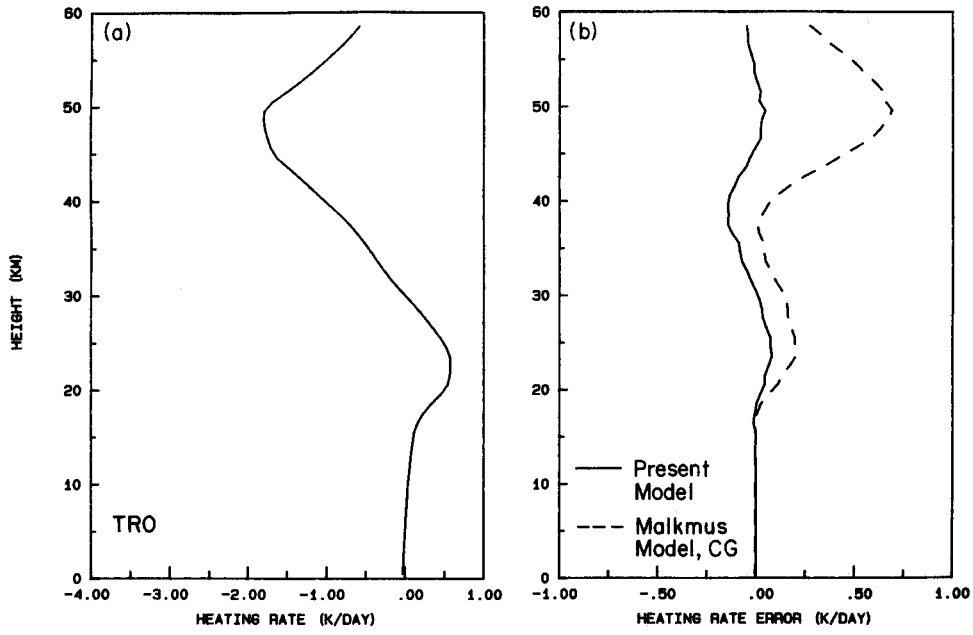

Fig. 3. Same as Figure 1, except for the tropical atmosphere (TRO).

slightly less than that calculated from the Lorentz line [*Dickinson*, 1972]. The improvement due to the three-parameter scaling between about 20 and 30 km is most evident in tropical atmospheres (Figure 3). The cooling rate calculations in the subarctic winter (Figure 2) are not sensitive to different scaling approximations. The errors produced by the Malkmus-Fels random model with the three-parameter scaling are less than 0.15 K/d. The maximum error occurs around 35 km.

Recently, *Chou and Kouvaris* [1991] employed a simple pressure and temperature scaling approximation for the calculations of transmittance in the O_3 9.6-μm band using precomputed tables. They showed that the differences between the heating rates computed from their model and a line-by-line method are less than 0.1 K/d. *Rosenfield* [1991] divided the O_3 9.6-μm band into two parts: a band center region and a band wing region. Each of the two regions is modeled with the Goody random model, with pressure- and temperature-dependent band parameters obtained by empirically fitting line by line results. The errors in heating rates are shown to be typically of the order of 0.1 K/d. Although both of the above parameterizations yield satisfactory results in terms of heating rate calculations, errors between 4 and 8% are produced in the downward flux at the surface. This appears to suggest that the scaling approximations used are inadequate for the calculations of transmittances in

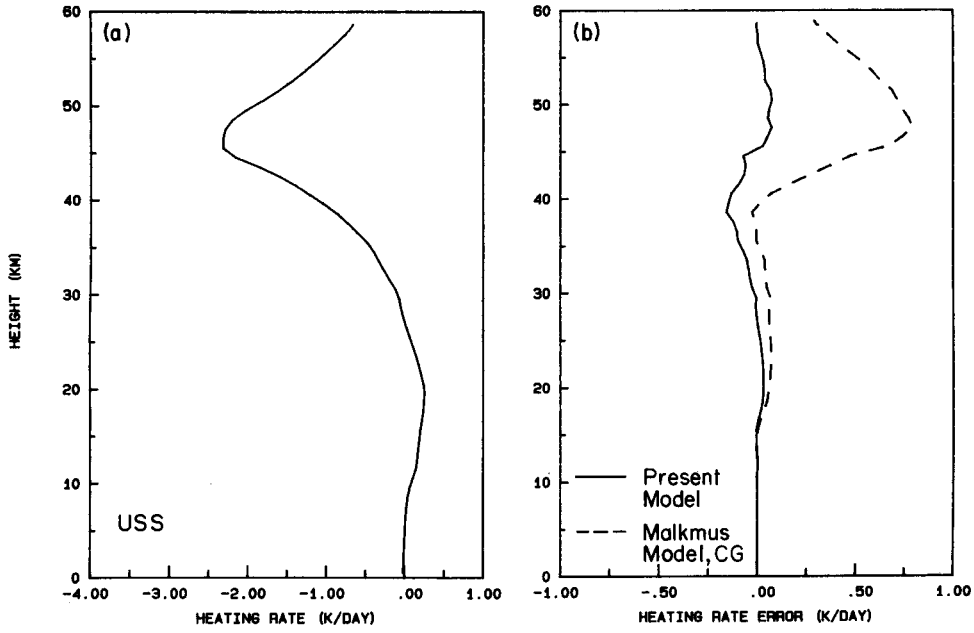

Fig. 4. Same as Figure 1, except for the U.S. Standard Atmosphere (USS).

nonhomogeneous atmospheres. Furthermore, both schemes improve heating rate calculations by fitting the line-by-line results so as to incorporate the effects of Voigt line shape.

6. CONCLUSIONS

A three-parameter scaling approximation in conjunction with a random model has been developed in which the Voigt line profile can be taken into account. A third parameter is introduced to reduce the scaled half width. In this manner, intermediate absorption may be largely covered by the scaling approximation. This approximation has been assessed by line-by-line calculations. Relative accuracies within ~1% can be achieved in the transmittance and flux calculations. Incorporating the three-parameter scaling into the Malkmus-Fels model, we demonstrate that the errors in heating rates due to the O_3 9.6-μm band produced by this model are within 0.15 K/d. The two-parameter CG scaling in conjunction with random band models has been widely used in infrared radiative transfer calculations. However, for the O_3 9.6-μm absorption band, the CG approximation introduces errors as large as 10% in transmittance and flux calculations and 0.2 K/d in cooling rate calculations. The use of Lorentz line shape produces a maximum error ~0.85 K/d in the Doppler broadening region. The present study shows that the transmittance and flux calculations are sensitive to different scaling methods, while the cooling rates are sensitive to the treatment of line shape. This study represents a first attempt to incorporate a three-parameter scaling approximation in the widely used random model for the computation of transmittance along nonhomogeneous atmospheres. Although we have applied this approximation to the O_3 9.6-μm band, it is equally applicable to H_2O and CO_2 bands. The present scaling approximation is efficient with little additional computer time required.

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