A Theory for Long-Lived Mesoscale Convective Systems

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ABSTRACT

It is proposed that certain long-lived mesoscale convective systems maintain themselves through an interaction between quasi-balanced vertical motions and the diabatic effects of moist convection. Latent heat release, evaporation and melting of precipitation, and thermal radiation are all shown to contribute to the creation of a positive potential vorticity anomaly in the lower troposphere. This anomaly can interact with a sheared environment so as to induce further lifting of low-level air and subsequent release of conditional instability.

1. Introduction

One of the most interesting questions of mesoscale meteorology is whether thunderstorms or groups of thunderstorms can influence the environment in such a way that their future existence is insured in the absence of externally provided support.

Two types of self-maintaining convective systems have been identified, namely supercell thunderstorms and squall lines. Supercell thunderstorms maintain themselves via an interaction between sheared environmental flow and a rotating updraft, which tends to produce upward motion on certain flanks of the storm (Rotunno and Klemp 1982, 1985). Squall lines, on the other hand, rejuvenate themselves by the interaction of precipitation-chilled outflows with a sheared environment (Rotunno, Klemp and Weisman 1988), and possibly, in addition, by lifting induced by storm-produced gravity waves (Raymond 1984, 1987).

The above mechanisms all operate in the fast manifold, i.e., they do not depend on the earth’s rotation. However, certain types of convective systems, called mesoscale convective complexes by Maddox (1980, 1983), last long enough that slow manifold effects might be important.

Slow manifold dynamics are often best thought of in terms of the potential vorticity distribution, and the associated invertibility principle by which the thermodynamic structure and flow is recovered (Hoskins, McIntyre and Robertson 1985). Thorpe and Emanuel (1985) showed that diabatic heating in the interior of a fluid volume neither creates nor destroys potential vorticity, but simply redistributes it. Haynes and McIntyre (1987) equivalently showed that the mass-integrated potential vorticity in a layer between two isentropic surfaces remains unchanged in the face of diabatic heating or mass transport in or out of this layer. Since the primary action of cumulus convection is to transport mass across isentropic surfaces, the effect of cumulus convection on the potential vorticity distribution is easily understood. If the mass-integrated potential vorticity remains constant between two isentropic surfaces, then evacuation of mass from this layer will decrease the mass and increase the potential vorticity. Conversely, where deposition of mass occurs the potential vorticity will be decreased.

Vertical mass flux profiles for thunderstorms vary greatly from case to case, but with minor exception mass is moved from lower levels to upper levels. Thus, one could expect the development of a positive potential vorticity anomaly at low levels and a negative anomaly at upper levels in regions with a high density of thunderstorms. As illustrated in Fig. 1, this would result in a cyclonic circulation at low levels and an anticyclonic circulation at upper levels. In addition, the air beneath the lower positive anomaly and above the upper negative anomaly would be cooler than the surroundings, and warmer than the surroundings between the two anomalies (Hoskins, McIntyre and Robertson 1985).

The structure represented in Fig. 1 is precisely what is seen in many long-lived mesoscale convective systems in midlatitudes, e.g., Bosart and Sanders (1981), Fritsch and Maddox (1981), Smull and Houze (1985), Menard and Fritsch (1989), etc. As Houze (1977) and others have shown, a similar structure occurs in strong tropical mesoscale systems. Simulations of mesoscale convective systems have also shown this structure, e.g., Zhang and Fritsch (1986, 1987, 1988), Wang and Orlanski (1987). However, little has been done to express these observations and computations in potential vorticity terms. Schubert, Fulton and Hertenstein (1989)
radius of influence is approximately equal to the ratio of the Coriolis parameter to the Brunt frequency or roughly 0.01 (Hoskins, McIntyre and Robertson 1985). For this reason, the negative anomaly induced by convection near the tropopause will have negligible influence near the surface. Thus, the lower positive anomaly will control the flow in and near the boundary layer where modest lifting can often induce further convection.

If the deformation of the lower potential vorticity anomaly by shear is slow enough that one can ignore the resulting evolution of isentropic surfaces, the vertical motion can be understood in terms of flow up and down these surfaces. Conceptually, two possibilities exist. As Fig. 2a shows, the ambient shear will cause air to flow underneath the potential vorticity anomaly. Since this region is cooler than its surroundings, air will be forced to rise as it follows the isentropic surface there. Descent will occur as the air exits the region under the anomaly. The second possibility is illustrated in Fig. 2b. In the case of westerly shear, the circulation due to the anomaly forces air up the ambient isentropic surfaces on the downshear or east side of the anomaly with the reverse effect on the upshear or west side. The net effect is, of course, a combination of the two and together they illustrate the well-known fact that ascent occurs on the downshear side of an upper-level low pressure area.

Two questions arise concerning this mechanism. First, can convection create a potential vorticity anomaly that is strong enough to cause significant lifting of unstable boundary layer air? Second, will the very shear that makes this lifting possible allow the anomaly to survive long enough for this mechanism to act? In order to answer these questions, a three-dimensional balanced model that advects potential vorticity and applies the inversion principle to obtain the pressure, potential temperature, and wind fields has been developed. Nonlinear balance was used as our balance condition. The model itself does not incorporate convection, but simply responds to the imposed potential vorticity anomaly.

In this paper, the model is used to follow the evolution of an initially postulated potential vorticity distribution embedded in a sheared environment. The shear approximates that seen in real mesoscale convective complexes, and the initial potential vorticity anomalies are comparable to that expected after an initial episode of convection in a rather large convective system. As later shown, the resulting lifting is adequate to produce further significant convection given a sufficiently unstable environment.

The plan of this paper is as follows: section 2 describes the nonlinear balance equations and the numerical model. Section 3 discusses the production of potential vorticity by convection. In section 4 the results of the numerical simulations are reported, and finally in section 5 the conclusions are presented.
2. Theory and numerical model

The validity of quasi-geostrophic theory for mesoscale convective systems is probably marginal given their tight gradients and short advective time scales. Semigeostrophic theory is useful when gradients normal to the predominant flow are quite tight as in fronts (Hoskins 1975). However, in vortices, the geostrophic momentum approximation breaks down due to strong centrifugal forces. Axially symmetric generalizations of semigeostrophic theory are available, e.g., Hack and Schubert (1986), but as the discussion in the Introduction shows, deviations from axial symmetry are probably at the heart of the dynamics of long-lived mesoscale convective systems making these theories inapplicable.

The nonlinear balance approximation seems to be the most appropriate balance assumption for this problem (Lorenz 1960; McWilliams 1985). Unlike the case of geostrophic balance, horizontal inertial forces are accurately treated as long as vertical velocities are weak. The nonlinear balance model used differs from those quoted above, in that potential vorticity conservation is assumed. The above models employ an approximate vorticity equation from which potential vorticity conservation cannot be consistently derived (McWilliams and Gent 1980). Since potential vorticity is of central importance to this analysis, we suggest that our approximation may be preferable to the traditional one.

Currently ignored are horizontal ageostrophic advections and vertical advections of perturbation quantities. With these additional approximations the model most closely resembles a quasi-geostrophic model enhanced to account for horizontal inertial forces, such as might be important in small intense vortices. The neglect of ageostrophic advections is probably that area in which the model is most vulnerable to criticism.

a. Derivation of nonlinear balance equations

The derivation begins with the hydrostatic primitive equations with minor approximation in the thermodynamic terms

\[
\frac{dv}{dt} + \frac{\theta_0}{f} k \times v = F, \quad (1)
\]

\[
\frac{\theta}{\partial \theta/\partial z} = b, \quad (2)
\]

\[
\nabla \cdot v + \rho_0^{-1}(\partial p_0/\partial z) = 0, \quad (3)
\]

\[
\frac{d b'}{dt} + N^2 w = gH/\theta_0, \quad (4)
\]

where \( \nabla \) is the horizontal gradient operator and \( z \) is the vertical geometrical coordinate. The velocity has horizontal components \( v = u_i + v_j \) and vertical component \( w \). The diabatic rate of potential temperature change is given by \( H \), and the external force per unit mass is \( F \). We set \( F = 0 \) in the numerical model, but retained here for completeness. The potential temperature is divided into ambient and perturbation parts, \( \theta = \theta_0(z) + \theta' \), and the buoyancy is defined \( b' = g\theta'/\theta_0 \), where \( g \) is the acceleration of gravity. The Exner function is similarly divided with \( \pi = \pi_0(z) + \pi' \), and the anelastic approximation is made with the ambient density \( \rho_0(z) \) used in the continuity equation. (The Exner function is defined \( \pi = C_p(p/p_0)^{\kappa} \), where \( p \) is the pressure, \( p_0 \) is a constant reference pressure, \( \kappa = R/C_p \), \( C_p \) is the specific heat at constant pressure, and \( R \) is the universal gas constant divided by the mean molecular weight of air. Note, also, that \( \pi = C_p T/\theta \), where \( T \) is the absolute temperature.) The analysis is done on an \( f \)-plane, i.e., the Coriolis parameter, \( f \), is assumed constant. The Brunt frequency is \( N(z) \).
The horizontal velocity components may be represented as the sum of solenoidal and irrotational parts:

\[ u = -\frac{\partial \psi}{\partial y} - \frac{\partial \phi}{\partial x} , \quad (5) \]
\[ v = \frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y} . \quad (6) \]

Inverting these equations yields an equation for the streamfunction \( \psi \),

\[ \nabla^2 \psi = \zeta_s = (\partial v / \partial x - \partial u / \partial y) \quad (7) \]

and the velocity potential \( \phi \),

\[ \nabla^2 \phi = -\delta = -(\partial u / \partial x + \partial v / \partial y) , \quad (8) \]

where \( \nabla^2 \) is the horizontal Laplacian, \( \zeta_s \) is the vertical component of relative vorticity, and \( \delta \) is the horizontal divergence.

Given \( u \) and \( v \), the determination of \( \psi \) and \( \phi \) is not unique because there exists, in general, a flow component that is both irrotational and solenoidal that can be assigned to either \( \psi \) or \( \phi \). However, we insist that

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi \, dx \, dy = 0 \quad (9) \]

and further demand that \( \phi \to 0 \) at infinity, then the ambiguous part is uniquely assigned to \( \psi \).

To the extent that vertical velocities are small, \( \phi \) can be ignored, and the \( x \) and \( y \) components of vorticity can be approximated as

\[ \zeta_x \approx -\frac{\partial^2 \psi}{\partial x \partial z} , \quad \zeta_y \approx -\frac{\partial^2 \psi}{\partial y \partial z} . \quad (10) \]

The potential vorticity \( q = \rho^{-1}(f + f k) \cdot \nabla \theta \) thus becomes, to this degree of approximation,

\[ q = \frac{1}{\rho_0} \left[ -\frac{\partial^2 \psi}{\partial x \partial z} \frac{\partial \theta}{\partial x} - \frac{\partial^2 \psi}{\partial y \partial z} \frac{\partial \theta}{\partial y} + (f + \nabla^2 \psi) \frac{\partial \theta}{\partial z} \right], \quad (11) \]

where the air density \( \rho \) has been approximated by its ambient profile, \( \rho_0 \). Rewriting (11) in terms of \( \theta_0 \) and \( \pi \), and splitting potential vorticity into ambient and perturbation parts, \( q = q_0(z) + q' \), where \( q_0 = f(d\theta_0 / dz) / \rho_0 \), yields

\[ \frac{\rho_0 q'}{\theta_0} = \frac{f}{\theta_0} \frac{\partial}{\partial z} \left( \theta_0^2 \frac{\partial \pi'}{\partial z} \right) + N^2 \nabla^2 \psi \]
\[ - \theta_0 \left( \frac{\partial^2 \psi}{\partial x \partial z} \frac{\partial \pi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial y \partial z} \frac{\partial^2 \pi}{\partial y \partial z} \right) . \quad (12) \]

The term \( (\partial \theta / \partial z) \nabla^2 \psi \) has been dropped even though it is formally of the same order as the other nonlinear terms. This is consistent with later approximations in which the perturbation vertical gradients are ignored compared to the ambient vertical gradients in vertical advection terms.

The nonlinear balance condition arises from replacing \( u \) and \( v \) in the horizontal momentum equations by \(-\partial \psi / \partial y \) and \( \partial \phi / \partial x \), and further assuming that the vertical velocity is negligible. The sum is then taken of the \( x \) derivative of the \( x \) equation and the \( y \) derivative of the \( y \) equation resulting in

\[ \nabla^2 (\theta_0 \pi' - f \psi) = 2 \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - 2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2 . \quad (13) \]

Equations (12) and (13) form a diagnostic pair, whereby the Exner function perturbation, \( \pi' \), and the streamfunction, \( \psi \), may be deduced from the perturbation potential vorticity, \( q' \). If the last major term of (12) and the right side of (13) are dropped the equations reduce to the quasi-geostrophic case. The difference between nonlinear balance and geostrophic balance thus lies in the inclusion of terms that are nonlinear in variables with \( x \) and \( y \) dependence.

The potential vorticity obeys the equation

\[ \frac{dq}{dt} = \rho^{-1} \nabla \cdot (H \theta_0 - \theta \nabla \times F) , \quad (14) \]

where \( \theta_0 \) is the absolute vorticity and \( H \), as before, is the diabatic rate of change of potential temperature, and \( F \) the applied force per unit mass. Expanding this in terms of the ambient and perturbation potential vorticity and considering only the streamfunction contribution to the horizontal velocity results in

\[ \frac{\partial q'}{\partial t} - \frac{\partial \psi}{\partial x} \frac{\partial q'}{\partial x} + \frac{\partial \psi}{\partial y} \frac{\partial q'}{\partial y} + w \frac{dq_0}{dz} \]
\[ = \rho^{-1} \nabla \cdot (H \theta_0 - \theta \nabla \times F) , \quad (15) \]

where the vertical advection of perturbation potential vorticity has been ignored. As long as the horizontal divergence is much less than the vertical component of absolute vorticity this approximation is justified.

To a similar degree of approximation, (4) can be written

\[ \frac{\partial \theta'}{\partial t} - \frac{\partial \psi'}{\partial x} \frac{\partial \theta'}{\partial x} + \frac{\partial \psi'}{\partial y} \frac{\partial \theta'}{\partial y} + w N^2 = g H / \theta_0 . \quad (16) \]

Defining a reduced potential vorticity perturbation,

\[ \eta' = q' - N^{-2}(dq_0 b'/dz) , \quad (17) \]

we find that \( \eta' \) obeys the prognostic equation

\[ \frac{\partial \eta'}{\partial t} - \frac{\partial \psi'}{\partial x} \frac{\partial \eta'}{\partial x} + \frac{\partial \psi'}{\partial y} \frac{\partial \eta'}{\partial y} \]
\[ = \rho_0^{-1} \left[ \nabla \cdot \left( H \theta'_0 - \theta_0 \nabla \times F \right) - \frac{f}{\theta_0} \frac{dq_0}{dz} \right] H , \quad (18) \]

which conveniently lacks a vertical advection term. The reduced potential vorticity perturbation is related to Charnay and Stern's (1962) pseudopotential vorticity or quasi-geostrophic potential vorticity. To first order it is actually the potential vorticity anomaly on the isentropic surface that intersects the specified elevation when \( b' = 0 \).
Rewriting (12) in terms of $q'$, we find

$$\frac{f^2 q'}{\theta_0 q_0} = \left( \frac{f^2 q_0}{N^2 \theta_0^2} \right) \frac{\partial}{\partial z} \left( \frac{\theta_0}{q_0} \frac{\partial q'}{\partial z} \right) + \frac{f}{\theta_0} \nabla^2 q'$$

$$- \frac{f}{N^2} \left( \frac{\partial^2 q'}{\partial x \partial z} \frac{\partial^2 q'}{\partial x \partial z} + \frac{\partial^2 q'}{\partial y \partial z} \frac{\partial^2 q'}{\partial y \partial z} \right). \tag{19}$$

b. Numerical model

The principal numerical problem in solving the above set of equations is in obtaining the streamfunction and Exner function perturbation from the reduced potential vorticity perturbation. A multigrid relaxation technique was used to solve this elliptic problem (Briggs 1987). The boundary condition $\theta_0 (\partial q' / \partial z) = b'$ is imposed at $z = 0$. The Exner function perturbation is also fixed at north and south walls and at the top of the domain. The condition $\theta_0 q' = f q$ is imposed at the north and south walls as well, which enforces geostrophic balance on the average. Periodic boundary conditions are applied at the east and west walls.

Relaxation of (13) is more easily done if an intermediate variable, $\Sigma$, which indicates departure from geostrophic balance, is introduced. With

$$\Sigma = \theta_0 q' - f q, \tag{20}$$

(13) may be written

$$\nabla^2 \Sigma - S_\Sigma = 0, \tag{21}$$

where

$$S_\Sigma = 2 \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} - 2 \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)^2, \tag{22}$$

and (19) becomes

$$\nabla^2 \Sigma' + \left( \frac{f^2 q_0}{N^2 \theta_0^2} \right) \frac{\partial}{\partial z} \left( \frac{\theta_0}{q_0} \frac{\partial q'}{\partial z} \right) - S_{\Sigma'} = 0, \tag{23}$$

where

$$S_{\Sigma'} = \frac{f^2 q'}{\theta_0 q_0} + \frac{1}{\theta_0} S_\Sigma + \frac{f}{N^2} \left( \frac{\partial^2 \psi}{\partial x \partial z} \frac{\partial^2 q'}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial y \partial z} \frac{\partial^2 q'}{\partial y \partial z} \right). \tag{24}$$

The nonlinear terms $S_\Sigma$ and $S_{\Sigma'}$ were frozen during each multigrid relaxation cycle, and recalculated afterwards. The process converged to a sufficiently accurate solution after 8 to 12 relaxation cycles. The method works as long as the nonlinear terms are small relative to the linear terms. This simplification is advantageous, in that true nonlinear multigrid techniques are quite complex. Physically this condition means that deviations from geostrophic balance are small, and only nonlinear balance problems that deviate moderately from the corresponding quasi-geostrophic problem can be solved by this technique.

The advection of reduced potential vorticity perturbation and buoyancy (the latter only at the surface) was done using a leapfrog scheme in time and centered differencing in space. No artificial viscosity or mode mixing were needed for the short integrations used in this paper. The vertical velocity was obtained by using the buoyancy equation (16) diagnostically in the interior of the domain. No attempt was made to diagnose the ageostrophic horizontal velocity components.

The equations were solved on a $17 \times 17 \times 17$ grid with a 100 km grid size in the horizontal and a 1 km grid size in the vertical. The domain is thus 1600 km by 1600 km in the horizontal, and 16 km in the vertical. A time step of 5 ks satisfied the Courant condition for these cases, and was found to be satisfactory.

3. Evolution of potential vorticity in mesoscale convective systems

Equation (14) can be used to demonstrate the conclusion of Thorpe and Emanuel (1985) and Haynes and McIntyre (1987) that potential vorticity (in a mass-integrated sense) cannot be created or destroyed, except at the earth’s surface. Density-weighted integration of (14) over some volume $\Gamma$ results in

$$\int_\Gamma (d q / dt) \rho dV = \int_\Gamma \nabla \cdot (H_\Sigma + \theta \nabla \times F) dV$$

$$= \int_{\partial \Gamma} (H_\Sigma + \theta \nabla \times F) \cdot n dA, \tag{25}$$

where the last integral is over the surface, $\partial \Gamma$, bounding $\Gamma$. The unit outward normal to $\partial \Gamma$ is $n$. If the volume is extended beyond the region in which diabatic heating and body forces exist, the surface integral is zero and the mass-integrated potential vorticity is conserved. However, if heating or stress is nonzero at the surface potential vorticity can flow into or out of the fluid through the bounding surface.

Evidently, $-(H_\Sigma + \theta \nabla \times F)$ is the nonadvective potential vorticity flux. Considering now only the contribution from diabatic heating, the strongest flux in a sheared environment is a horizontal one normal to the local shear. However, all this can do is move potential vorticity from one side of a convective system to the other. The vertical part causes downward transport of potential vorticity in proportion to the product of the diabatic heating and the vertical component of absolute vorticity. Ignoring lateral transport, we therefore approximate (14) by

$$dq / dt = -1 \partial (H_\Sigma + \theta \nabla \times F) / \partial z. \tag{26}$$

Since convective heating is intimately related to the vertical mass flux (see, for instance, Ooyama 1971), (26) is consistent with the picture presented in the introduction, in which the result of convection is to enhance the potential vorticity at low levels and deplete it at high levels. In particular, if $n$ is the upward volume flux of mass in convection per unit area per unit time,
then neglecting radiative effects and evaporation of condensate in the high-level outflow, we have

\[ H = m (\partial \theta_p / \partial z) \]  
(27)

Estimates of vertical mass flux derived from mesoscale or synoptic-scale sounding arrays are a composite of convective and nonconvective effects, and measurements of pure convective fluxes uncontaminated by anvil or other mesoscale effects are rare. Frank and Foote (1982) and Miller, Harris and Fankhauser (1982) measured vertical mass fluxes in severe convective systems in Colorado using multiple-Doppler radar and aircraft, and found peak upward fluxes near 7 km MSL with near-zero values at storm top and in the boundary layer. In some cases the net flux was downward in the boundary layer due to strong downdrafts. Raymond and Wilkening (1985) found in a composite study using aircraft measurements that summer thunderstorms in New Mexico had a maximum vertical mass flux near 5 km MSL with a secondary peak near 9 km. Starting from 5 km, mass fluxes tended to decrease nearly linearly to zero at the surface. The mass flux below 5 km was inferred to be a combination of a nearly constant updraft mass flux with height and a downdraft mass flux that increased linearly (in magnitude) from 5 km to just above the surface.

These results combined with (27) suggest that the downward potential vorticity flux due to convection increases monotonically with height from the boundary layer up to roughly the middle troposphere. As a consequence, the lower half of the troposphere has its potential vorticity increased by convection with the precise vertical profile being determined by the details of the convection in the particular case.

Nonconvective contributions to \( H \) in the form of latent heat release in stable ascent, radiation, and evaporation and melting of anvil precipitation are also possible in mesoscale convective systems. The precipitation effects should result in an upward flux of potential vorticity terminating near the base of the anvil, which presumably marks the highest level at which precipitation can evaporate. Thus, potential vorticity would be dumped at the base of the anvil. Radiation, on the other hand, would tend to remove potential vorticity from the anvil and deposit it at higher and lower levels. This is because anvil tops radiatively cool, corresponding to an upward flux of potential vorticity out of the top of the anvil, while anvil bases warm, corresponding to a downward flux out of the base (Webster and Stephens 1980). As Zhang and Fritsch (1987) have pointed out the heating due to stable ascent should be concentrated at lower levels since that is where most condensation occurs. This would produce a downward flux of potential vorticity in the lower half of the troposphere, which would tend to cancel the contribution from evaporation of precipitation.

For the thick anvils typical of mesoscale convective systems with bases near the freezing level, radiation and precipitation effects together would tend to create a layer of enhanced potential vorticity just below the base of the anvil. Radiation by itself can be quite significant in inducing a flow of potential vorticity. Webster and Stephens (1980) calculated heating rates at the base of thick anvil clouds in the tropics to be of order 20 K day\(^{-1}\). If the corresponding potential vorticity flux is dumped in a layer 2 km thick in the middle troposphere, this corresponds to a potential vorticity generation rate of order 2 pvu day\(^{-1}\). (The potential vorticity unit, or pvu, was suggested by Hoskins, McIntyre and Robertson (1985) and equals 10\(^{-6}\) K m\(^2\) kg\(^{-1}\) s\(^{-1}\).) Since the ambient potential vorticity at middle levels is of order 0.6 pvu, this is quite a significant contribution considering that mesoscale anvils can be spread over tens of thousands of square kilometers. Leary and Houze (1979) inferred cooling rates due to evaporation and melting of anvil precipitation in tropical systems between 5\(^{\circ}\) and 170\(^{\circ}\) K day\(^{-1}\). Undoubtedly, the higher cooling rates are spread over smaller areas, but it is probable that this can also be a significant source of potential vorticity transport. The heating rate due to stable, saturated ascent is simply

\[ H = w (\partial \theta_p / \partial z), \]  
(28)

where \( \theta_p \) is the potential temperature of a moist parcel. If \( \partial \theta_p / \partial z = 6^\circ\)K km\(^{-1}\) and \( w = 1\) cm s\(^{-1}\), then \( H \approx 5^\circ\)K day\(^{-1}\), which shows that this factor is also potentially important.

It is clear from the above results that the convective and anvil regions have the same qualitative effect on potential vorticity distributions, i.e., potential vorticity in the lower half of the troposphere is increased, whereas potential vorticity near the tropopause is decreased. However, further work is needed in determining the contribution from each effect in a variety of mesoscale systems. In particular, the spatial distributions of such effects as radiation and evaporative cooling need to be determined, as well as the prevalence of stable lifting.

Measurements of diabatic heating based on mesoscale and large-scale sounding arrays present a composite of convective and mesoscale effects, and are therefore useful in obtaining an overall view of a mesoscale system. Thompson, Payne, Recker and Reed (1979) showed that convection in the tropical Atlantic and the tropical Pacific had very different characteristics. Whereas the heating in the Pacific systems peaked near 7 km, the Atlantic systems peaked between 2 and 5 km. The positive potential vorticity anomaly produced by Atlantic systems should therefore be concentrated at much lower levels than in the Pacific systems.

Bosart and Sanders (1981) measured the characteristics of a long-lived mesoscale convective complex using a time compositing technique. They found that about 2 \times 10^7 m^3 s\(^{-1}\) of air ascended through middle levels over a circle roughly 400 km in diameter on the average over a four day period. (Peak volume fluxes exceeded this by as much as a factor of 10 over short periods.) This corresponds to a mean ascent rate over
large area. The observed mean potential vorticity anomaly over the same volume was due mostly to positive relative vorticity and amounted to about 0.2 pu. Given the inferred potential vorticity import rate into this volume, the observed anomaly could have been produced in a little more than one day.

All of the above arguments suggest that mesoscale potential vorticity anomalies with strengths comparable to the ambient potential vorticity can be created within a few hours to a day. We now investigate the effect of such anomalies on the flow.

4. Simulation results

Simulations were performed on the evolution of various potential vorticity anomalies in a moderately sheared environment characteristic of those in which mesoscale convective complexes occur. (See, for instance, McAnelly and Cotton 1986.) Figure 3a shows the westerly component of the ambient wind chosen for the simulations (the southerly component was set to zero), while Fig. 3b shows the assumed ambient potential temperature as a function of height. The ambient wind was assumed to be independent of $y$, but the tropopause was assumed to be 12 km at the south side of the domain and 10 km at the north side. This results in the ambient potential temperature and potential vorticity patterns in the $y$-$z$ plane shown in Fig. 4. As expected, the isentropes slope up to the north in the troposphere, and downward in the stratosphere. Only weak horizontal gradients of potential vorticity exist in the troposphere.

Anomalies in $\eta'$ were assumed to have the form

$$\eta' = E \exp\left(-\frac{x}{r_0}^2 - \frac{y}{r_0}^2 - \frac{z}{z_0}^2\right),$$

where $x$, $y$, and $z$ are positions relative to the center of the anomaly. In this paper we discuss the results of

![Fig. 3. Ambient conditions in the center of the computational domain. (a) Westerly wind as a function of height. (b) Potential temperature as a function of height.](image)

![Fig. 4. Ambient conditions in a vertical, north–south plane. The solid lines show contours of potential temperature with a contour interval of 10 K. The numbers show ambient potential vorticity (pu), and vertical hatching shows where the potential vorticity exceeds 1.5 pu, roughly indicating the stratosphere.](image)
a configuration similar to that attained by the long-lived convective system discussed by Bosart and Sanders (1981). Two anomalies were assumed, a lower one with $E = 0.3$ pvu, and an upper one with $E = -1.0$ pvu. The vertical radius of both anomalies was assumed to be $r_0 = 3$ km, while both were assumed to have a horizontal radius of $r_0 = 200$ km. The lower anomaly was centered at 3 km elevation, while the upper was placed at 10 km. Both were started in the center of the 1600 km by 1600 km domain and allowed to evolve for a period of 80 ks, or nearly one day.

Figure 5 shows the potential vorticity and temperature anomalies and the flow in the vertical, east-west plane centered on the initial anomaly positions at $y = 800$ km. Figure 5a shows the situation at a time of 10 ks or about 3 h after the start of the simulation, whereas Fig. 5b is taken at a time of 80 ks. During this interval the upper anomaly advects about 400 km to the east, whereas the lower anomaly is advected about 200 km to the west and deformed significantly by the shear. However, note that upward motion of similar magnitude exists at both times on the downshear-side of the lower anomaly in agreement with the discussion in the Introduction. As previously discussed, the region between the potential vorticity anomalies is warmer than the surroundings, while the regions above and below are cooler. At $t = 80$ ks, the potential temperature structure is somewhat modified by advection normal to the plane of the figure.

Figure 6 shows contours of potential temperature, horizontal flow, and vertical velocity at $z = 3$ km. The upward motion is correlated with warm advection caused by the circulation around the potential vorticity anomaly. Thus, the lifting mechanism illustrated in Fig. 2b is acting.

**FIG. 6.** Flow and potential temperature at $z = 3$ km and $t = 10$ ks. Contours indicate constant values of potential temperature at 1 K intervals, with cooler regions to the north. Vectors show the horizontal flow with a scale of 3 m s$^{-1}$ per 100 km. Vertical hatching shows updrafts in excess of 0.5 cm s$^{-1}$, while downdrafts less than -0.5 cm s$^{-1}$ are indicated by horizontal hatching.

The peak vertical velocity predicted by the simulation is of order 1 cm s$^{-1}$, which seems rather weak. However a trajectory analysis shows that it can cause ascent of up to 700 m over a 24-h period. Figure 7 shows trajectories of those parcels starting at $z = 2$ km at the initial time, which either ascend or descend more than 500 m. In agreement with the vertical velocity
patterns those parcels east of the lower potential vorticity anomaly ascend as they move cyclonically around the anomaly, while those on the west side descend a similar amount.

Under the right conditions, a 500 m ascent would be sufficient to release conditional instability, especially since the ascent occurs in the rather deep layer from 1 to 5 km (see Fig. 5). Thus, the quasi-balanced vertical motions induced by the lower potential vorticity anomaly interacting with the ambient shear could produce additional convection that would reinforce the anomaly.

To get a better notion of the amount of mass being moved vertically, the upward mass current, \( M_u \), was calculated as

\[
M_u = g \rho_0 \int_{h=0} \omega \, dA. \tag{30}
\]

Figure 8 shows \( M_u \) as a function of height at time 10 ks. The maximum current has a magnitude of \( 1.8 \times 10^8 \) m\(^2\) mb s\(^{-1}\) or approximately \( 1.8 \times 10^9 \) kg s\(^{-1}\). This is of the same order of magnitude as found by Bosart and Sanders (1981) using kinematic analysis. The simulated mass current peaks at an elevation of 2 km and drops off to near zero above 6 km. This contrasts with the observed peak current near 5 km. However, the difference can be attributed to the neglect of penetrative convection in the simulation—presumably, if unstable air were lifted above the level of free convection near 2 km, it would ascend to the tropopause. If significant entrainment occurred, the level of maximum vertical mass flux could then be pushed upward.

Figure 9 shows a horizontal section at \( z = 10 \) km and \( t = 10 \) ks. The anticyclonic circulation around the upper potential vorticity anomaly with the corresponding jet around the north side is well represented. This confirms the findings of Maddox, Perkey and Fritsch (1981) that this circulation is caused by the convective system itself and not by some external mechanism.

Finally in Fig. 10, the effect of the circulation from the lower potential vorticity anomaly on the surface temperature distribution and flow is shown. Warm surface air is advected to the north around the east side of the lower anomaly, while cold air is advected to the south around the west side. This resembles the spinup of a surface cyclone by an existing low pressure area aloft as described by Petterssen and Smebye (1971), except that the scale is small and the wind anomalies due to the modified surface temperature distribution are weak. This effect wasn't observed at the surface by Bosart and Sanders (1981), which suggests that the potential vorticity anomaly did not reach quite as low as postulated in the model.

5. Discussion

In this paper we show that large long-lived convective systems can produce mesoscale potential vorticity anomalies of a strength comparable to the ambient potential vorticity. Furthermore, the interaction of these anomalies with a sheared environment can produce enough low-level lifting to feed the observed convection. We therefore, postulate the existence of self-maintaining mesoscale convective systems based on an interaction between convection and slow manifold dynamics.
We suggest that the description of this process in terms of "potential vorticity thinking" is a clear and economical way of characterizing what is happening. In particular, it shows that convection and anvil processes influence slow manifold dynamics solely through their effects on the potential vorticity distribution. This understanding can be useful both in the interpretation of complex numerical models and in the development of strategies for observational programs.

In the latter area much remains to be done. Though it is clear that latent heat release, melting and evaporation of precipitation, and thermal radiation effects are all potentially important to the dynamics of mesoscale convective systems, their distributions and integrated intensities have not been well measured. Making these measurements is clearly an important goal for improving the understanding of these systems.

Further work also needs to be done in the theoretical arena. Perhaps the weakest assumption in the present theory is that the tiny, but persistent quasi-balanced upward motions are more important in the long term than the much larger, but transient motions due to fast manifold effects. This assumption is clearly not true in certain circumstances. For instance, although satellite photographs of the Johnstown flood storm described by Bosart and Sanders (1981) showed the nearly stationary (and probably slow manifold) system that produced the flood, they also showed a rapidly moving squall line that was most likely driven by a density current or gravity wave. Both of these systems were reproduced in the numerical simulations of Zhang and Fritsch (1986, 1987). It would be interesting to diagnose the potential vorticity in such a model, seek its origins in the various modeled diabatic processes, and

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**Fig. 9.** Horizontal section at $z = 10$ km and time $= 10$ ks. The hatching shows regions with the reduced potential vorticity anomaly $\eta ' < -0.33$ pvu. The scale on the velocity vectors is $5$ m s$^{-1}$ per 100 km.

Bosart and Sanders (1981) suggested a mechanism that is quite similar to that postulated in this paper. They proposed that boundary layer air was forced to ascend as it moved over the cold dome underneath their observed system, implying possible further release of convective instability. This is precisely the mechanism described in Fig. 2a.

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**Fig. 10.** Surface flow and potential temperature pattern. Potential temperature contours at $2^\circ$K intervals are shown with the cooler air to the north. The velocity scale is $10$ m s$^{-1}$ per 100 km. (a) Time $= 10$ ks. (b) Time $= 80$ ks.
finally determine if the predicted slow manifold vertical velocities match the directly computed velocities in some average sense. In this way the relative importance of slow manifold and fast manifold processes could be assessed.

Finally, a fully predictive quasi-balanced model could be created, in which diabatic processes were parameterized directly in terms of their effects on the potential vorticity distribution. If such a model captured the most significant effects of convection (relative to large scale dynamics), it would be quite useful, considering the simplified initialization and computational economies available to balance models.

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REFERENCES


Lorenz, E. N., 1966: Maximum simplification of the dynamic equation. Tellus, 12, 243-254.


