The sea-breeze circulation

Part II: Effect of Earth’s rotation

Rotunno (1983)

- Quasi-2D analytic linear model
- Heating function specified over land
  - Becomes cooling function after sunset
  - Equinox conditions
- Cross-shore flow $u$, along-shore $v$
- Two crucial frequencies
  - Heating $\omega = 2\pi$/day (period 24h)
  - Coriolis $f = 2\Omega\sin\Phi$ (inertial period 17h @ 45°N)
- One special latitude… where $f = \omega$ (30°N)
Heating function

FIG. 1. (a) The heating function $\tilde{H}(\xi, \zeta)$ given by (22) with $\tilde{A} = 1$ and $\xi_0 = 0.2$; and (b) $\partial \tilde{H}(\xi, \zeta)/\partial \xi$ given by (23).

Rotunno’s analytic model lacks diffusion so horizontal, vertical spreading built into function.
Streamfunction $\psi$

$$u = \frac{\partial \psi}{\partial z} \quad w = -\frac{\partial \psi}{\partial x}$$
Circulation $C$

$C \equiv \oint \vec{V} \cdot d\vec{l}$

Integrate CCW as shown

$C \equiv \int (u_{sfc} - u_{top}) dx + \int (w_{land} - w_{sea}) dz$

Take $w \sim 0$;

$u_{top} \sim 0$

$C \approx \int_{x=-\infty}^{x=+\infty} u_{sfc} dx$

Integrate from ±infinity, from sfc to top of atmosphere
Rotunno’s analytic solution

\[ N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x} \]

- \( N \) = Brunt-Vaisalla frequency (buoyancy frequency)
- \( Q \) = heating function
- \( f \) = Coriolis parameter (inertial frequency)
- \( \omega \) = diurnal forcing frequency
- \( \psi \) = streamfunction (gradients are velocities)
Rotunno’s analytic solution

\[ N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x} \]

If \( f > \omega \) (poleward of 30°) equation is elliptic

- sea-breeze circulation spatially confined
- circulation in phase with heating
- circulation, onshore flow strongest at \textbf{noon}
- circulation amplitude decreases poleward

If \( f < \omega \) (equatorward of 30°) equation is hyperbolic

- sea-breeze circulation is extensive
- circulation, heating out of phase
- \( f = 0 \) onshore flow strongest at \textbf{sunset}
- \( f = 0 \) circulation strongest at \textbf{midnight}
Rotunno’s analytic solution

\[ N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x} \]

If \( f = \omega \) (30°N) equation is **singular**

- some friction or diffusion is needed
- circulation max at **sunset**
- onshore flow strongest at **noon**
f > \omega \text{ (poleward of } 30^\circ\text{)} \text{ at noon}

Note onshore flow strongest at coastline \((x = 0)\); this is day’s max

**Fig. 2.** (a) The streamfunction \(\tilde{\psi}(\xi, \zeta)\) at \(\tau = \pi/2\) for \(f > \omega\) [Eq. (24)]; (b), as in (a) except for \(\tilde{u}(\xi, \zeta)\); (c), as in (a) except for \(\tilde{w}(\xi, \zeta)\). Dashed lines indicate negative values in all figures.
\[ f < \omega \text{ (equatorward of 30°)} \psi \text{ at three times} \]

- Sunrise
- Noon (reverse sign for midnight)
- Sunset

Note coastline onshore flow max at sunset
Paradox?

- Why is onshore max wind at sunset and circulation max at midnight/noon?
  - While wind speed at coast strongest at sunset/sunrise, wind integrated along surface larger at midnight/noon

$$C \approx \int_{x=-\infty}^{x=+\infty} u_{sfc} \, dx$$

**Fig. 5.** The streamfunction $\tilde{\psi}(\xi, \zeta, \tau)$ at $\tau = 0, \pi/2$ and $\pi$ for $f < \omega$ for the heating function displayed in Fig. 1.
Effect of linear friction

Time of circulation maximum

As friction increases, tropical circulation max becomes earlier, poleward circulation max becomes later.
DTDM long-term sea-breeze strategy

- Incorporate Rotunno’s heat source, mimicking effect of surface heating + vertical mixing
- Make model linear
- Dramatically reduce vertical diffusion
- Simulations start at sunrise
- One use: to investigate effect of latitude and/or linearity on onshore flow, timing and circulation strength
The rotunno_seabreeze namelist implements a lower tropospheric heat source following Rotunno (1983), useful for long-term integrations of the sea-land-breeze circulation.

iseabreeze (1 = turn Rotunno heat source on; default is 0)
sb_ampl - amplitude of heat source (K/s; default = 0.000175)
sb_x0 - controls heat source shape at coastline (m; default = 1000.)
sb_z0 - controls heat source shape at coastline (m; default = 1000.)
sb_period - period of heating, in days (default = 1.0)
sb_latitude - latitude for experiment (degrees; default = 60.)
sb_linear (1 = linearize model; default = 1)
input_seabreeze.txt
&rotunno_seabreeze section

&rotunno_seabreeze
iseabreeze = 1,
sb_ampl = 0.000175,
sb_x0 = 1000.,
sb_z0 = 1000.,
sb_period = 1.0,
sb_latitude = 30.,
sb_linear = 1,
$

sb_latitude ≠ 0 activates Coriolis
sb_linear = 1 linearizes the model

Other settings include:

\text{timend} = 86400 \text{ sec}
\text{dx} = 2000 \text{ m}, \text{dz} = 250 \text{ m}, \text{dt} = 1 \text{ sec}
\text{dkx} = \text{dkz} = 5 \text{ m}^2/\text{s} (\text{since linear})
Caution

• Don’t make model anelastic for now
  – Make sure ianelastic = 0 and ipressure = 0
  – Didn’t finish the code for anelastic linear model
  – iseabreeze = 1 should be used alone (i.e., no thermal, surface flux, etc., activated)
Solution strategy

• Model starts at sunrise (6 am)
• Equinox presumed (sunset 6 pm)
• Heating max at noon, zero at sunset
• Cooling at night, absolute max at midnight
Heat source sb_hsrc

set mproj off
set lev 0 4
set lon 160 240 [or set x 80 120]
d sb_hsrc
Heating function vs. time

Period fixed as of version 1.2.2
Time series using GrADS

> open seabreeze.rotunno.30deg
> set t 1 289
> set z 1
> set x 100
> set vrange -0.0003 0.0003
> set xaxis 0 24 3
> d sb_hsrc
> draw xlab hour
> draw ylab heating function at surface
> draw title heating function vs. time
30°N linear case
(5 m²/s diffusion)

Shaded: vertical velocity; contoured: cross-shore velocity
30°N linear case

Circulation max @ sunset

set t 1 289
set z 1
set xaxis 0 24 3
d sum(u,x=1,x=199)
30°N linear case

Onshore flow max ~ 4pm

Note non-calm wind @ 24h... should run several days to spin-up
Variation with latitude
Cross-shore flow and vertical motion at noon

Cross-shore flow at noon (lat = 60 deg)  

Cross-shore flow at noon (lat = 30 deg)  

Cross-shore flow at noon (lat = 00 deg)  

(set t 72)
Cross-shore flow and vertical motion at sunset

cross-shore flow at sunset (lat = 60 deg)

cross-shore flow at sunset (lat = 30 deg)

cross-shore flow at sunset (lat = 00 deg)
Cross-shore flow and vertical motion at midnight
Cross-shore near-surface wind at coastline (linear model)
Circulation vs. time

- Circ magnitude decreases w/ latitude (expected)
- 30N circ max at sunset (expected)
- Poleward circ max later than expected (noon)
- Equator circ max earlier than expected (midnite)
- Consistent w/ existence of some friction?
Recall: linear friction effect

Time of circulation maximum

As friction increases, tropical circulation max shifts earlier, poleward circulation max becomes later.
Hovmoller diagrams
(seabreeze_hov.gs)

What is missing?
Questions to ponder

• Why is the onshore flow at 30N strongest in midafternoon? (Heating max was noon)
• Why does onshore max occur earlier at 60N?
• Why does onshore flow magnitude decrease poleward?
• Why does friction shift circulation max time?
• Note no offshore flow at equator at all. Why?
• How does linear assumption affect results?
Further exploration

• Run model longer… how long until statistically steady?
• Make model nonlinear
• Compare to data
  – Collect surface data along N-S coastline
  – Compare to “data” from more sophisticated model
Tips for nonlinear runs

• Set `sb_linear = 0`
• Increase `dkx, dkz` to avoid computational instability
• Example with `dkx = 500, dkz = 50 m²/s` (probably excessive)
Cross-shore wind at coastline
linear vs. nonlinear

Why are they different?
Linear vs. nonlinear at noon

linear - 30 deg - cross-shore flow at noon

nonlinear - 30 deg - cross-shore flow at noon
Linear vs. nonlinear at sunset