The sea-breeze circulation

Part II: Effect of Earth's rotation

Reference: Rotunno (1983, *J. Atmos. Sci.*)

Rotunno (1983)

- Quasi-2D analytic linear model
- Heating function specified over land
  - Becomes cooling function after sunset
  - Equinox conditions
- Cross-shore flow *u*, along-shore *v*
- Two crucial frequencies
  - Heating $\omega = 2\pi$/day (period 24h)
  - Coriolis $f = 2\Omega\sin\Phi$ (inertial period 17h @ 45˚N)
- One special latitude… where $f = \omega$ (30˚N)

Heating function

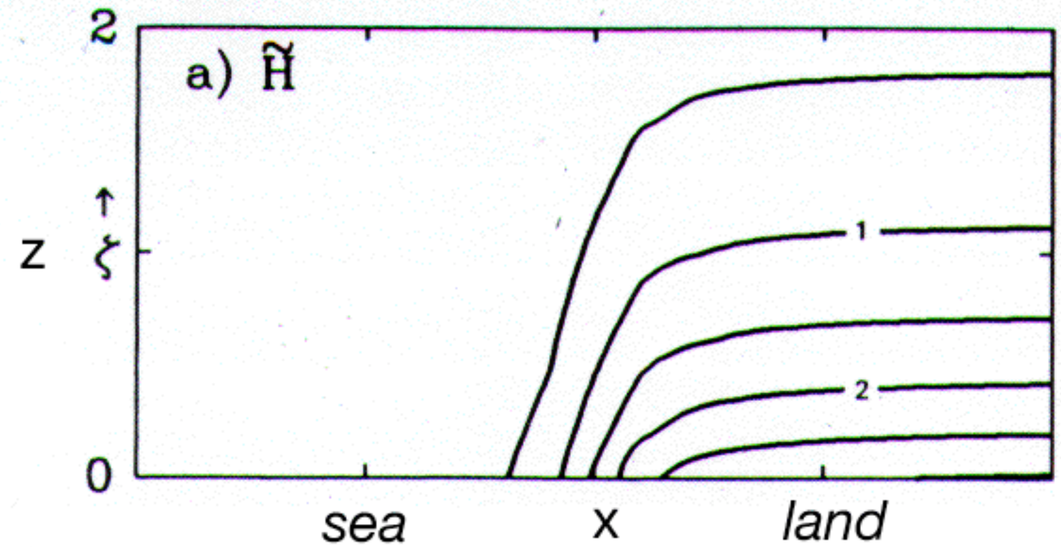
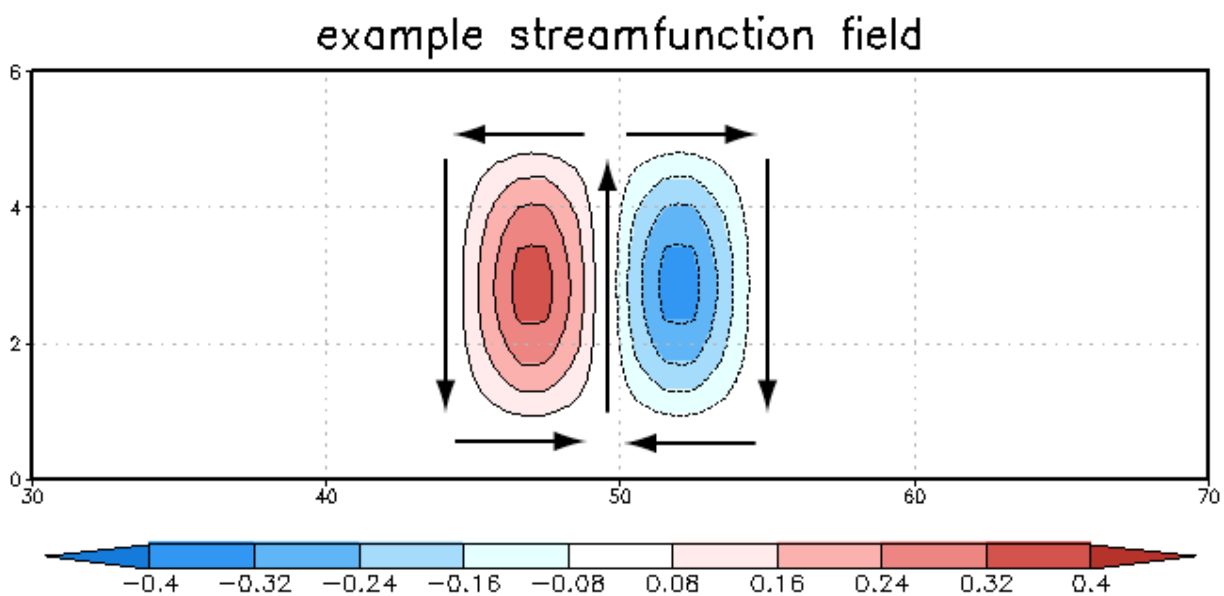


FIG. 1. (a) The heating function $\tilde{H}(\xi, \zeta)$ given by (22) with $\tilde{A} = 1$ and $\xi_0 = 0.2$; and (b) $\partial\tilde{H}(\xi, \zeta)/\partial\xi$ given by (23).

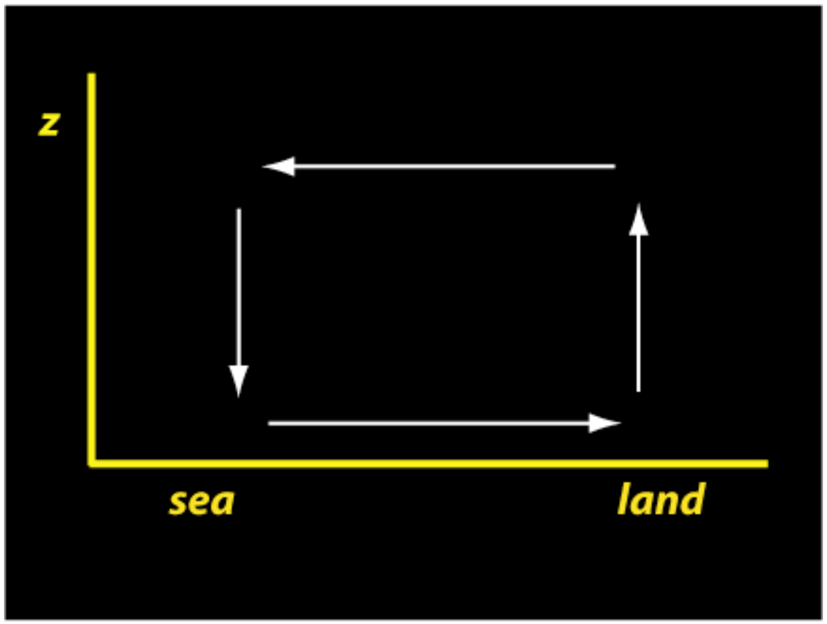
Rotunno's analytic model lacks diffusion
so horizontal, vertical spreading built into function

Streamfunction ψ

$$u = \frac{\partial \psi}{\partial z} \quad w = -\frac{\partial \psi}{\partial x}$$



Circulation *C*



Integrate from ±infinity,
from sfc to top of atmosphere

$$C \equiv \oint \vec{V} \cdot d\vec{l}$$

Integrate CCW as shown

$$C \equiv \int (u_{sfc} - u_{top})dx + \int (w_{land} - w_{sea})dz$$

Take $w \sim 0$;
$u_{top} \sim 0$

$$C \approx \int_{x=-\infty}^{x=+\infty} u_{sfc} dx$$

Rotunno's analytic solution

$$N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x}$$

- N = Brunt-Vaisalla frequency (buoyancy frequency)
- Q = heating function
- f = Coriolis parameter (inertial frequency)
- ω = diurnal forcing frequency
- ψ = streamfunction (gradients are velocities)

Rotunno's analytic solution

$$N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x}$$

If $f > \omega$ (poleward of 30˚) equation is elliptic

- sea-breeze circulation spatially confined
- circulation in phase with heating
- circulation, onshore flow strongest at **noon**
- circulation amplitude decreases poleward

If $f < \omega$ (equatorward of 30˚) equation is hyperbolic

- sea-breeze circulation is extensive
- circulation, heating out of phase
- $f = 0$ onshore flow strongest at **sunset**
- $f = 0$ circulation strongest at **midnight**

Rotunno's analytic solution

$$N^2 \frac{\partial^2 \psi}{\partial x^2} + (f^2 - \omega^2) \frac{\partial^2 \psi}{\partial z^2} = -\frac{\partial Q}{\partial x}$$

If $f = \omega$ (30˚N) equation is <u>singular</u>

- some friction or diffusion is needed
- circulation max at **sunset**
- onshore flow strongest at **noon**

f > ω (poleward of 30˚) at noon

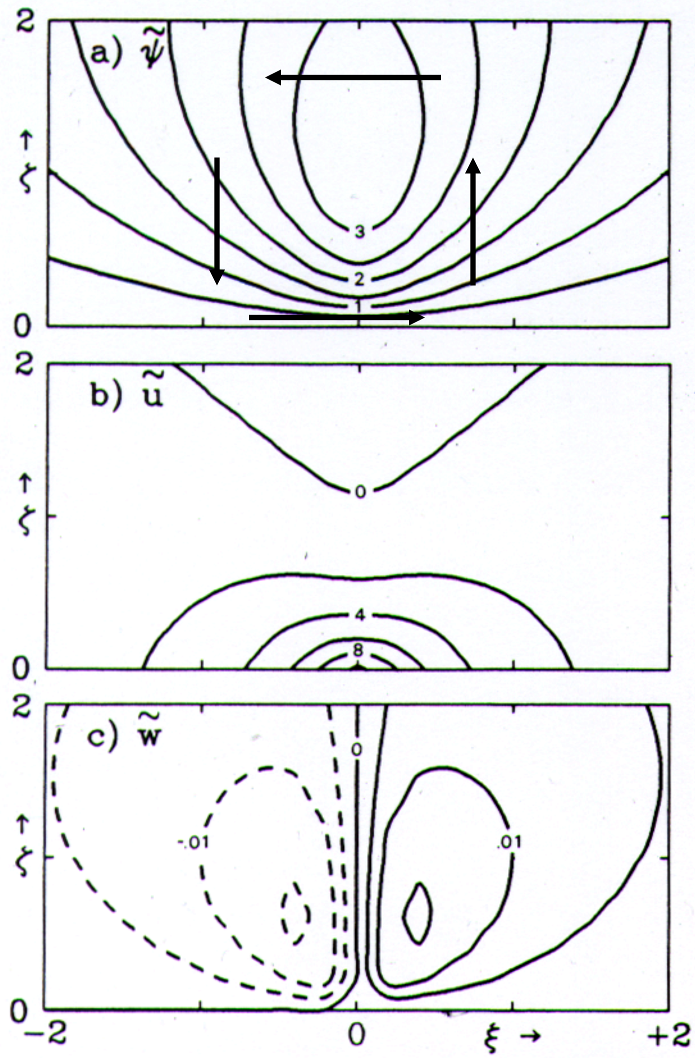


FIG. 2. (a) The streamfunction $\tilde{\psi}(\xi, \zeta)$ at $\tau = \pi/2$ for $f > \omega$ [Eq. (24)]; (b), as in (a) except for $\tilde{u}(\xi, \zeta)$; (c), as in (a) except for $\tilde{w}(\xi, \zeta)$. Dashed lines indicate negative values in all figures.

Note onshore flow strongest at coastline (x = 0);
this is day's max

f < ω (equatorward of 30˚) ψ at three times

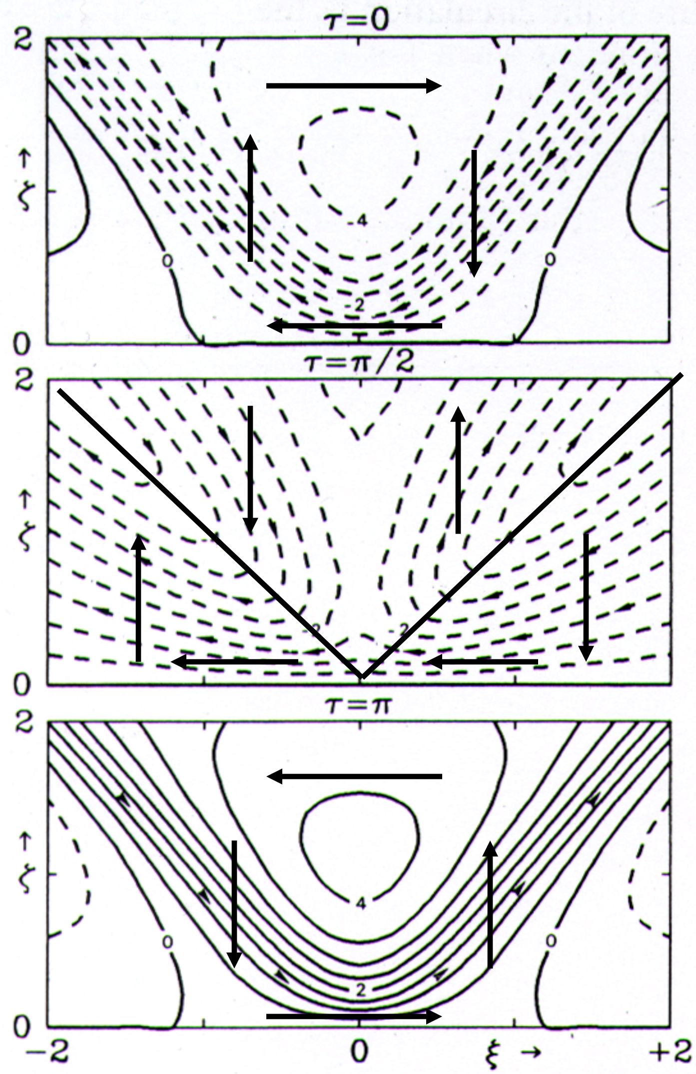


FIG. 5. The streamfunction $\tilde{\psi}(\xi, \zeta, \tau)$ at $\tau = 0, \pi/2$ and π for $f < \omega$ for the heating function displayed in Fig. 1.

sunrise

Noon
(reverse sign for midnight)

sunset

Note coastline onshore flow max at sunset

Max |C| noon & midnight

Paradox?

- Why is onshore max wind at sunset and circulation max at midnight/noon?
  - While wind speed at coast strongest at sunset/sunrise, wind integrated along surface larger at midnight/noon

$$C \approx \int_{x=-\infty}^{x=+\infty} u_{sfc} dx$$

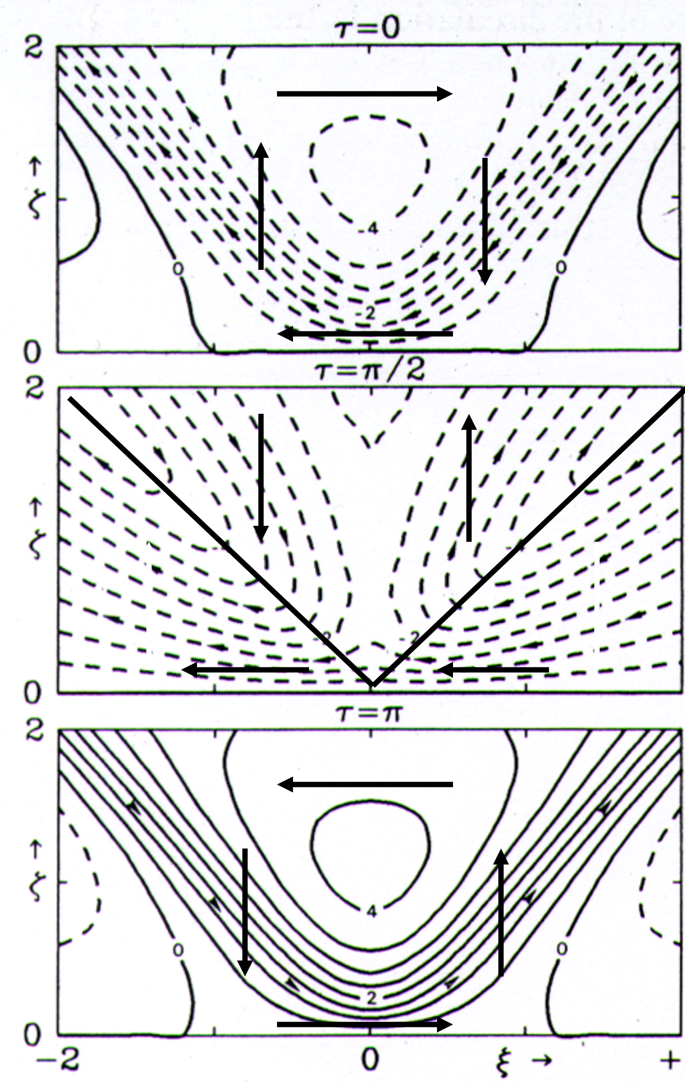


FIG. 5. The streamfunction $\tilde{\psi}(\xi, \zeta, \tau)$ at $\tau = 0, \pi/2$ and π for $f < \omega$ for the heating function displayed in Fig. 1.

Effect of linear friction

Time of circulation maximum

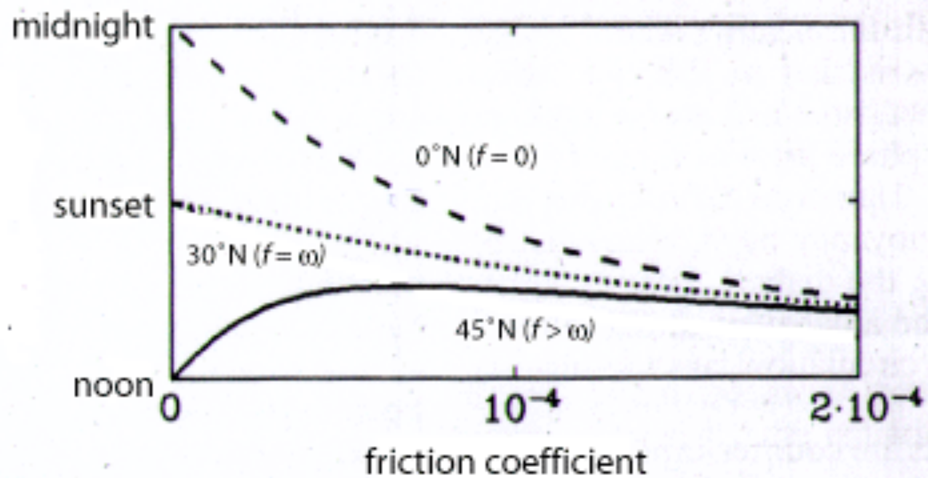


FIG. 6. The phase lag of the circulation relative to the heating as a function of the linear friction parameter α [Eq. (45)], for $f = 0$ (dashed line), $f = \omega$ (dotted line) and $f = 10^{-4}$ s^{-1} (solid line).

As friction increases, tropical circulation max becomes earlier, poleward circulation max becomes later

DTDM long-term sea-breeze strategy

- Incorporate Rotunno’s heat source, mimicking effect of surface heating + vertical mixing
- Make model linear
- Dramatically reduce vertical diffusion
- Simulations start at sunrise
- One use: to investigate effect of latitude and/or linearity on onshore flow, timing and circulation strength

input_seabreeze.txt
&rotunno_seabreeze section

```
c=====================================================================
c
c The rotunno_seabreeze namelist implements a lower tropospheric
c  heat source following Rotunno (1983), useful for long-term
c  integrations of the sea-land-breeze circulation
c
c  iseabreeze (1 = turn Rotunno heat source on; default is 0)
c  sb_ampl - amplitude of heat source (K/s; default = 0.000175)
c  sb_x0 - controls heat source shape at coastline (m; default = 1000.)
c  sb_z0 - controls heat source shape at coastline (m; default = 1000.)
c  sb_period - period of heating, in days (default = 1.0)
c  sb_latitude - latitude for experiment (degrees; default = 60.)
c  sb_linear (1 = linearize model; default = 1)
c
c=====================================================================
```

input_seabreeze.txt
`&rotunno_seabreeze` section

```
&rotunno_seabreeze
       iseabreeze = 1,
       sb_ampl = 0.000175,
       sb_x0 = 1000.,
       sb_z0 = 1000.,
       sb_period = 1.0,
       sb_latitude = 30.,
       sb_linear = 1,
$
```

`sb_latitude` ≠ 0 activates Coriolis
`sb_linear` = 1 linearizes the model

Other settings include:
`timend` = 86400 sec
`dx` = 2000 m, `dz` = 250 m, `dt` = 1 sec
`dkx` = `dkz` = 5 m^2/s (since linear)

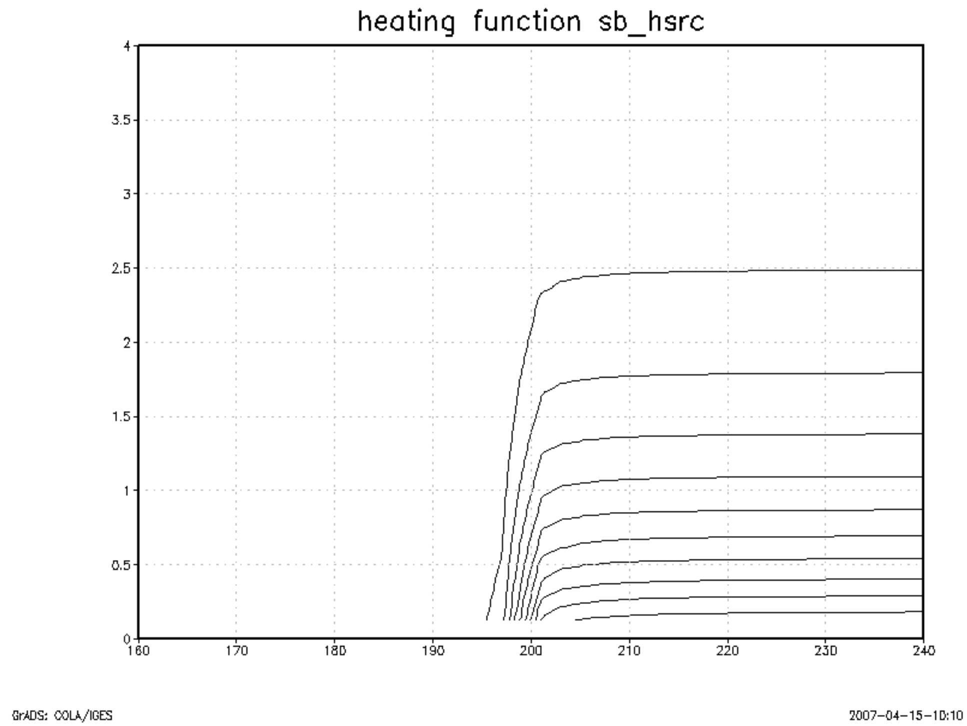
Caution

- Don't make model anelastic for now
  - Make sure `ianelastic = 0` and `ipressure = 0`
  - Didn't finish the code for anelastic linear model
  - `iseabreeze = 1` should be used alone (I.e., no thermal, surface flux, etc., activated)

Solution strategy

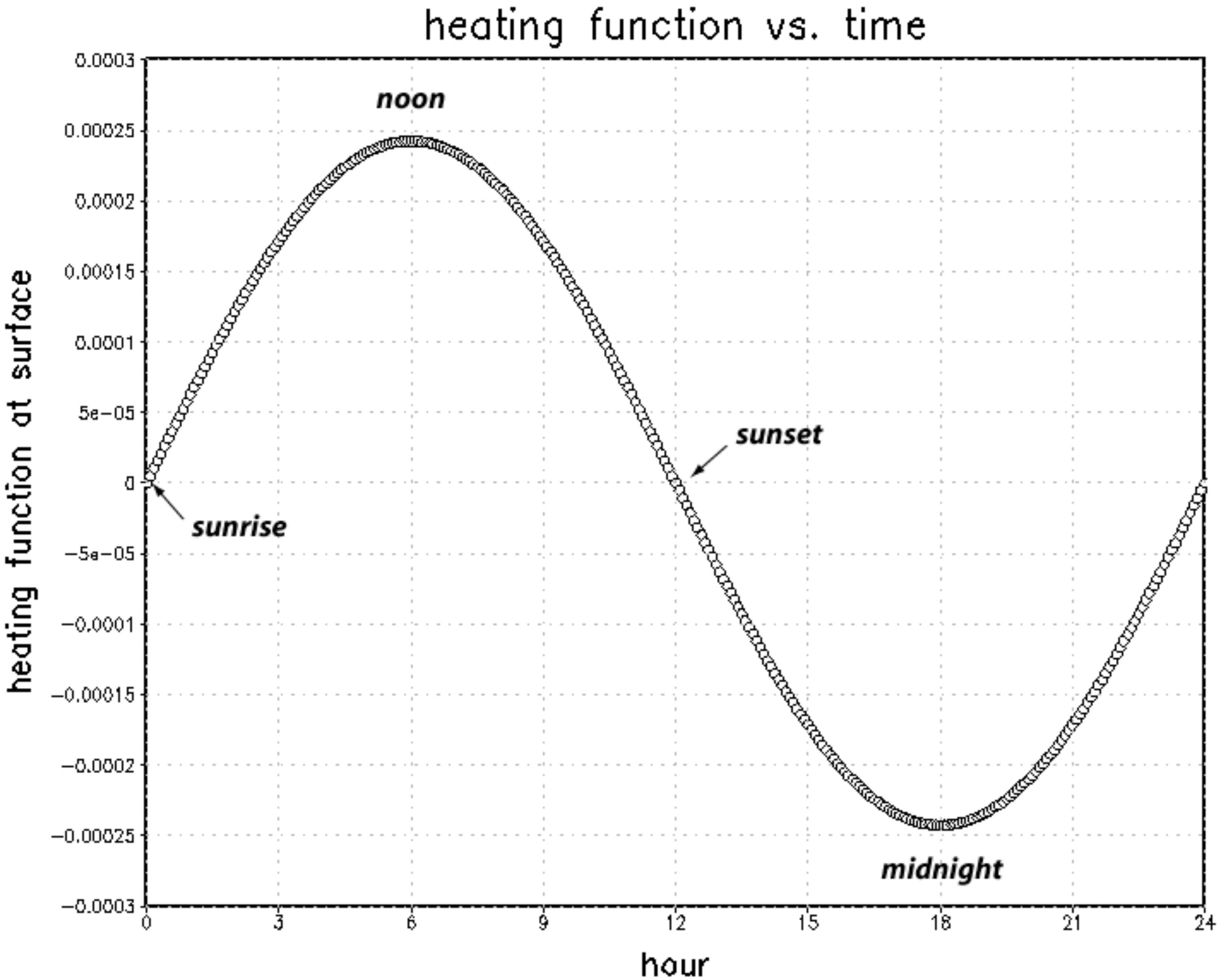
- Model starts at sunrise (6 am)
- Equinox presumed (sunset 6 pm)
- Heating max at noon, zero at sunset
- Cooling at night, absolute max at midnight

Heat source `sb_hsrc`



set mproj off
set lev 0 4
set lon 160 240 [or set x 80 120]
d sb_hsrc

Heating function vs. time



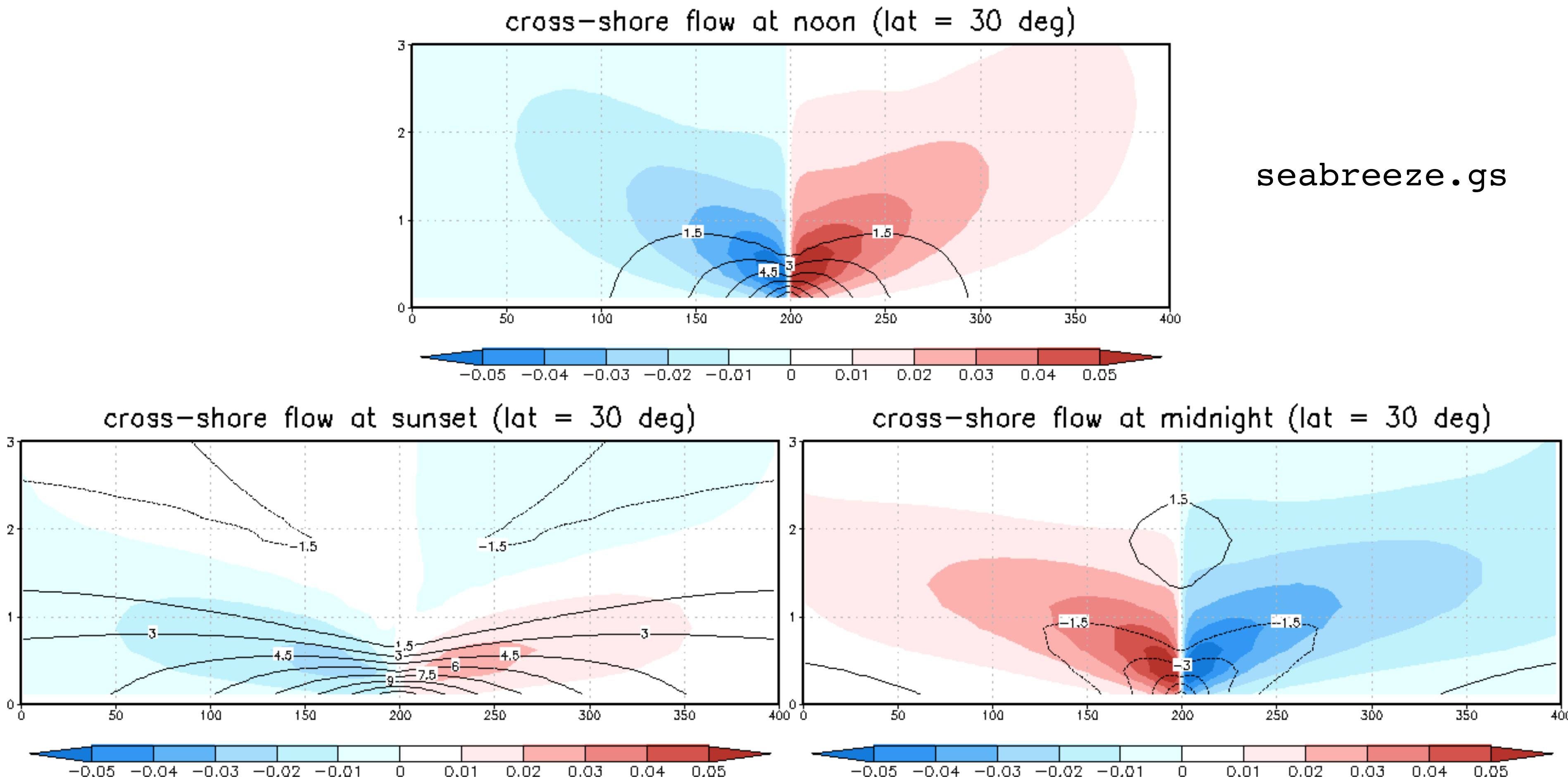
Period fixed as of version 1.2.2

Time series using GrADS

```
> open seabreeze.rotunno.30deg
> set t 1 289
> set z 1
> set x 100
> set vrange -0.0003 0.0003
> set xaxis 0 24 3
> d sb_hsrc
> draw xlab hour
> draw ylab heating function at surface
> draw title heating function vs. time
```

30˚N linear case

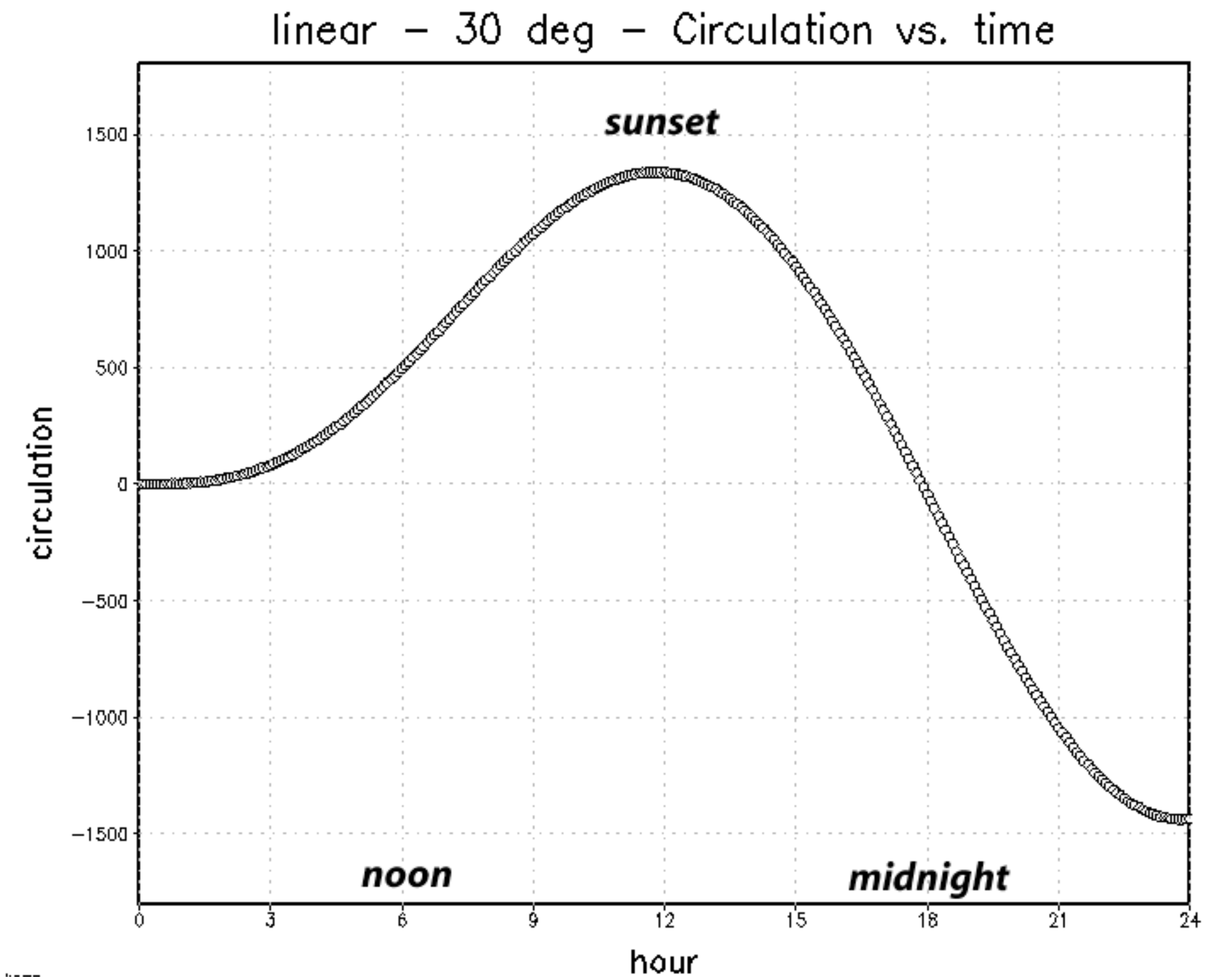
(5 m^2/s diffusion)



seabreeze.gs

Shaded: vertical velocity; contoured: cross-shore velocity

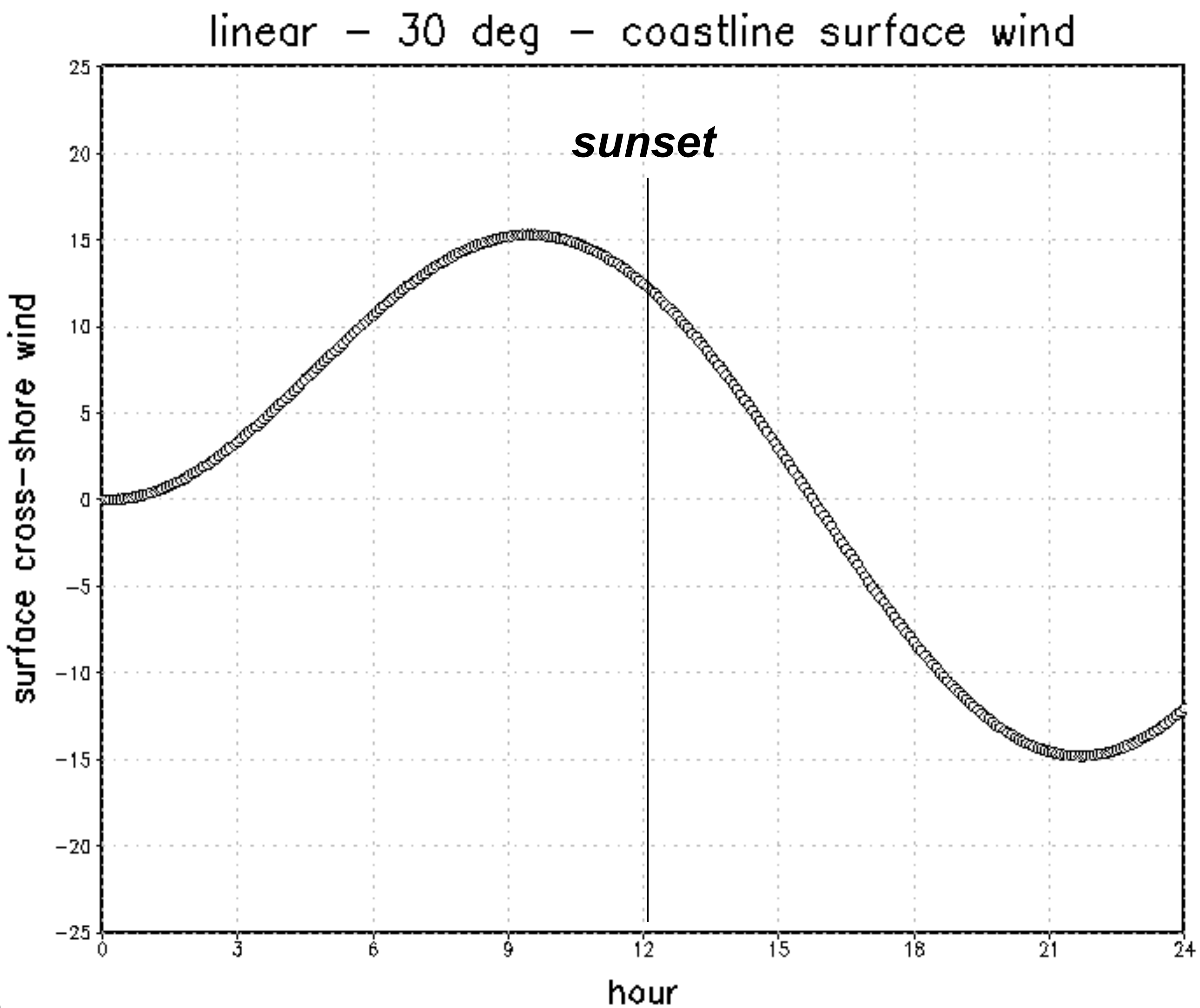
30˚N linear case



Circulation max @ sunset

set t 1 289
set z 1
set xaxis 0 24 3
d sum(u,x=1,x=199)

30˚N linear case

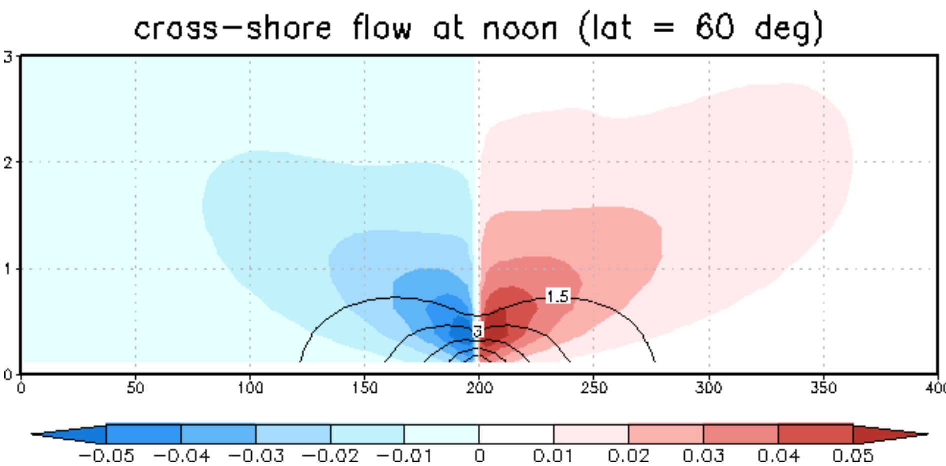


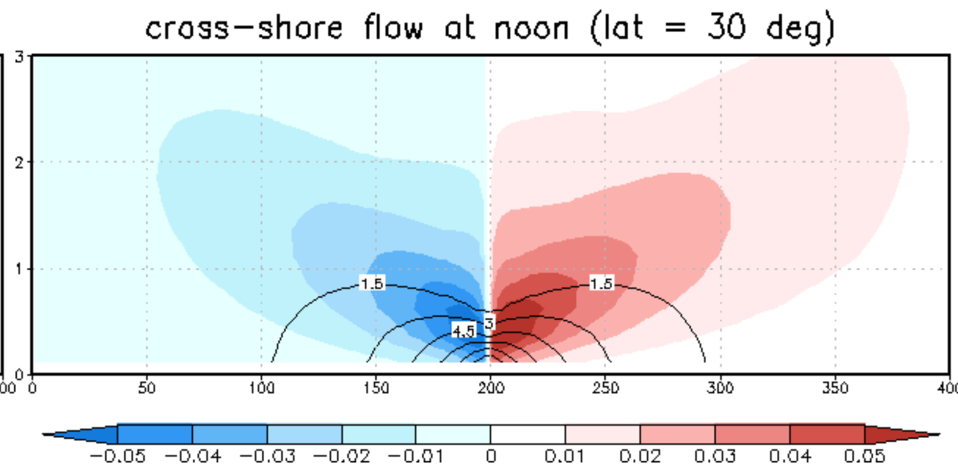
Onshore flow max ~ 4pm

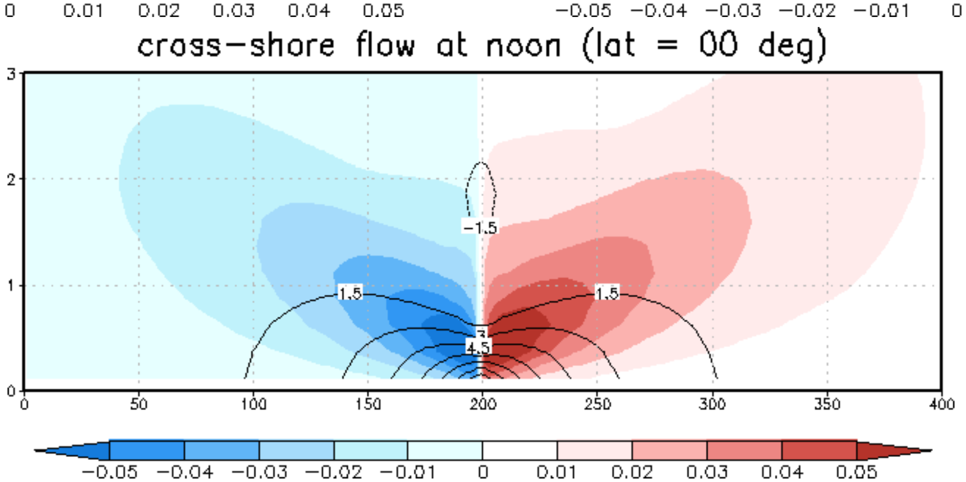
Note non-calm wind @ 24h… *should run several days to spin-up*

Variation with latitude

Cross-shore flow and vertical motion at noon



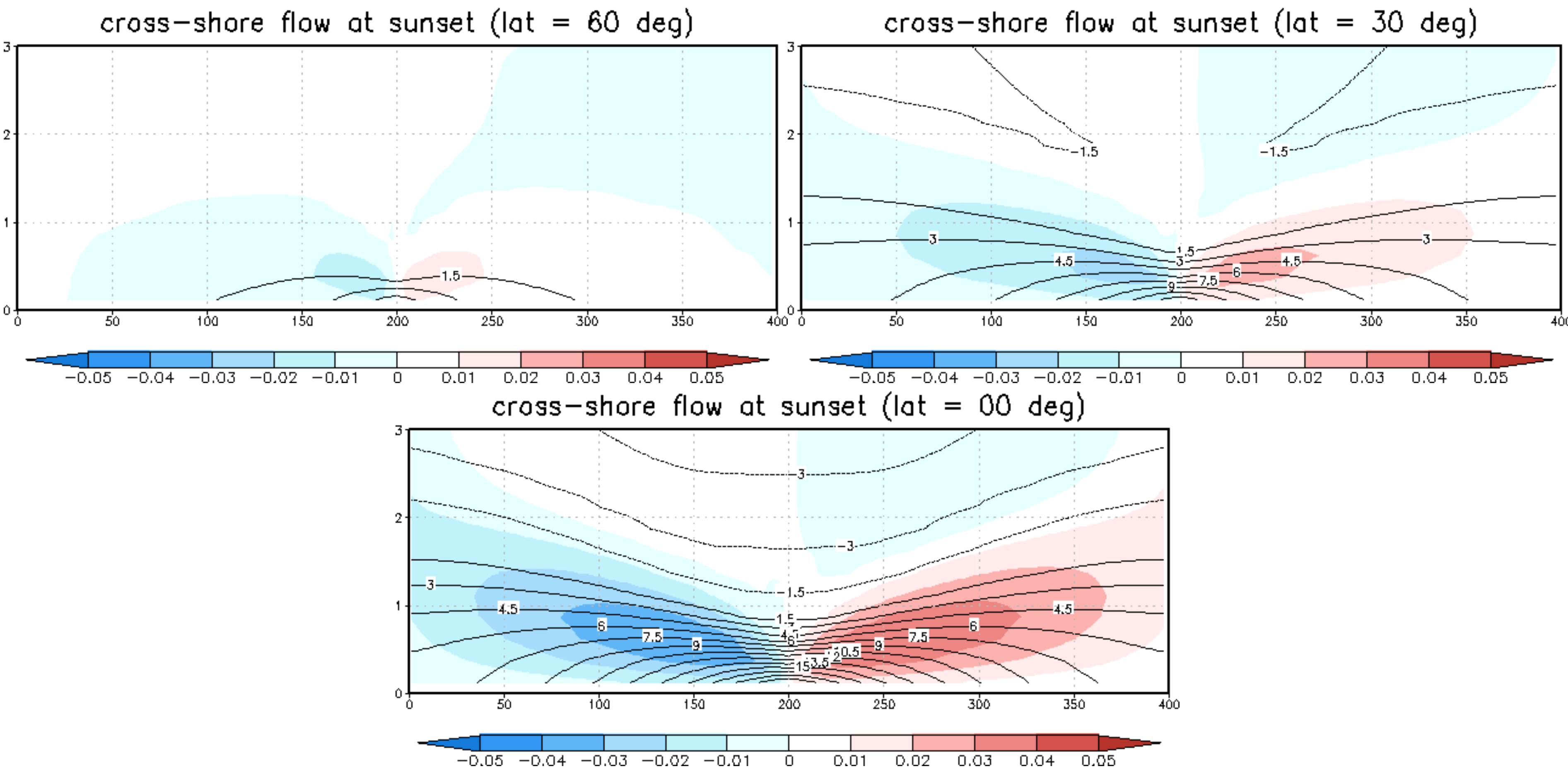




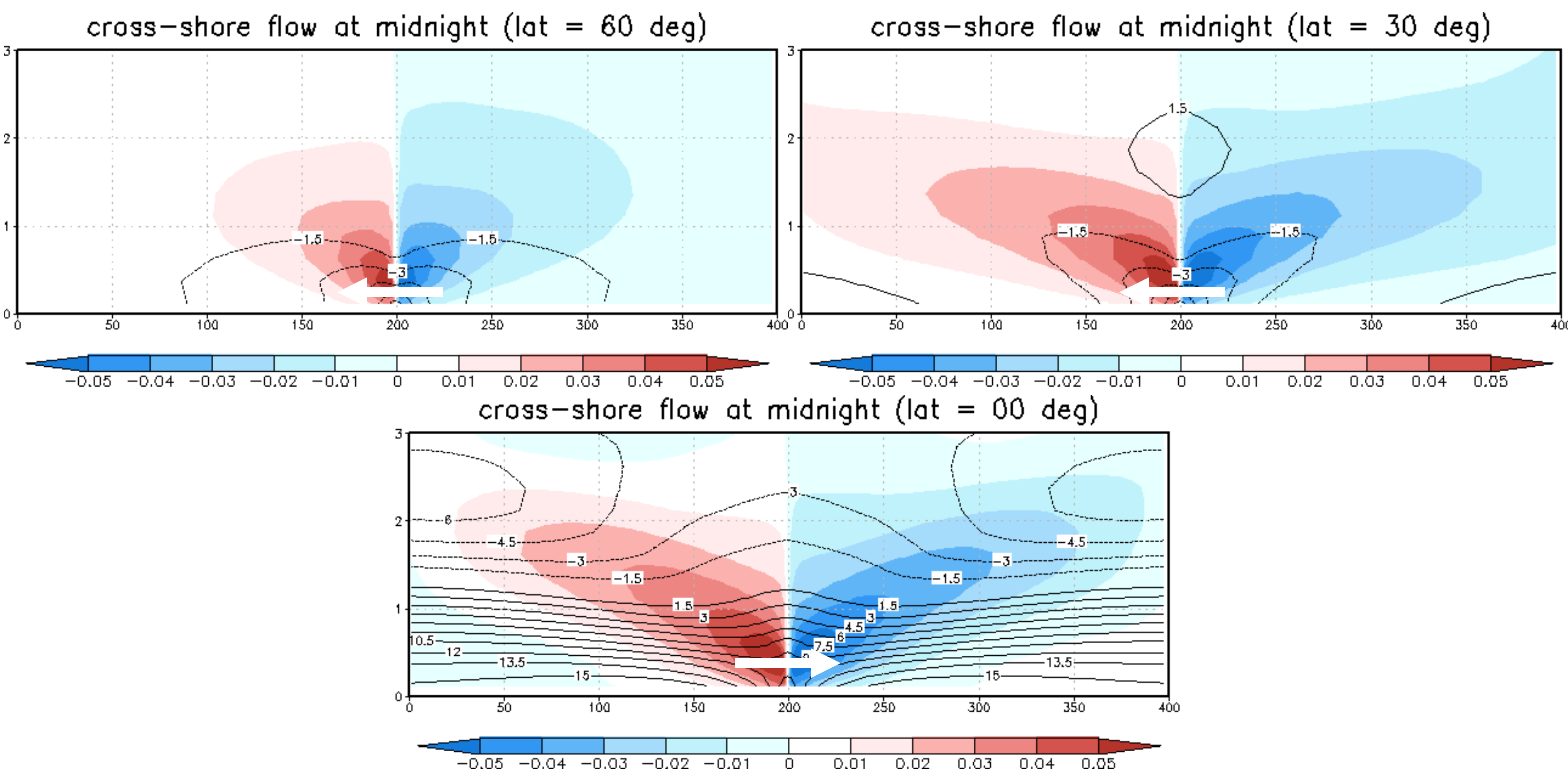
(seabreeze.gs)

set t 72

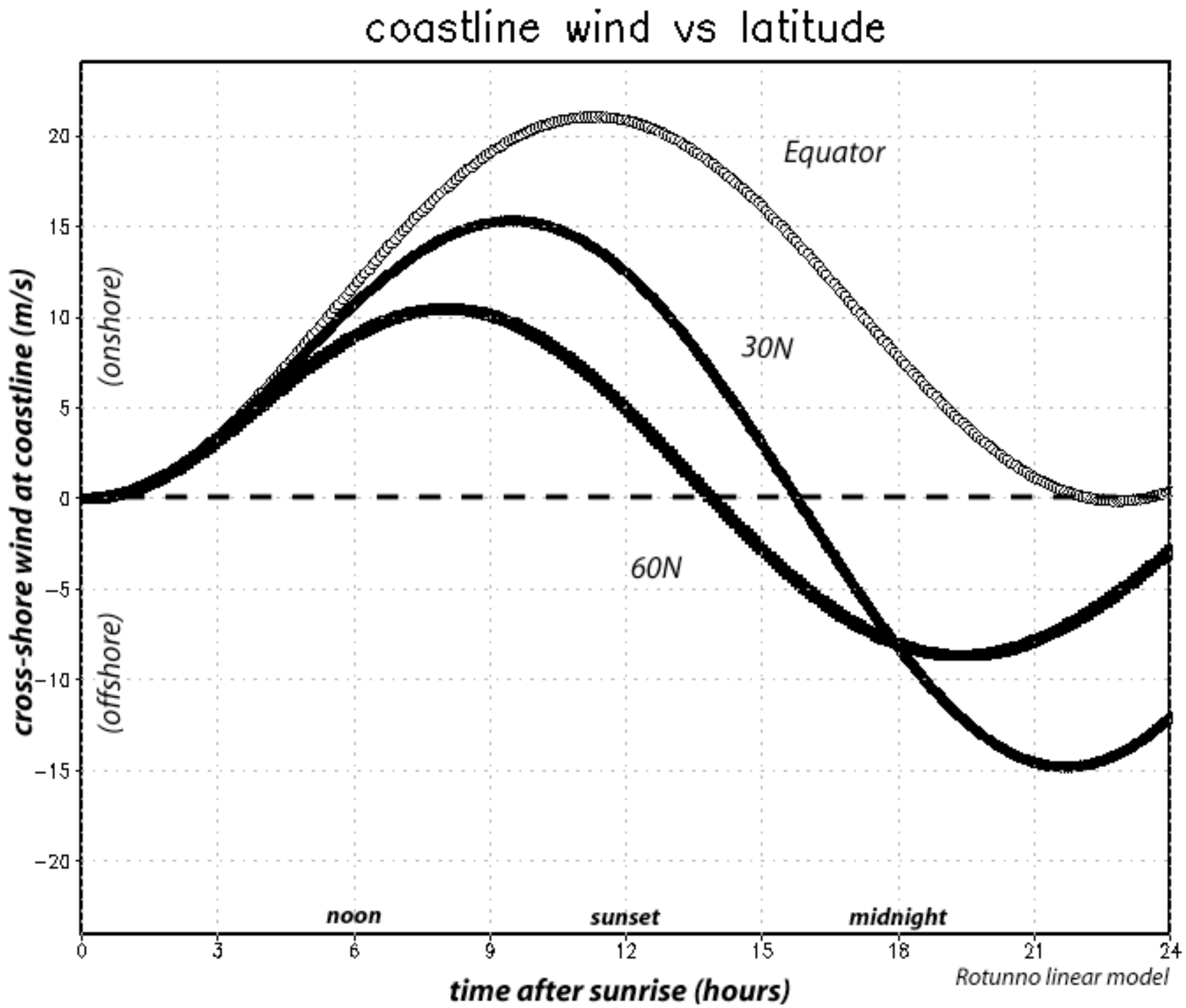
Cross-shore flow and vertical motion at sunset



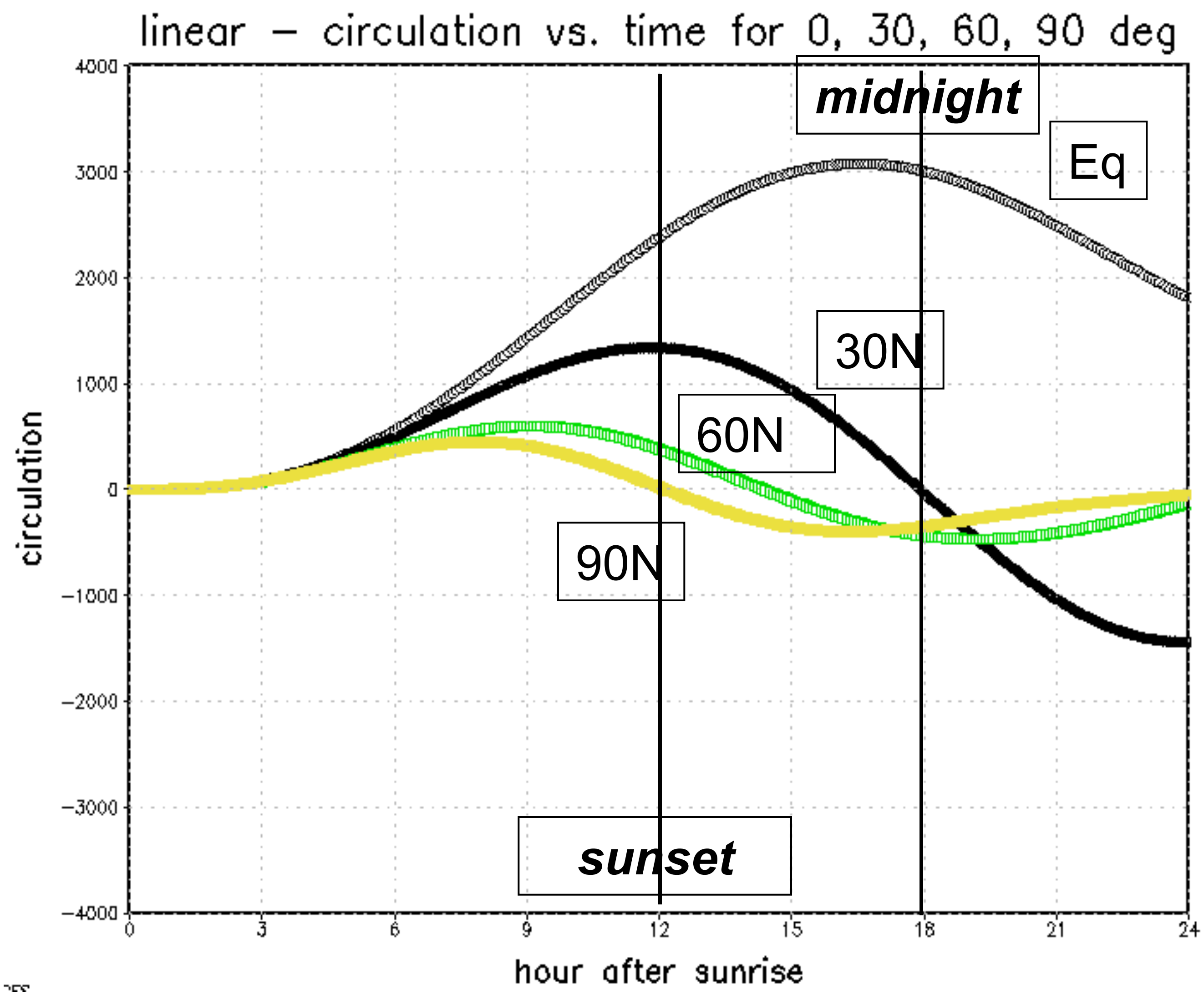
Cross-shore flow and vertical motion at midnight



Cross-shore near-surface wind at coastline (linear model)



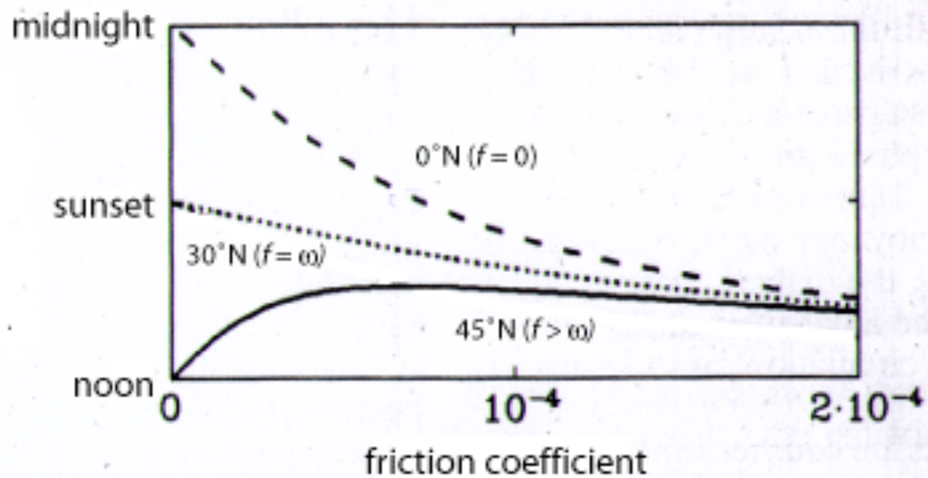
Circulation vs. time



- Circ magnitude decreases w/ latitude (expected)
- 30N circ max at sunset (expected)
- Poleward circ max later than expected (noon)
- Equator circ max earlier than expected (midnite)
- Consistent w/ existence of some friction?

Recall: linear friction effect

Time of circulation maximum



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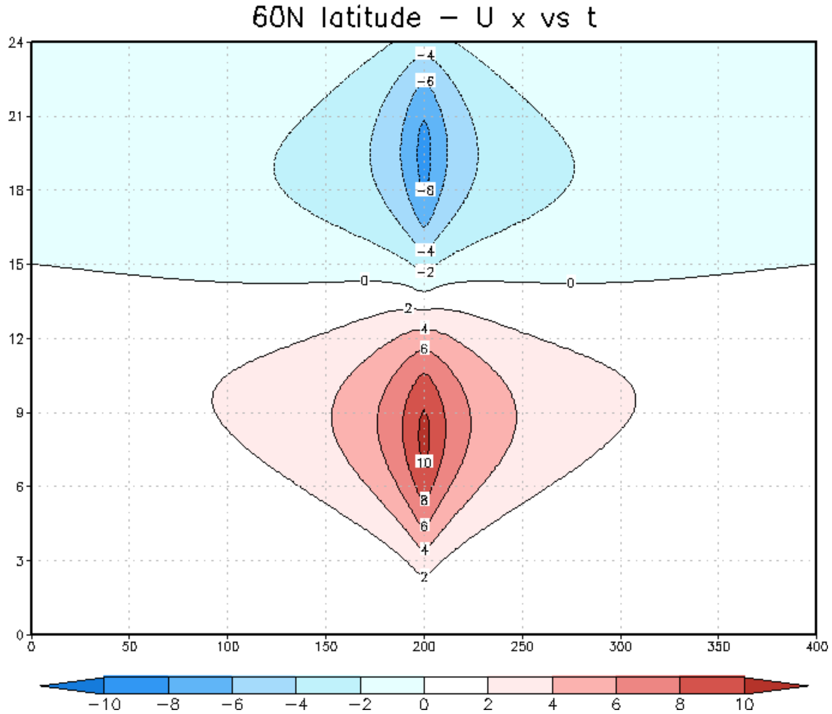
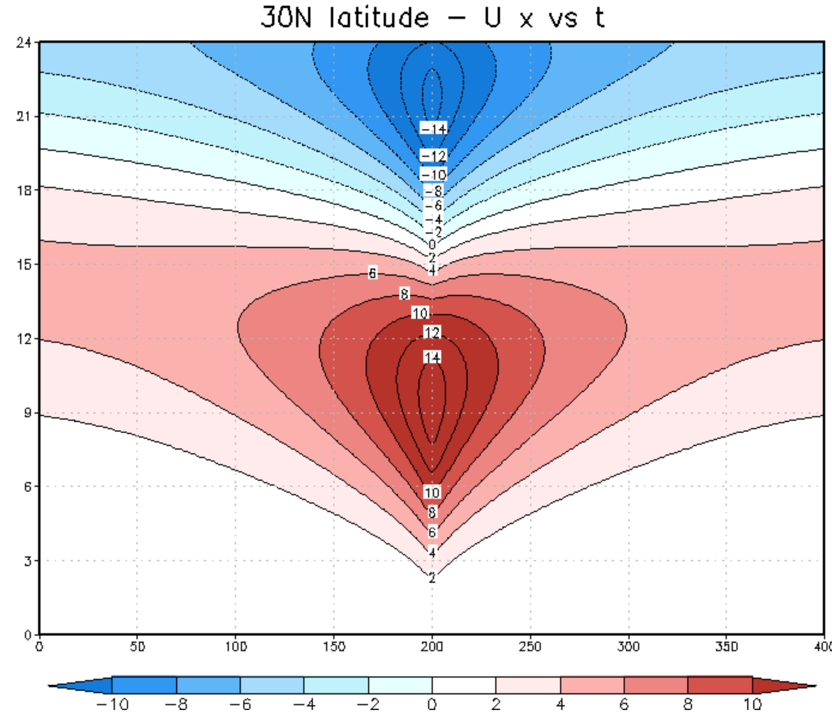
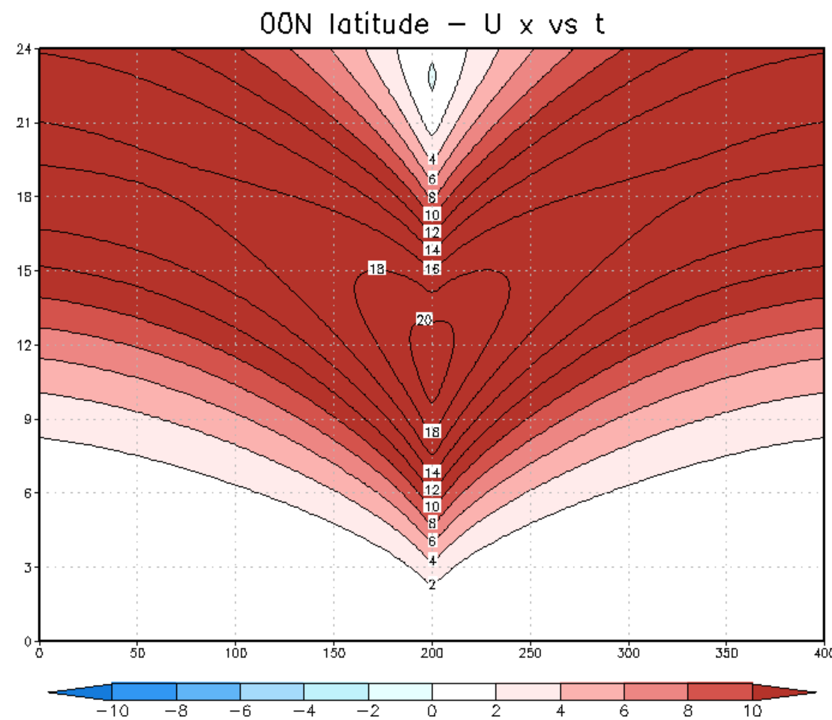




Hovmoller diagrams

(seabreeze_hov.gs)

What is missing?



Questions to ponder

- Why is the onshore flow at 30N strongest in midafternoon? (Heating max was noon)
- Why does onshore max occur earlier at 60N?
- Why does onshore flow magnitude decrease poleward?
- Why does friction shift circulation max time?
- Note no offshore flow at equator at all. Why?
- How does linear assumption affect results?

Further exploration

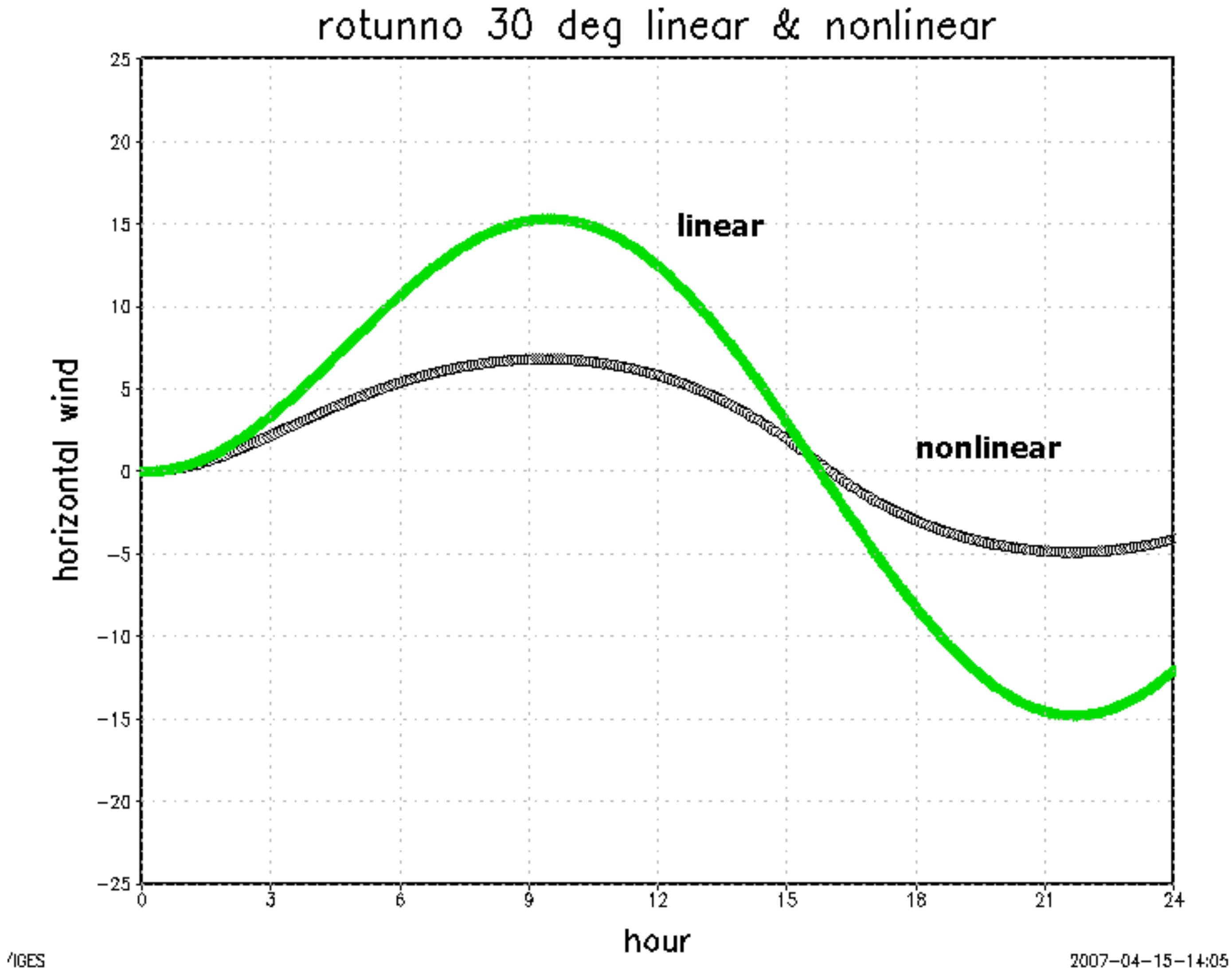
- Run model longer… how long until statistically steady?
- Make model *nonlinear*
- Compare to data
  - Collect surface data along N-S coastline
  - Compare to “data” from more sophisticated model

Tips for nonlinear runs

- Set `sb_linear = 0`
- Increase `dkx`, `dkz` to avoid computational instability
- Example with `dkx = 500`, `dkz = 50` m^2/s (probably excessive)

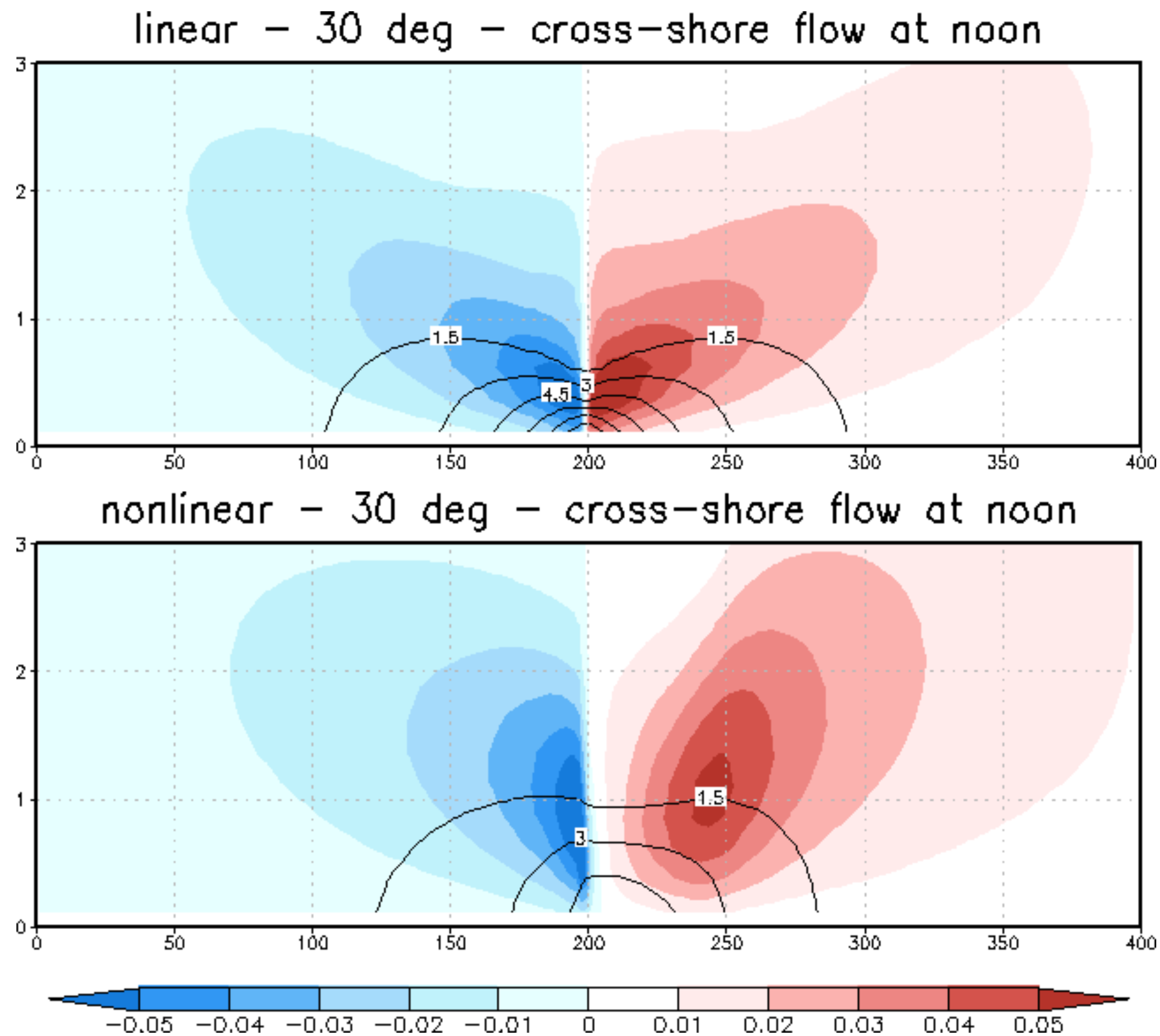
Cross-shore wind at coastline

linear vs. nonlinear



Why are
they different?

Linear vs. nonlinear at noon



Linear vs. nonlinear at sunset