A&OS 101 Entropy production associated with water phase changes

Winter, 2008 – Fovell

Suppose we take 2 grams of ice initially at -10°C and convert it to steam at 100°C without changing the pressure. What is the entropy change associated with this process? We can divide this into four steps, together illustrating every kind of process that can affect water substance. Keeping in mind phase changes occur at constant temperature, the four steps are:

1. Δs_1 due to warming ice from -10°C (T_1) to 0°C (T_2)
2. Δs_2 due to melting ice at 0°C
3. Δs_3 due to warming liquid from 0°C (T_2) to 100°C (T_3)
4. Δs_4 due to vaporizing liquid at 100°C

For step 1, we need to relate the heat absorbed by the ice to temperature change via the specific heat of ice c_i. Ice is not an ideal gas, but since the process is isobaric this relationship still has a simple form

$$\delta Q = mc_i dT$$

Therefore, the entropy change for the first step is

$$\Delta s_1 = \int_{T_1}^{T_2} \frac{\delta Q}{T} = \int_{263K}^{273K} mc_i \frac{dT}{T}.$$

$c_i = 2106$ J kg^{-1} K^{-1} $= 2.106$ J g^{-1} K^{-1}. You should find $\Delta s_1 = 0.157$ J K^{-1}.

Step 2's ice-liquid transition takes place at constant temperature T_2 as well as pressure and involves the latent heat of fusion, $L_f = 3.34 \times 10^5$ J kg^{-1} $= 334$ J g^{-1}:

$$\Delta s_2 = \frac{\delta Q}{T} = \frac{mL_f}{T_2}.$$

This yields $\Delta s_2 = 2.447$ J K^{-1}.

Heating the liquid to the boiling point is also a simple calculation despite not involving an ideal gas. The specific heat of liquid water, c_l, is 4218 J kg^{-1} K^{-1} $= 4.218$ J g^{-1} K^{-1}. As the process is isobaric, $\delta Q = mc_l dT$. So the entropy change is

$$\Delta s_3 = \int_{T_2}^{T_3} \frac{\delta Q}{T} = mc_l \ln \frac{T_3}{T_2},$$

or $\Delta s_3 = 2.632$ J K^{-1}.

Finally, the liquid is converted to steam at 100°C, another isothermal process. This time, the latent heat of vaporization, $L_v = 2.25 \times 10^6$ J kg^{-1} $= 2250$ J g^{-1}, is used. (In other problems, we have used $L_v = 2.5 \times 10^6$ J kg^{-1}. L_v is a function of temperature, and that value was valid for processes occurring at temperatures closer to 0°C.) The entropy change for this step is:

$$\Delta s_4 = \frac{\delta Q}{T} = \frac{mL_v}{T_4},$$

or $\Delta s_4 = 12.064$ J K^{-1}.

Thus, the total entropy production for the process is 17.3 J K^{-1}, of which 70% occurred in the final step.