A&OS 101 Coriolis acceleration due to equatorward displacement

Fall, 2003 – Fovell

Assume Earth is a sphere of radius r. An initially resting parcel starts out at latitude ϕ_0 in the Northern Hemisphere. This parcel has angular momentum (AM) owing to the Earth's spin of ΩR_0^2, where R_0 is the parcel's distance from the spin axis. Call this quantity AM_0.

This parcel is then impelled to travel southward towards target latitude is $\phi_0 + d\phi$ (where $d\phi < 0$). The distance traversed along the surface is simply the arc length $dy = r d\phi$. (Note dy is negative and points south owing to the sign of $d\phi$). This takes the parcel farther from the spin axis by distance dR. See Figure 1.

Yet, the parcel cannot continue traveling straight south. Resting parcels located at the target latitude have more AM. Meanwhile, the moving parcel's AM has to be conserved (we've not identified any sources or sinks). Since the parcel has moved away from the spin axis, its spin velocity must decrease (by $d\Omega < 0$) in compensation so AM conservation is assured.

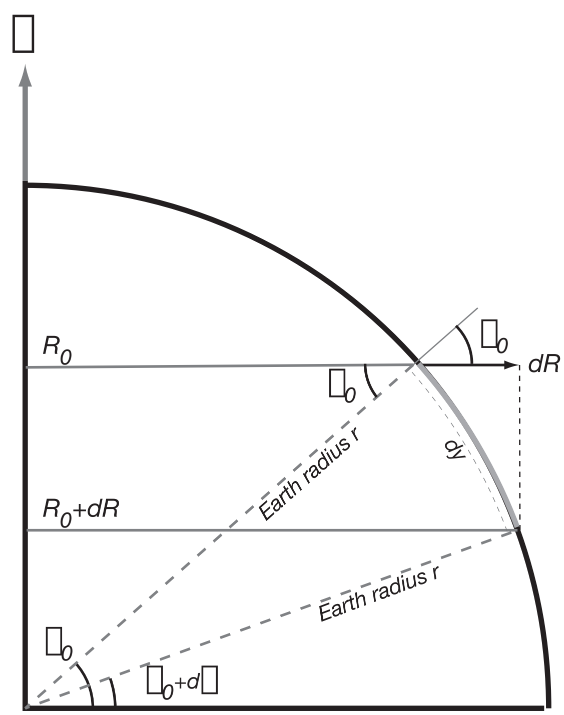

Figure 1: Southward displacement carrying parcel farther from spin axis.

To *decrease* its Earth-relative spin, the parcel must turn *westward*, against the direction of the Earth's spin. Thus, the parcel develops deflection velocity $du < 0$ by latitude $\phi_0 + d\phi$. Recalling that the spin axis distance at that latitude is $R_0 + dR$, we note du is simply the tangential velocity associated with angular velocity $d\Omega$ there. Thus

$$d\Omega = \frac{du}{R_0 + dR}.$$

Equating the initial and subsequent AMs, we have:

$$\Omega R_0^2 = [\Omega + d\Omega](R_0 + dR_0)^2.$$

Substitute the expression for $d\Omega$ found above, and cancel out all higher order terms as they appear. This yields a simple expression for du in terms of dR, namely

$$du = -2\Omega dR. \tag{1}$$

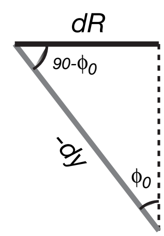

Figure 2: The dR-dy right triangle for small dy.

What is dR? Figure 2 illustrates the geometric situation presuming dy is small enough that the surface path is not curved. The angle between dR and dy is $90°$-ϕ_0, and so the remaining acute angle must also be ϕ_0. Thus we know that[1]

$$dR = -dy \sin\phi_0.$$

Substituting in the arc length definition, this becomes

$$dR = -rd\phi \sin\phi_0.$$

We're interested in the westward acceleration owing to the Coriolis force, so we differentiate (**??**) with respect to time, yielding

$$\begin{aligned} [\frac{du}{dt}]_{Corio} &= -2\Omega\frac{dR}{dt} \\ &= 2\Omega r \sin\phi_0 \frac{d\phi}{dt} \\ &= 2\Omega v \sin\phi_0 \end{aligned}$$

In the last line, we recognized that $r\frac{d\phi}{dt}$ is simply the north-south velocity component v. You can think of v as representing tangential velocity at constant radius r, the radius of the Earth.

[1] Writing the path length as $-dy$ since $dy < 0$ and dR is positive. Alternatively, we could accept ϕ_0 as negative owing to its quadrant and recognize $\sin(-\alpha) = -\sin\alpha$.