Assume Earth is a sphere of radius $r$. An initially resting parcel starts out at latitude $\phi_0$ in the Northern Hemisphere. This parcel has angular momentum (AM) owing to the Earth’s spin of $\Omega R_0^2$, where $R_0$ is the parcel’s distance from the spin axis. Call this quantity $\text{AM}_0$.

This parcel is then impelled to travel southward towards target latitude $\phi_0 + d\phi$ (where $d\phi < 0$). The distance traversed along the surface is simply the arc length $dy = rd\phi$. (Note $dy$ is negative and points south owing to the sign of $d\phi$). This takes the parcel farther from the spin axis by distance $dR$. See Figure 1.

Yet, the parcel cannot continue traveling straight south. Resting parcels located at the target latitude have more AM. Meanwhile, the moving parcel’s AM has to be conserved (we’ve not identified any sources or sinks). Since the parcel has moved away from the spin axis, its spin velocity must decrease (by $d\Omega < 0$) in compensation so AM conservation is assured.

To decrease its Earth-relative spin, the parcel must turn westward, against the direction of the Earth’s spin. Thus, the parcel develops deflection velocity $du < 0$ by latitude $\phi_0 + d\phi$. Recalling that the spin axis distance at that latitude is $R_0 + dR$, we note $du$ is simply the tangential velocity associated with angular velocity $d\Omega$ there. Thus

$$d\Omega = \frac{du}{R_0 + dR}.$$
Equating the initial and subsequent AMs, we have:

\[ \Omega R_0^2 = [\Omega + d\Omega](R_0 + dR_0)^2. \]

Substitute the expression for \( d\Omega \) found above, and cancel out all higher order terms as they appear. This yields a simple expression for \( du \) in terms of \( dR \), namely

\[ du = -2\Omega dR. \quad (1) \]

![Figure 2: The dR-dy right triangle for small dy.](image)

What is \( dR \)? Figure 2 illustrates the geometric situation presuming \( dy \) is small enough that the surface path is not curved. The angle between \( dR \) and \( dy \) is \( 90^\circ - \phi_0 \), and so the remaining acute angle must also be \( \phi_0 \). Thus we know that\(^1\)

\[ dR = -dy \sin \phi_0. \]

Substituting in the arc length definition, this becomes

\[ dR = -rd\phi \sin \phi_0. \]

We’re interested in the westward acceleration owing to the Coriolis force, so we differentiate (??) with respect to time, yielding

\[
\frac{du}{dt}_{\text{Corio}} = -2\Omega \frac{dR}{dt}
= 2\Omega r \sin \phi_0 \frac{d\phi}{dt}
= 2\Omega v \sin \phi_0
\]

In the last line, we recognized that \( r \frac{d\phi}{dt} \) is simply the north-south velocity component \( v \). You can think of \( v \) as representing tangential velocity at constant radius \( r \), the radius of the Earth.

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\(^1\)Writing the path length as \(-dy\) since \( dy < 0 \) and \( dR \) is positive. Alternatively, we could accept \( \phi_0 \) as negative owing to its quadrant and recognize \( \sin(-\alpha) = -\sin \alpha \).