AOS 103 - Physical Oceanography

Calculus Refresher

Disclaimer: *AOS 103 is not necessarily a math-heavy course, but the mathematical concepts presented here will only help you in understanding the physical phenomena discussed in the class. Using these mathematical tools, you should be aiming to apply systems-level thinking to the concepts rather than memorization.*

Why do we need calculus?

Calculus, very simply, is concerned with **change**. It explains how fields (ocean temperature, salinity, velocity, etc.) change along a certain dimension. For most geophysical purposes, we are concerned with changes of these fields along the **space** and **time** dimensions. Representing the evolution of the fields in a mathematical form (as **partial differential equations**) allows us to 1) prove or disprove the mechanisms that drive oceanic flows and 2) explain and/or predict behavior in the ocean.

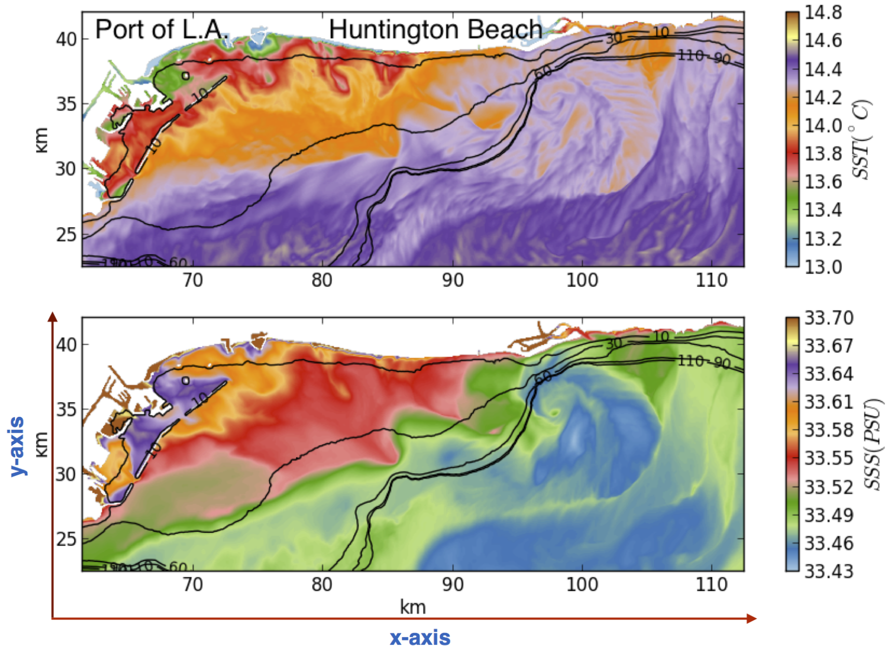

Figure 1: Snapshot of sea surface temperature and salinity off the coast of Southern California from a ROMS (Regional Oceanic Modeling System) simulation. The top panel shows sea surface temperature (SST) and the bottom panel shows sea surface salinity (SSS).

The figure above provides a visual example of how variable (changing) oceanic fields are; this snapshot is an illustration of variation in space (as opposed to time). This figure is a snapshot of an instance in time of the Regional Oceanic Modeling System (developed here at UCLA). The model solves a set of partial differential equations for fluid flow in the ocean and outputs various fields (temperature, salinity, velocities, etc.). The snapshot is a visualization of the surface temperature and salinity outputs at a certain point in time. The domain of the model comprises Santa Monica Bay and Orange County. What is shown is the southern portion of the domain (Long Beach down to Newport Beach). The black contours in each panel denote the depth of the water (in meters). The top panel shows sea surface temperature and the bottom panel shows sea surface salinity. The colors correspond to different values, defined on the right. What you should notice are the **changes in colors** throughout the domain. For example, looking from offshore (bottom of each panel) to the coast (white) we see a general decrease in surface temperature (purple to red/green) and increase in surface salinity (blue/green to red).

Imagine for a moment that the snapshot actually showed an ocean surface that had a constant temperature and salinity (one color covering the ocean surface for each panel). It turns out that this would lead to a pretty tame ocean because horizontal (x, y) differences (gradients) in temperature and salinity are very indicative of ocean currents (and in certain situations the gradients are the driver of a class of ocean current). The main point here being that **if nothing in these snapshots was changing in space, we would have no need for calculus to figure out the physical mechanisms at play**. However, mostly everything that drives ocean flows is changing in both space and time, from the amount of radiation the ocean receives due to the sun, to the temperature differences across the ocean surface, to vertical differences in density in a water column.

While the above snapshots show only 2D variation in temperature and salinity, ocean fields (velocity, density, pressure, nutrient concentration, etc.) are variable in 4 dimensions $(x, y, z, time)$. Furthermore, multiple fields influence each other to bring about the behavior of the system (e.g. temperature gradients can drive velocity and velocity can advect temperature). Thus if we want to describe the full spectrum of ocean physics we would need a system of partial differential equations (PDEs) that take into account all of the pertinent fields and their behavior in all 4 dimensions (but don't worry, we won't be solving 4D PDEs). What we usually do in physical oceanography is simplify our system based on some assumptions, and with those assumptions in mind describe (i.e write an equation) behavior in one, two, or three dimensions.

Calculus Tools and Concepts

The following topics are the calculus tools and concepts that should give you the appropriate mathematical foundation for understanding the physical phenomena presented in this course:

- Differentiation and integration in one variable
- Partial derivatives
- Area and volume integrals (control volume)
- Gradient, Divergence, Curl
- Divergence theorem

The Math 3B/31B pre-req for the course should mean that everyone is familiar with these concepts, however it is possible that some may have forgotten or become unfamiliar with some of these items. Control volumes are probably something you may have never seen before but they are just a way to think about volume integrals in the ocean. The divergence theorem may be new *as well.*

Differentiation and Integration in One Variable

Differentiation

Suppose that we can represent the temperature field at the surface of the ocean as a function and for simplicity we assume that the temperature is only variable in the x direction (you can geographically think of the x-axis as the bottom axis of the ROMS snapshots):

$$T(x) = 3x^4 + e^{-2x} + \sin(3x + 1) \tag{1}$$

This a completely made up function, but it is being presented to remind everyone of the rules for taking derivatives.

The derivative of $T(x)$ with respect to x is as follows:

$$\frac{d}{dx}T(x) = 12x^3 - 2e^{-2x} + 3\cos(3x + 1) \tag{2}$$

If you are unsure of how that derivative was calculated, revisit power rule (first term), exponent rule(second term), and trigonometric derivative rules (third term) in your calculus textbook or Google.

The meaning of the derivative is quite simple: **it just describes the rate of change of the temperature along the x-direction.** Thinking back to the sea surface temperature plot, imagine calculating the change in temperature (color) starting at the 25km hashmark on the vertical axis and moving from left to right along the horizontal axis. The difference in temperature from each point to the next is the derivative along that axis (at that point along the vertical axis). It is very common for temperature changes (gradients) in space to drive ocean currents.

Integration

1D integration can be simply described as taking a sum over a certain range. Everyone is probably familiar with the description of **1D integration as the calculation of the area under a curve**; let's apply that to an oceanographic variable and give a physical interpretation. Say that we wanted to calculate the amount of biomass across a transect in an ocean basin; imagine a line (transect) running across the Pacific ocean from Asia to North America at a depth of 100m below the ocean surface. If we have a biomass profile ($B(x)$) along that transect given by the (again made up) function:

$$B(x) = 2x^5 + \cos(x) \tag{3}$$

We can calculate the total amount of biomass across a certain range [a,b] along the transect by integration:

$$\int_a^b B(x)dx = \int_a^b 2x^5 + \cos(x)dx = \frac{x^6}{3} + \sin(x)\Big|_a^b = B_{tot} = \text{total biomass} \tag{4}$$

Note that this calculation is only telling us the biomass at a certain depth. If we wanted to know the biomass along the line covering all depths (not just 100m), we would redo our calculation using an area integral, which will be discussed later.

Partial Derivatives

Partial derivatives are used for **fields that are multidimensional** (again, think back to the temperature and salinity from the ROMS snapshot above). We can take derivatives of multi-dimensional functions with respect to each dimension. Just like before, let us imagine that the temperature field is described by a made up function, but this time it is variable in both x and y:

$$T(x, y) = T = (\cos(2x + 5))^2 + \sin(3y + 2)$$

The partial derivatives that result from differentiation in x and y are as follows:

$$\frac{\partial T}{\partial x} = -4\cos(2x + 5)\sin(2x + 5) \quad (6)$$

$$\frac{\partial T}{\partial y} = 3\cos(3y + 2) \quad (7)$$

(6) and (7) are derived by regular 1D differentiation with x treated as a variable and y treated as a constant for (6) and vice versa for (7). The meaning of each is simply **the temperature change along each respective axis**. We will see that temperature is related to pressure and that the spatial structure of horizontal pressure gradients (spatial structure of pressure partial derivatives in x and y) combined with the rotation of the Earth (Coriolis force) and a few other assumptions gives rise to a very important class of 2D (x, y) currents known as geostrophic currents.

Area Integrals and Control Volumes

Figure 2 is a from ROMS simulation (again, done here at UCLA) of the Gulf Stream, a western boundary current in the Atlantic Ocean visualized by the warm (red) water transported north.

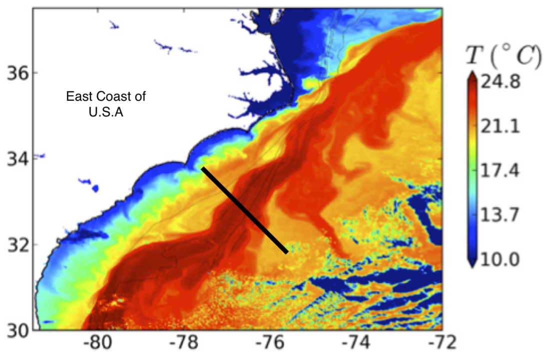

Figure 2: Sea surface temperature (colors) from a ROMS simulation of the Gulf Stream. The hashmarks on the bottom axis denote longitude and the hashmarks on the vertical axis denote latitude. The black line denotes the horizontal width of the area where we wish to calculate transport through.

Let's suppose that we wanted to know how much of some ocean property (water, mass, salt, nutrients, oil!) is transported across some part of the Gulf Stream, a very fast and globally important current. And let's say that we want to know the transport that happens across some horizontal distance (denoted by the black line in the figure) and down to a depth of 1km. **We can do this calculation using an area integral.**

Area Integrals

We can set up and visualize our transport calculation in Figure 3. We are essentially dropping a rectangular 'frame' (what we call a 'cross-section') into the water from above the ocean and letting it sit in the (x, z) plane (i.e one axis in the horizontal, one in the vertical going down to depth).

Transport is achieved by advection (bulk movement due to fluid flow). Thus, all we need to calculate transport are the following:

- **Area of the domain** to calculate transport through (width times height of our cross-section) $= dx \times dz$
- **Property transported** (let's keep it simple and let it be water; but we could just as easily calculate transport of heat, salt, nutrients, etc.)
- **Velocity normal to the face of the cross-section** (i.e flowing through the cross-section) $= v$

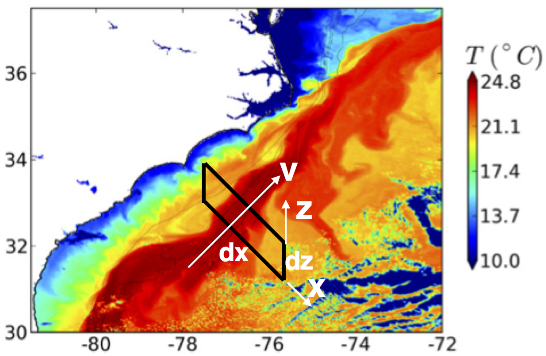

Figure 3: Illustration of set up for calculating transport. The 2D domain has a vertical height dz and horizontal width dx, with a velocity v transporting (advecting) material through it.

Since we have decided to calculate the amount of water transported, we are calculating the **volume transport**, which will give us units of (m^3/s). Volume transport has the following, very simple, equation:

$$volume\ transport = velocity\ \times\ area\ ===> \frac{m}{s} \times m^2 = \frac{m^3}{s} \tag{8}$$

Now, in reality, the **velocity in our cross-section can be variable** with depth (z) and variable along the width of our domain (x). In other words, we can imagine that the velocity in one location in our cross-section is different than in another location in our cross-section. The way we take this into account in our transport calculation is to separately calculate the transport for different sub-domains within the cross-section, each with its own velocity and area:

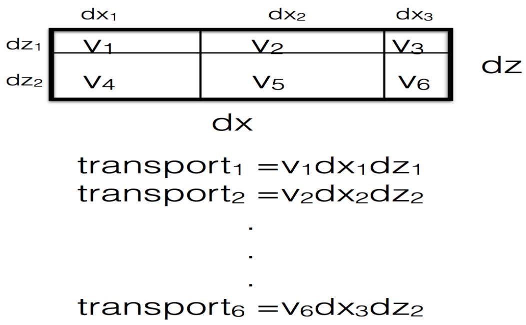

Figure 4: Illustration of our cross-section with different velocities flowing through sub-areas and the transport calculation for each. Each sub-area has its own velocity and area. The velocities are flowing into or out of the page.

We then calculate our total transport by summing all of our sub-transports:

$$total\ transport = transport_1 + transport_2 + transport_3 + transport_4 + transport_5 + transport_6 \tag{9}$$

We can expand this idea further by assuming that we have N subdomains, each with its own velocity and area and we can then calculate the total transport by the following summation:

$$total\ volume\ transport = \sum_{i=1}^{N} v_i dx_i dz_i \tag{10}$$

Where $\sum_{i=1}^{N}$ denotes a summation over the N subdomains. If the velocity throughout the cross-section is a continuous function in (x, z) (e.g. $v(x, z) = x^2 + e^{-z}$) we can formally write this as an area integral (after all, an integral is just another way to take a sum):

$$total\ volume\ transport = \int\int v(x, z)dxdz \tag{11}$$

The double integral is used because we are summing over 2 dimensions: x and z. We can easily extend this formulation for mass transport by adding in a density profile, $\rho(x, z)$:

$$total\ mass\ transport = \sum_{i=1}^{N} v_i \rho_i dx_i dz_i = \int\int v(x, z)\rho(x, z)dxdz \tag{12}$$

Which will give us units of (kg/s). *Density has units of* $\frac{kg}{m^3}$. We can do analogous formulations for heat, nutrient concentration, salt and other ocean properties (all we would do is put in the appropriate factor in the integral and get different transport units).

Control Volume

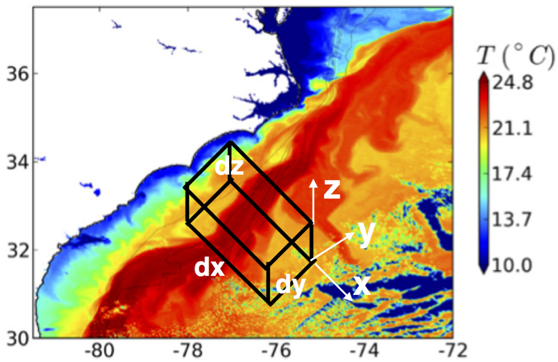

Figure 5: Illustration of set up for control volume.

We can add complexity to our cross-section setup in Figure 3 by creating a 3D box that has a length dy (shown in Figure 5). In fluid mechanics, this imaginary box is often called a **control volume** and it can be rigid or deformable depending on the assumptions you make for your system. With our area integral method for calculating transport, we could get the transport through each face of the box. However, this would mean that we will have to take into account velocities flowing normal to (through each) face (not just the $v(x, z)$ from before).

Once we have the transport though each face, we could add them up. If that sum is not 0, that would mean the water flowing in does not equal the water flowing out of the control volume. It actually turns out that, we frequently assume that the sum must be 0. We say that the fluid in the control volume (and more importantly in the ocean) is **conserved** (in other words, there are no holes in the box). To fully understand this concept we need to talk about 3D fluid velocities and their properties...

Gradient, Divergence, Curl

Before we begin our discussion of 3D fluid flows, we need to establish the difference between **scalars** and **vectors**.

- **Scalars** are fully described by a numerical value **(magnitude)**
  - Ocean scalars include temperature, salinity, density, nutrient concentration
- **Vectors** are fully described by BOTH a **magnitude and direction**
  - Ocean vectors include velocity and gradients of the above scalar fields

We write the ocean's 3D velocity ($\mathbf{U}$) as a vector:

$$\mathbf{U} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{13}$$

With the following coordinate conventions in space:

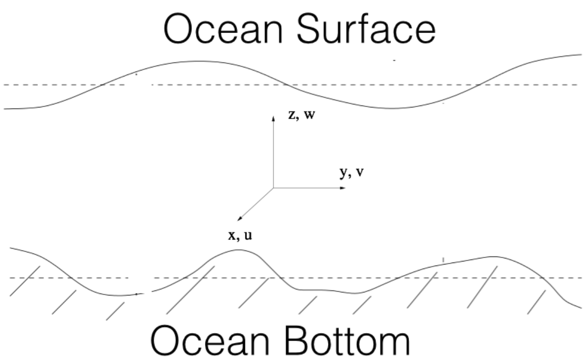

Figure 6: Illustration of the coordinate system corresponding to the ocean 3D velocity. The horizontal velocities (u,v) flow through the page and to the right/left on the page, respectively. Vertical velocity (w) flows up or down on the page.

Gradient

As has been stated, spatial differences in temperature can drive ocean currents. We can calculate (and subsequently visualize) these differences by calculating the gradient of a temperature field. If our temperature field is 3D in space (as it is in the ocean), our resulting gradient will have 3 vector components, each corresponding to the rate of change of temperature along a dimension (x, y, z). If our temperature is a 3D function, the gradient is calculated by simply taking the partial derivative along each dimension and representing the partial derivatives in vector form:

$$T(x, y, z) = 2x + \sin(y) + e^{-z} \tag{14}$$

We use ∇ (or grad) to denote the gradient operator:

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \tag{15}$$

$$\mathrm{gradT} = \nabla\mathrm{T} = \frac{\partial \mathrm{T}}{\partial \mathrm{x}}\mathbf{i} + \frac{\partial \mathrm{T}}{\partial \mathrm{y}}\mathbf{j} + \frac{\partial \mathrm{T}}{\partial \mathrm{z}}\mathbf{k} = 2\mathbf{i} + \cos(\mathrm{y})\mathbf{j} - \mathrm{e}^{-\mathrm{z}}\mathbf{k} \tag{16}$$

You can think about what ∇T means geographically as follows: ∇T **will be a vector that is aligned with the direction along which T changes most rapidly**. You can see an example of this in Figure 7 (below), in which ∇T is aligned across the regions where temperature (colors) change most rapidly (the snapshot below shows an example of a 'temperature front'). The convention used is to point the vector in the direction in which temperature is rising most quickly (at that point). So, for the point denoted by the 'x', the most rapid temperature cold to warm (orange to purple) temperature change is in the onshore to offshore direction.

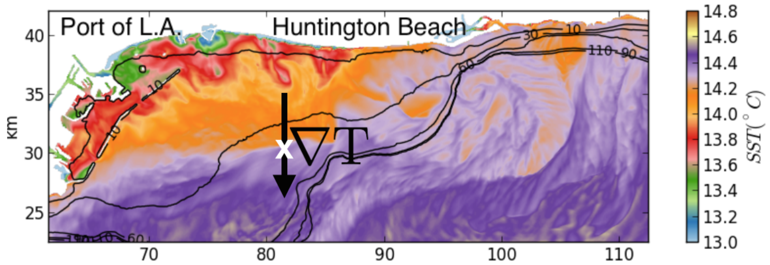

Figure 7: Illustration of temperature gradient vector at a point X. The gradient vector points in the direction in which temperature changes most quickly

When we want **the spatial variation of a scalar we use the gradient** operator. If we want to look at the full spatial variation of a vector field, we need to use the gradient operator combined with vector operations ($\nabla\cdot$ and $\nabla\times$). *This is not to say that we cannot take the gradient of, say, the x-component of velocity (u), which by itself can be thought of as a scalar...but curl* ($\nabla\times$) *and divergence* ($\nabla\cdot$) *of vector fields are typically more meaningful.*

Divergence

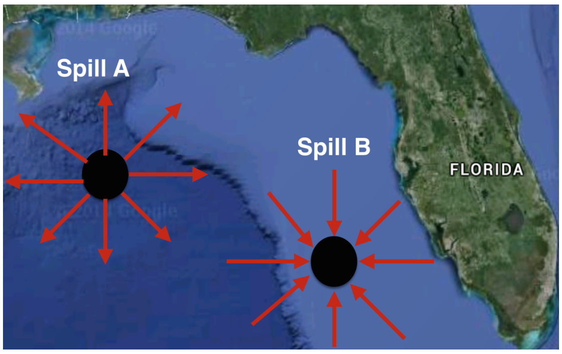

Figure 8: Oil patches with separate velocity fields surrounding each, denoted by the red arrows.

Imagine that there is are 2 oil spills at separate locations in the surface of the ocean, and for simplicity, imagine that this oil on the surface of the ocean (oil is less dense than ocean water, therefore it will float at the surface) has an initial shape of a perfect circle and it is not diffusing through the water. That is, the only way the oil will move is through advection by the ocean's velocity field. Let's further simplify things and say that the oil's viscosity has no effect on the ocean's flow. Also, because oil is substantially less dense than ocean water, we can safely assume that the only velocity advecting the oil is the horizontal (x, y) velocity (vertical velocity will not transport the oil). This scenario is crudely depicted in Figure 8. The final assumption we will make (again for the sake of the example) is that the velocity fields surrounding each patch of oil (red arrows) are independent of each other; that is, the arrows on Spill A do not change the arrows on Spill B, and vice versa.

We can do a simple thought experiment to predict what will happen to each patch of oil given each velocity field. Again, remember that the only way the oil moves is through advection by velocity. For Spill A, given the outward pointing arrows, we can easily imagine that the oil will spread symmetrically outward from its initial point source (circle). Conversely for Spill B, we can imagine that the oil will remain in place (and the circle will possibly shrink in diameter) due to the velocity pushing in on the patch. We can say that the velocity field surrounding Spill A is **divergent** and the velocity field surrounding Spill B is **convergent**.

All we are doing when we calculate the divergence of a vector field (in most cases fluid velocity), is mathematically stating whether more fluid is flowing away from that point than towards it.

The divergence operator ($\nabla\cdot$ or div) can be formally defined as follows:

Given a vector field:

$$\mathbf{U} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{17}$$

The **divergence is defined as the dot product of the gradient operator with the vector field**:

$$\nabla \cdot \mathbf{U} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle u, v, w \rangle = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{18}$$

Let's say that we have a horizontal 2D velocity field defined by:

$$\mathbf{U}_{2D} = x^2 y\mathbf{i} + y^2 x\mathbf{j} \tag{19}$$

The divergence is calculated as follows:

$$\mathrm{div}\mathbf{U_{2D}} = \nabla \cdot \mathbf{U}_{\mathrm{2D}} = \frac{\partial}{\partial \mathrm{x}}(\mathrm{x}^2\mathrm{y}) + \frac{\partial}{\partial \mathrm{y}}(\mathrm{y}^2\mathrm{x}) = 2\mathrm{xy} + 2\mathrm{xy} = 4\mathrm{xy} \tag{20}$$

Note that the **divergence is a scalar**, not a vector.

Now let's add in a vertical velocity and take the divergence of a fully 3D fluid velocity field:

$$\mathbf{U}_{3D} = x^2 y\mathbf{i} + y^2 x\mathbf{j} - 4xyz\mathbf{k} \tag{21}$$

$$\nabla \cdot \mathbf{U}_{3D} = \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(y^2 x) + \frac{\partial}{\partial z}(4xyz) = 2xy + 2xy - 4xy = 0 \tag{22}$$

The fact that the divergence of the 3D fluid velocity equals 0 is not by mistake, but is done to bring up an important assumption made in physical oceanography. A very general and important assumption in physical oceanography is that we assume **the fluid in the ocean is incompressible**. Mathematically this assumption results in the divergence of the 3D velocity summing to zero:

$$\nabla \cdot \mathbf{U} = 0 \tag{23}$$

Physically, the assumption states that **water parcels preserve their infinitesimal volume and mass as they follow the flow**. A slightly less abstract way to state this is **that there can be no net transport of water towards or away from any given point in space, as this would imply compression or expansion of water at that point**. The incompressible assumption stems from the relatively small magnitude of variations of density in the ocean. We'll elaborate on this and hopefully make it less abstract and more intuitive when we talk about the physical manifestation of the divergence theorem below.

Curl

In the ocean, currents often exhibit rotational tendencies. This is visualized in the figure below, which shows a vortex off the coast of Malibu. The red colors represent a quantity called **relative vorticity**. Vorticity is a very important quantity in physical oceanography as it can be used to easily visualize flow fields and it is used a great deal in the formulation of theories for fluid flow in the ocean. Vorticity is calculated by taking a component of the curl of the velocity field.

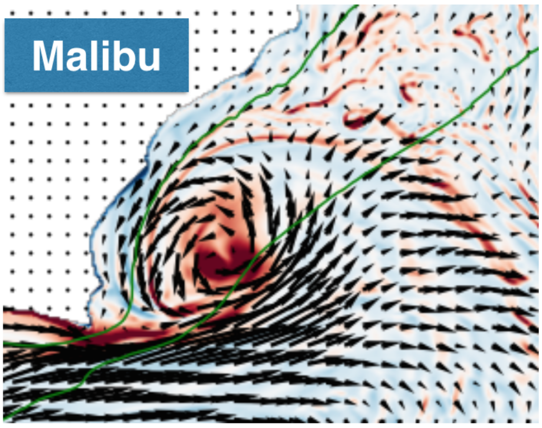

Figure 9: Surface horizontal velocity (arrows) and surface relative vorticity (colors) off the coast of Malibu. The velocity field is from the output of ROMS simulation.

Simply put, **the curl is a calculation of the amount of rotation in a flow(vector) field**. Unlike the divergence, the result of a the curl operator is a vector that describes rotation in each plane that there is fluid flow. The curl operator ($\nabla\times$ or curl) for a 3D flow field is defined as follows:

Given the same 3D velocity field from before:

$$\mathbf{U} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{24}$$

The **curl is the cross product of the gradient operator with the vector field**:

$$\text{curl}\mathbf{U} = \nabla \times \mathbf{U} = \left\langle \frac{\partial}{\partial \mathrm{x}}, \frac{\partial}{\partial \mathrm{y}}, \frac{\partial}{\partial \mathrm{z}} \right\rangle \times \langle \mathrm{u}, \mathrm{v}, \mathrm{w} \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \tag{25}$$

$$\nabla \times \mathbf{U} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k} \tag{26}$$

Again **note that $\nabla \times \mathbf{U}$ is a vector**, not a scalar.

The surface vorticity shown in Figure 8 is just the last component $\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k}$ of the curl operator (it is also normalized by the rotation of the Earth, but it is the curl operation that gives the spatial structure of the colors in the figure). Vorticity is an indicator of rotation in the (x, y) plane *about (around)* the vertical (z) axis; it is also a great indicator of **velocity shear**. Shear in velocity fields can be caused by velocity vectors opposing each other, or, more generally, gradients in the strength and/or direction of the velocity itself. Two examples of shear flows are shown in Figure 10.

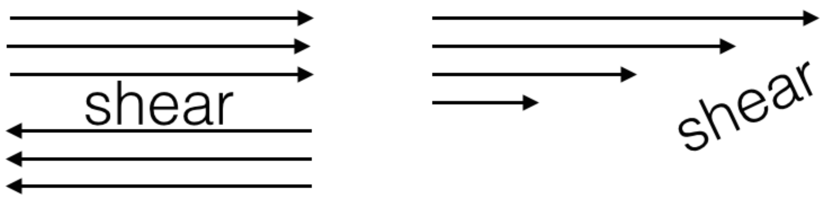

Figure 10: Flow profiles that can lead to velocity shear. The flow profile on the left has equal velocity magnitude with opposing directions. The flow profile on the right has velocity with uniform direction, but a top to bottom drop off in magnitude.

Divergence Theorem

Let's revisit our control volume in Figure 5. We know that a 3D velocity field leads to flow through each face of the box. Depending on the direction of the flow relative to the face, **fluid is flowing in (sources) or out of (sinks) the box** at each point on each face. We also know that the **divergence of the velocity vector at each point gives us the strength of the sources and sinks of fluid at each point**.

We could figure out the total amount of fluid flowing in our out of the box by **summing (integrating) the divergence at every point in the interior of the box**. This would just result in a **volume integral** of the divergence of the 3D fluid velocity:

$$\text{volume total of fluid sources and sinks} = \iiint_V (\nabla \cdot \mathbf{U})\ dV \tag{27}$$

Keeping in mind that our control volume is a **closed region**, we can actually say that the volume total of fluid sources and sinks in our control volume is **equivalent to the net flow of fluid across each face (boundary)**. This statement is the divergence theorem and is mathematically expressed by **equating our volume integral** (27) to the **surface integral of flow normal to each boundary**:

$$\iiint_V (\nabla \cdot \mathbf{U})\ dV = \oiint_S (\mathbf{U} \cdot \hat{\mathbf{n}})\ dS \tag{28}$$

Where $\oiint_S$ indicates a surface integral over the boundary of the volume V. The circle enclosing the 2 integral symbols is to denote that the volume V is a closed surface.

To explicitly write an equation in words to get the point across again, we can say that:

sum of fluid sources and sinks in control volume = sum of flow across control volume boundaries

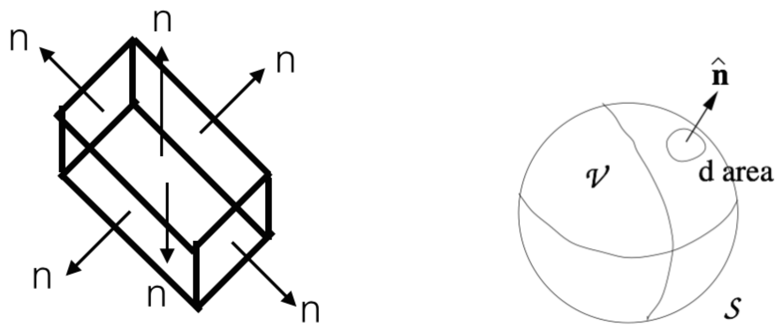

Figure 11: Left: Control volume from Figure 5 with surface normals (n). Right: Illustration of a water parcel with infinitesimal volume (V), surface (S) and normal (n). The divergence theorem is applicable to both shapes.

<u>Physical Implications</u>

The divergence theorem is applicable to a macro-scale control volume box like the one shown on the Gulf Stream in Figure 5 or a 'water parcel of infinitesimal volume' (both shown in Figure 11), and all closed shapes in between. Using the **incompressible assumption** combined with the **divergence theorem** we can perform a thought experiment with our control volume to explain behavior commonly observed in the ocean (sea surface elevation and depression).

First, assume that the top of the box is flush with the ocean surface and that the water at surface on top of the box is initially flat. Then, imagine that the fluid flow through each face (sides, and bottom, no flow through the top) is INTO the box. In other words, there is a **net convergence** of water. The divergence theorem tells us the relation between flow in through each face and the sources and sinks of fluid within the box. The incompressible assumption tells us that the volume of water in the box must remain the same (no holes in the box). Mathematically the combination of the divergence theorem and incompressible assumption is expressed as:

$$\iiint_V (\nabla \cdot \mathbf{U})\ dV = \oiint_S (\mathbf{U} \cdot \hat{\mathbf{n}})\ dS = 0 \tag{29}$$

Since $\nabla \cdot \mathbf{U} = 0$ due to the incompressible assumption. You can interpret this as a statement of **no net normal flow through the box's boundary,** in other words **IN = OUT**. With this in mind, we can expect that if there is a net convergence of fluid into the box, there will be a mound of water above the initially flat surface. Conversely, if we have the opposite situation with a net divergence of water, the surface will depress. Sea surface elevations and depressions are are indicative (and sometimes the main driver) of classes of ocean currents.